Calculating and Presentating 3D-objects with the TI-89/92

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With the TI-89/92 we have a wonderful tool to perform many calculations which have been preserved for the PC only for a long time. As there is also a powerful programming and graphing tool available it is a challenge to combine the calculating and graphing abilities using a program (package).
(*You are invited to ask for the package*)

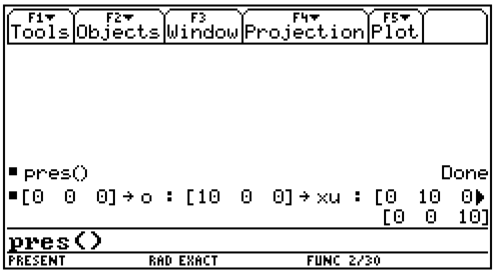
We will see that it is very easy to not only calculate 3D-problems from intersections of lines and planes through differential geometry in space but also to illustrate the results by attractivegraphic representations.

We will start with some introductory exercises to get accustomed with my program `pres()`.

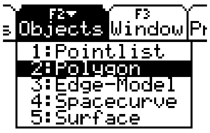
Settings: Set your device in Angle mode RADIAN and Exact/Approx AUTO, Switch OFF Axes and Grid in the GRAPH-Window and take care that plots declared in the Y= - Editor might get lost.

1 Introduction

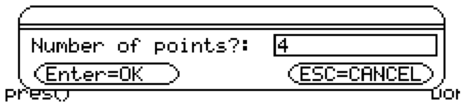
(1) **Represent the system of coordinates** given by the origin O and three points on the axes with a distance of 10 units from O.



Start `pres()` and you will face a newly created menu bar. The Home Screen is not cleared by the program, because sometimes you might be glad to refer to your calculations done earlier on the Home Screen.
As you can see on the screen shot, I entered the four points which define the axes. This way is recommended for later working with the points using variables.
For representation purpose only you could enter the data points from within the program using `F2 Objects`.

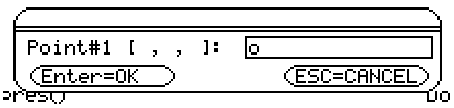


We have to choose `F2 Objects` to create an object, built up by the four points. Now we have to decide how to build the axes: as a 3D-polygon (a sort of space "curve" or as a list of edges. Let´s choose `2:Polygon` and see what will happen.

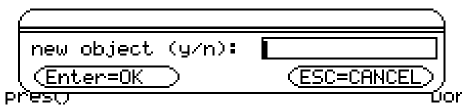


You are asked for an object name: take `axes` (or any other one of your choice). Then you are asked for the number of points which are necessary to produce `axes`. Answer with 4.

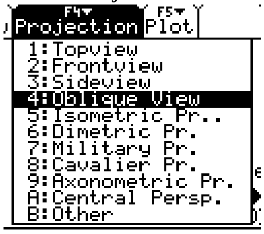
You are asked to enter the four points. You can enter either the variable names of the points (having them defined earlier in the Home Screen) or the coordinates as a vector (between brackets):



Enter also point#3 and point#4 (yu and zu). Note the order of entering the points, because later you have to address the points by their numbers. Unfortunately it is not possible to form a list of vectors on the TI, so I had to overcome this obstacle with a trick. At the other hand this is a good opportunity to train the students´ 3D-imagination. Find a closed way passing all given points to describe the axes: [From point#1 to point#2, back to point#1 and to point#4,.....]. So we enter the vector [1,2,1,4,1,3].

