NEW FORMS OF TEACHING PROVOKE AND REQUIRE NEW FORMS OF ASSESSMENT

JOSEF BOEHM

In the very early nineties the Austrian Government purchased the general license of *DERIVE* for all Austrian General Secondary Schools. Several people – school authorities and experienced teachers – didn't want to leave the colleagues alone with this great challenge and change in mathematics education (as it has been done when the ordinary pocket calculators had entered school some years before). So the ACDCA (Austrian Centre for Didactics of Computer Algebra) was founded.

4 major projects were started and finished since then:

CAS I: 1993 - 1994 DERIVE Project

CAS II: 1997 - 1998 TI - 92 Project

70 classes on 44 Secondary Schools, (65 teachers, 680 female and 1570 male students)

CAS III: 1999 - 2000 2. TI - 92 Project

Electronic Learning Media in Maths-Teaching

- Influence on Teaching, Learning, Curriculum and Assessment, (94 teachers, 2000 students)

CAS IV: 2001 - 2002 CAS Project

New Media and Methods

- New Culture of Problems, Supervision & Support, Bilingual Teaching,

(140 classes, more than 2200 students)

CAS V: 2003 - 2005 CAS Project

has just started

- e- & online Learning, Self responsible Learning, Standards, Teaching in "Laptop-Classes"

The first two projects investigated the teachers' and students' acceptance of new teaching methods using technology. They focused on changes in students' learning success, on changes in their attitude towards maths and science, on differences in knowledge comparing the traditional teaching with the technology supported methods,

Now we tried to find new fields for investigation and research:

in CAS III we decided to work in the following five research areas:

***** Electronic Teaching- and Learning Media

Testing and evaluating existing software for using CAS and dynamic geometry in classroom.

***** TIMS Study - Quality Control for Maths Teaching

Preparing tests to control the learning success and running the tests in CAS and non-CAS classes.

***** Preparing a comment to the Upper Secondary Level Curriculum with special regard to a CAS-supported teaching

Teaching selected teaching sequences according to a recommended process and giving written reports.

★ Influence of CAS on the assessment-situation

Trying and observing new forms of assessments with one or more classes.

***** New Learning Culture with CAS

Preparing workstations for "open learning" for some selected items, as

Direct and indirect proportion, Introducing the function concept, Simultaneous equations, Power- and root-functions, Calculus (differentiating), Preparation for the end-examination.

** Influence of CAS on the assessment-situation **

12 teachers (6 female, 6 male; 9 from General Secondary schools,

3 from Colleges for Business Administration) - 16 classes (aged 15 to 17)

I joined the last group, because during the last years when I taught technology supported (TI-92) I felt uncomfortable with the given situation which did in no way consider new and additional competences which we wanted the students to acquire and which couldn't be assessed in the traditional tests which consisted more or less of recipe following calculations.

The situation until now was and in most cases still is: Fixed number of written tests, oral assessments possible and on demand of the students. Thus no flexibility in posing the task (usually 4 or five problems) and in changing the form of the assessment.

We found some fields worth to be investigated:

- (1) "Continuous" carrying on the traditional form of tests including CAS.
- (2) Problem solving tests supported by textbooks, notebooks, any materials....
- (3) Total assessment time for year can be divided in shorter tests for basics and longer problem solving tests; the assessment times are fixed by the teachers together with the students.
- (4) Presentations of selected chapters (problems) individually or in groups.
- (5) One part of the written tests can be substituted by a "project work" (= "Facharbeit")
- (6) Cross curriculum written test
- (7) Written test as group work
- (8) Instead of a fixed number of one-hour-tests have more shorter tests announced and not announced to measure the increase of knowledge
- (9) "Inner distinguishing" in the assessment situation.the "Must" / other task(s) for a better mark / extra credits for the gifted.

My class: IIIc, 27 not very bright – but cooperating and friendly - students (age 16 - 17).

They formed 7 groups for "Facharbeiten" – presentations (including preparing hand outs and home exercises for the colleagues). The presentations will be graded.

Usually we have two assessments (written tests) à 50 minutes per semester = 200 min /year. Additionally the teachers needs some "notes" about the students to give the final mark. If a student is in danger not to pass the year an oral assessment (15 minutes) is compulsory.

Helmut Heugl – the chair of the ACDCA, school inspector in Lower Austria – made possible an "experimental year" for all colleagues in our group so that we could work outside of the laws with respect to any of the 9 changes in assessment habits given above. We had to carefully and very detailed explain what we were intending to do and we needed the agreement of students and parents.

We – the students and I - agreed on

having 3 short tests (basics, with and/or without the TI-92) and one problem solving test (1 hour) for the first semester.

Second semester: 2 short basics tests à 20 minutes and one extended problem solving test with teamwork.