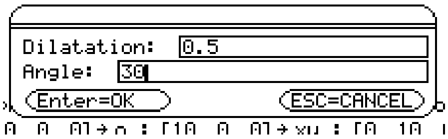


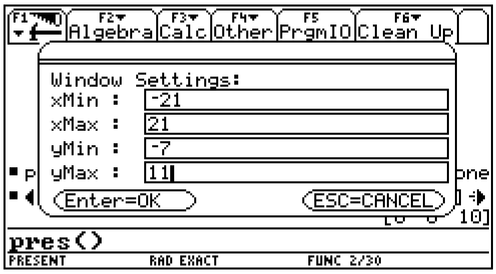
As we don´t want to create a new object at the moment we answer with "`n`". The most important step is done. We have to choose a projection, adjust the WINDOW-settings and then `Plot` the object `axes`. Press `F4:Projections` and plenty of projections are offered:



For the first example lets choose `Oblique view`. We know that oblique view representation needs two parameters: dilatation of one axis and the angle between the projection of this axis and the horizontal direction.

We take dilatation 0.5 and angle 30°.

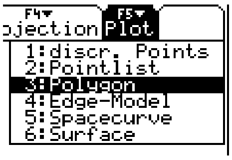




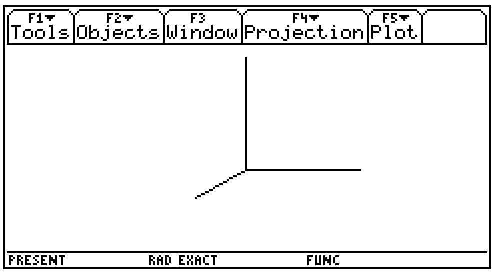
Before plotting we have to set appropriate WINDOW-values. It is important to keep in mind the ratio of the graphic window´s lengths:

If you want to have an undistorted plot then set your values in such a way that (`xMax-xMin`) : (`yMax – yMin`) = 7 : 3. Don´t forget to fix all answers by pressing ENTER.

Press `F5 Plot` and choose option `3:Polygon` (because the axes are built as a polygon), and then enter the name of the object.



Now the great moment is very near. After the last ENTER you should find the axes in oblique view. If we had more objects to represent, then we could add them now.

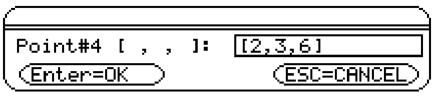


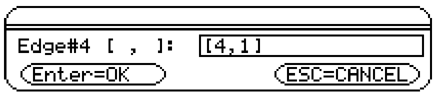
We immediately proceed with

(2) **Add a rectangle standing vertically on the x-y-plane**. Define the rectangle as an edge model.

Let the quadrilateral be defined by ABCD [(2,3,0), (4,-3,0), (4,-3,6), (2,3,6)].

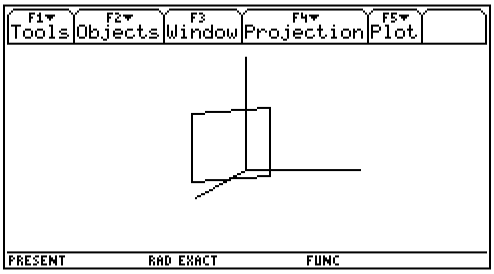
Press `F2 Objects` and then `3:Edge-Model`, `Name of object: Rect, Number of points: 4`. Enter points A through D, point by point:





`Number of edges: 4`, Then define the 4 edges by the points (addressed by their numbers as vectors in brackets): Edge#1: [1,2], Edge#2: [2,3], Edge#3: [3,4] and Edge#4: [4,1], and finally `new object: n`.

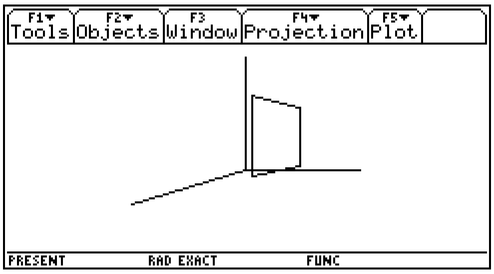
`F5 Plot, 4:Edge-Model, Name of object: rect.`
You should see the following figure:



Choose another projection and represent the same figures. It might be necessary to adapt the Window-values. Change `Projection` via `F4` and repeat the `F5-Plot`-procedure.

`F1 Tools, 1:Clear Screen` will clear the screen.

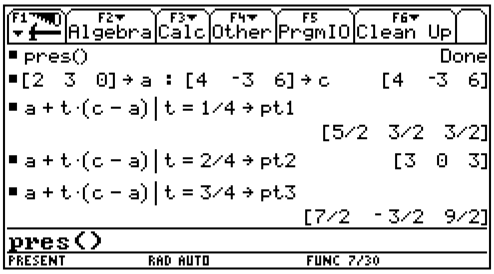
The figure below shows a perspective view.



In the next example CAS will enter the stage.

(3) **Represent one diagonal** of the rectangle by 5 equidistant points. Show the rectangle together with the points in isometric representation. We take diagonal AC.

Leave `pres()` by `F1 Tools, 2:Quit`.



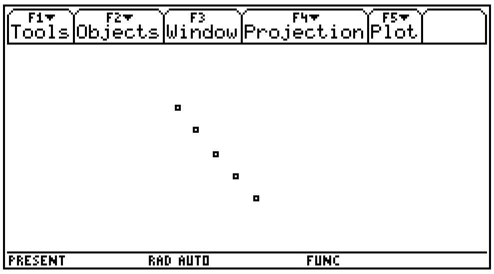
Switch to the home screen and perform the calculation, store the points as pt1, pt2 and pt3.

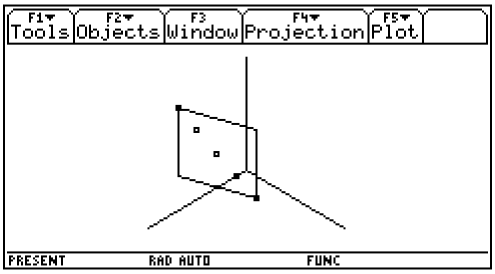
Load `pres()`. Choose `F2 Objects 1:Pointlist`.
Name of object: `diago` (not diag, because this is an implemented function)

`Number of points: 5`, As Point#1 through Point#5 enter a, pt1, pt2, pt3 and c.

Then `F4 Projections, 5:Isometric` and `F5 Plot, 1:discr. Points, Pointlist: diago`

You should see the 5 points, then add the rectangle `rect` via `F5, 4, rect` and the axes (`F5, 3, axes`)

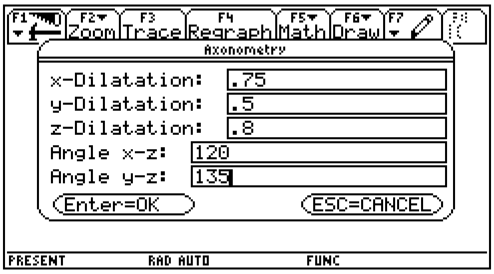


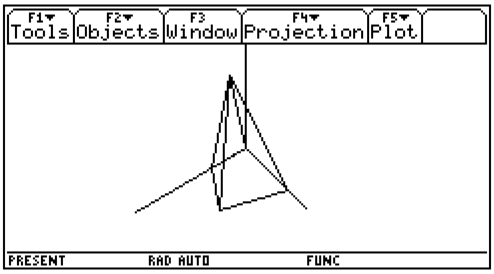


Clear the screen and quit `pres()`. Switch to the Home Screen and be ready for example (4).

(4) **Create and represent a pyramid in a chosen projection.** The pyramid is given by its base ABCD [(3,0,0), (5,5,0), (0,7,0), (0,0,0)] and by its top S (3,3,8). We need this solid for an extended problem in the main part of this workshop.

Enter points A through S in the Home Screen. Load `pres()`, define object "`pyra`" as a polygon or as list of edges and then you shold find out a nice representation of you pyramid. (eg axonometric proj.).



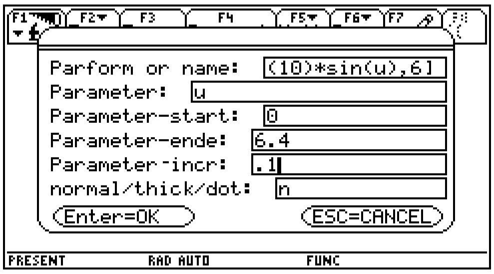


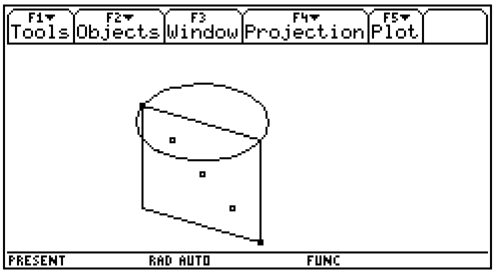
(5) **The first example of a space curve:** Plot a circle with one side of the rectangle as diameter.

One possible solution : m = ½ *(c + d), r = |d – c|/2 with C(4,-3,6) and D(2,3,6).

The parameter form of the circle is [3 + r*cos(u), r*sin(u), 6] with r = √10 and parameter u (0 ≤ u ≤ 2π).

Load `pres(), F5 Plot, 5:Spacecurve`



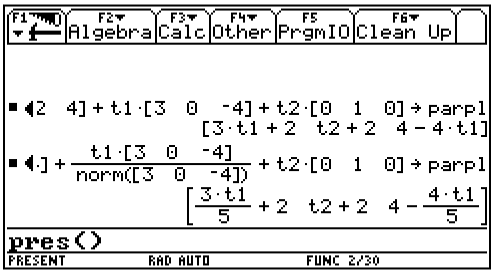


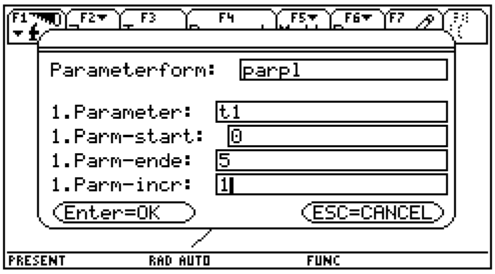
You see the circle together with the rectangle and the points from (2) and (3).

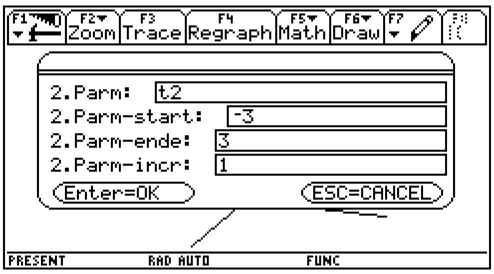
(6) **Present the plane :** (2,2,4) + t_1(3,0,-4) + t_2(0,1,0) and add a 1 × 1 grid.

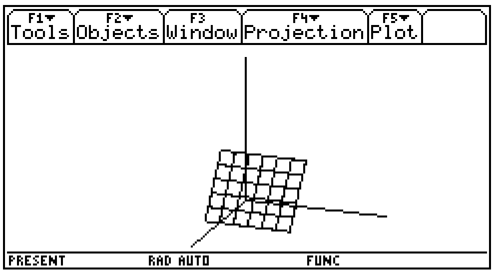
Save this plane as parpl, because we will need it in the future.

Choose any projection, define the Window-values and then `F5 Plot, 6:Surface`





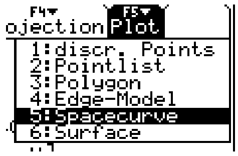




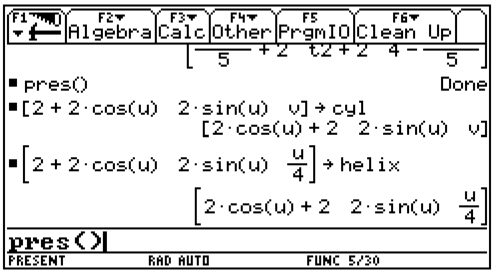
The picture above is in dimetric projection with adjusted Window-values.

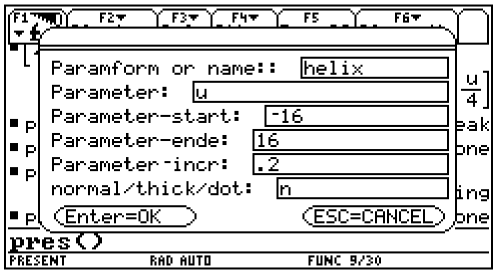
Attention: Don´t use parameter t to define surfaces!

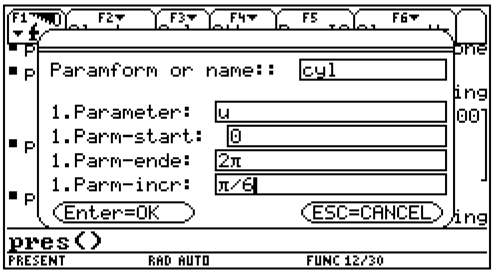
(7) **Present the cylinder with a circle as base together with a helix.** M(2,0,0); r = 2; height 8 units. The cylinder should be divided by the x-y-plane in two equal parts.

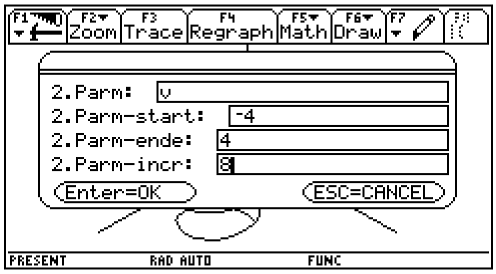


cyl = [2 + 2*cos(u), 2*sin(u), v] with 0 u ≤ 2π; -4 ≤ v ≤ 4
helix = [2+2*cos(u), 2*sin(u), 0.25u] with -16 ≤ u ≤ 16.

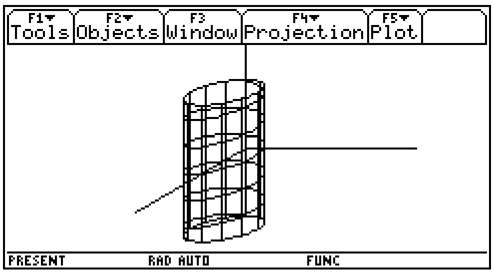








In oblique view together with the axes:



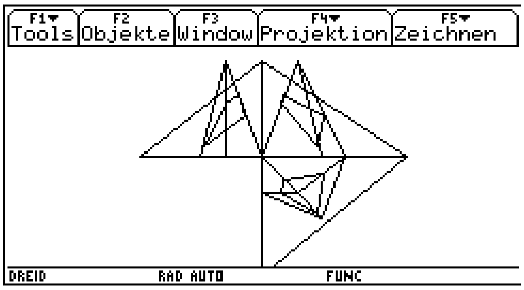
2 Advanced Workshop

(8) **Intersect pyramid pyra from (4) with a plane given by three points [XE(10,0,0), YE(0,12,0), ZE(0,0,8)].**

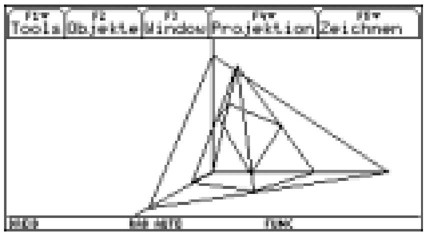
You can easily follow the script. Axes and pyra have been defined in the first part of this workshop. So you can skip repeating this.

```
:Script for intersection pyra - plane
 :Enter the given points
C:[3,0,0]»a:[5,5,0]»b:[0,7,0]»c:[0,0,0]»d:[3,3,8]»s
C:[10,0,0]»xe:[0,12,0]»ye:[0,0,8]»ze
 :
 :The axes:
 :
C:[0,0,0]»o:[12,0,0]»xu:[0,12,0]»yu:[0,0,12]»zu
 :
 :Call pres(), define the pyramid (pyra - pointlist), the plane and the
   axes.
 :
C:pres()
```

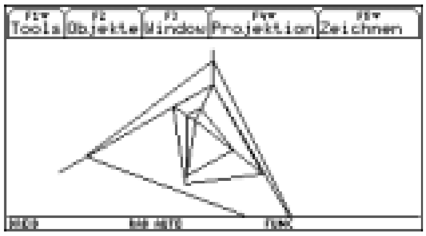
```
 :
 :Intersection edges - plane
 :
 :
C:p1+t*(p2-p1)»lin(p1,p2)
C:p1+u*(p2-p1)+v*(p3-p1)»pla(p1,p2,p3)
 :
C:a+t*(s-a)=xe+u*(ye-xe)+v*(ze-xe)
 :
 :or
 :
C:lin(a,s)=pla(xe,ye,ze)
```



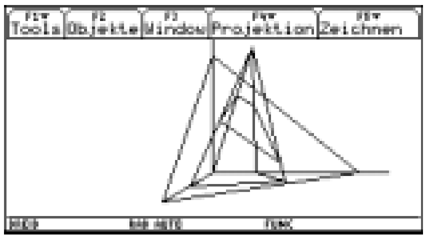

```
 :
C:rref([0,ª10,ª10,ª7;3,ª12,0,0;8,0,ª8,0])
 :t = 14/25
 :
 :with the TI-92PLUS & 89 it works directly
 :
C:solve(3=ª10*u-10*v+10 and 3*t=12*u and 8*t=8*v,{t,u,v})
 :
 :
C:a+t*(s-a)|t=14/25»a1
 :
C:lin(a,s)|t=14/25
 :
C:lin(b,s)=pla(xe,ye,ze)
C:b+t*(s-b)=xe+u*(ye-xe)+v*(ze-xe)
C:rref([2,ª10,ª10,ª5;2,12,0,5;8,0,ª8,0])
```



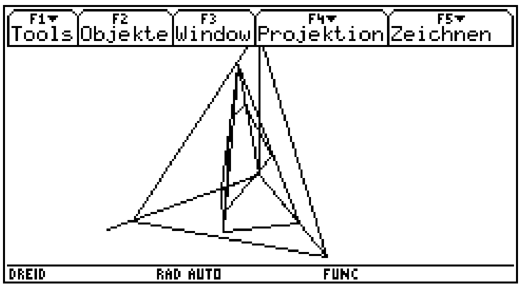
```
 :
 :
C:lin(b,s)|t=5/38»b1
C:b+t*(s-b)|t=5/38»b1
 :
 :do the same with  SC
 :
C:lin(c,s)=pla(xe,ye,ze)
C:c+t*(s-c)=xe+u*(ye-xe)+v*(ze-xe)
C:rref([3,10,10,10;4,12,0,7;8,0,ª8,0])
```



```
 :
 :c1 doesn't work, system variable! cc1
 :
C:lin(c,s)|t=25/58»cc1
C:c+t*(s-c)|t=25/58»cc1
 :
C:lin(d,s)=pla(xe,ye,ze)
C:d+t*(s-d)=xe+u*(ye-xe)+v*(ze-xe)
C:rref([3,10,10,10;3,ª12,0,0;8,0,ª8,0])
```

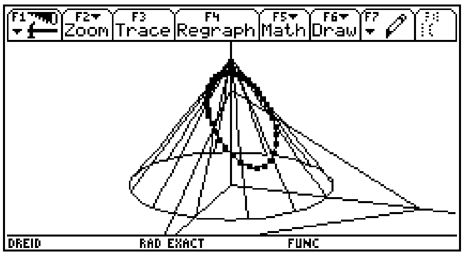


```
 :
C:lin(d,s)|t=20/31»d1
C:d+t*(s-d)|t=20/31»d1
 :
 :
 :define list [a1,b1,cc1,d1,a1] and present it
 :
C:pres()
```



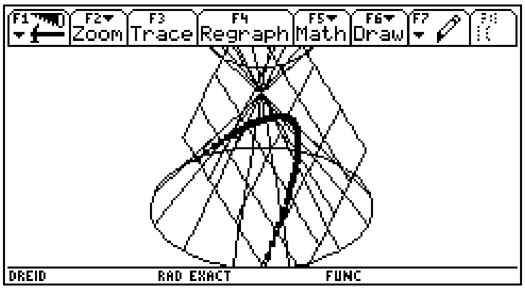
(9) **Produce an elliptic and a parabolic intersection of a cone.**

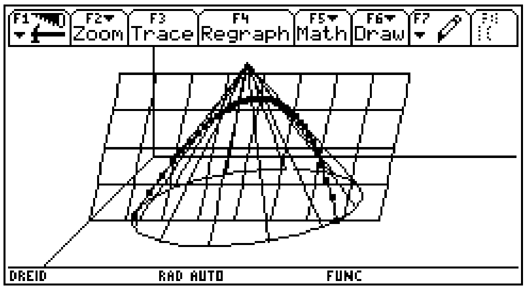
Follow the script below:




```
:Definition of the cone
 :Window: ª4 œ x œ 10, ª3 œ y œ 3
 :
C:[(3u-3)*cos(v),(3u-3)*sin(v),4u]»cone
 :
 :0œuœ2 » double cone; 0œuœ1 » cone
 :intersect.pts plane ª axes
 :
C:[5,0,0]»e1:[0,5,0]»e2:[0,0,3]»e3
 :
 :define plane1
 :
 :{1,2,3,1}»pla1
 :
 :
C:e3+t1*(e1-e3)+t2*(e2-e3)»pla1
C:pla1=cone
 :
C:[[5*t1=3*(u-1)*cos(v),5*t2=3*(u-1)*sin(v),ª3*t1-3*t2+3=4*u]]
C:expand(ans(1))
 :
 :solve for t1,t2 and u
 :
 :with the PLUS we solve the system in one step:
 :
C:solve(5*t1=3*(u-1)*cos(v) and 5*t2=3*(u-1)*sin(v) and ª3*t1-
3*t2+3=4*u,{t1,t2,u})
 :
 :[[5*t1=3*u*cos(v)-3*cos(v),5*t2=3*u*sin(v)-3*sin(v),ª3*t1-
3*t2+3=4*u]]
 :
 :Put the equations in the right order and read off the coefficients:
 :
C:rref([5,0,ª3cos(v),ª3cos(v);0,5,ª3sin(v),ª3sin(v);3,3,4,3])»ell
 :
C:ell™[4,3]
 :
C:cone|u=ans(1)»ellipse
 :
C:ellipse
 :
 :
 :present Objects cone, pla1, ax and ellipse in any projection
 :
 :create a parabolic intersection:
 :
 :we shift the cone's base center to
 :[4,4,0] » cone
 :
C:[(3u-3)*cos(v)+4,(3u-3)*sin(v)+4,4u]»cone
 :
 :plane passes [2,2,4] and has direction vectors [3,0,ª4] and
[0,1,0]:
 :
C:[2,2,4]+t1*[3,0,ª4]+t2*[0,1,0]»pla2
 :0.2œt1œ1 step 0.2; ª1œt2œ step 1  or use parpl from (6)
 :graph. representation of the plane
 :
C:crossp([3,0,ª4],[0,1,0])
 :
C:dotp([x,y,z],ans(1))=dotp([2,2,4],ans(1))
 :intersections with the axes:
 :
```

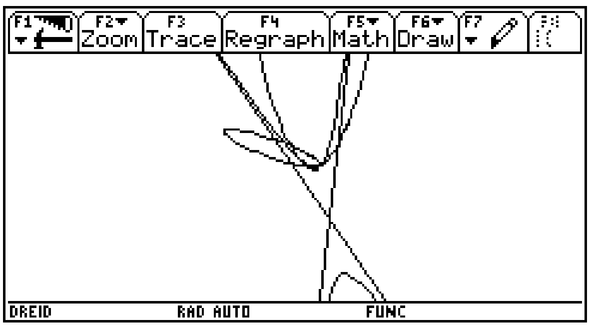
```
C:[0,0,20/3]»zs:[5,0,0]»xs
 :
C:pla2=cone

 C:rref([3,0,ª3*cos(v),2-3*cos(v);0,1,ª3*sin(v),2- 3*sin(v);
ª4,0,ª4,ª4])»para
 C:para™[4,3]
 C:cone|u=ans(1)»parabel
 :
 :or
 :
C:solve(ans(1)[1,1] and ans(1)[1,2] and ans(1)[1,3],{u,t1,t2})
C:cone|u=(3cos(v)+1)/(3*(cos(v)+1))»parabel
 :
C:zeros(getNum(parabel[1,3]),v)
C:approx(ans(1))
 :
 :take ª1.91 and 1.91 as parameter boundaries
 :
 :hyperbolic intersection !!?
 :
```





The next screen shot shows another way to introduce the various conics. Take any circle and create its central projections. Choose appropriate eye points to obtain ellipse, parabola and hyperbola as projections of a circle (= intersection cone - projection plane).

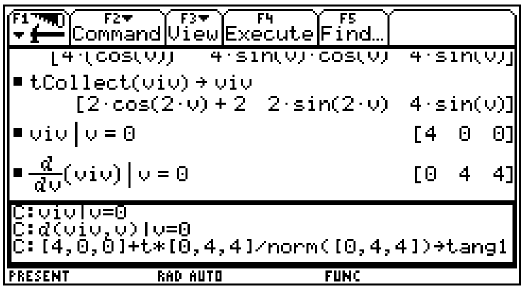
The given circle lies horizontal and osculates the projection plane.



(10) **Intersect a cylinder and a sphere to obtain the "Hippopede" or the "Window of Viviani". Present the space curve together with the tangents in its double point. Find the parameter form of the surface of tangents and show the surface.**

In this example we can combine differential geometry with graphic representation. Here also we meet bulky expressions as before, but CAS does the nasty calculation work for us. The script can be followed easily:

```
:The cylinder
C:[[2*cos(u)+2,2*sin(u),v]]»cyl
 :The sphere
C:[[4*cos(u)*cos(v),4*cos(u)*sin(v),4*sin(u)]]
 :
 :sphere in Cartesian coordinates
C:x^2+y^2+z^2=16|x=cyl[1,1] and y=cyl[1,2] and z=v
C:solve(ans(1),v)
C:cyl|v=2*§(ª2(cos(u)-1))
C:ans(1)|u=2v
C:ans(1)»viv
C:tExpand(viv)
 :[[4*(cos(v))^2,4*sin(v)*cos(v),4*abs(sin(v))]]
 :
C:[[4*(cos(v))^2,4*sin(v)*cos(v),4*sin(v)]]»viv
C:tCollect(viv)»viv
 :pres()
 :Tangents in double point (t=0,t=Œ)
C:viv|v=0
C:¶(viv,v)|v=0
C:[4,0,0]+t*[0,4,4]/norm([0,4,4])»tang1
C:viv|v=Œ
C:¶(viv,v)|v=Œ
C:[4,0,0]+t*[0,4,ª4]/norm([0,4,ª4])»tang2
 :pres()
 :
 :Surface of tangents
 :
C:¶(viv,v)
C:viv+u*¶(viv,v)/norm(¶(viv,v))»tsurf
C:tCollect(tsurf) »tsurf
 :pres()
```



Parameters:

Cylinder: $0 \le u \le 2\pi$, $-4 \le v \le 4$
Sphere: $0 \le u \le 2\pi$, $0 \le v \le \pi$

Space curve viv: $0 \le v \le 2\pi$

Tangents: $-5 \le t \le 5$

Surface of tangents: $0 \le v \le 2\pi$, $0 \le u \le 4$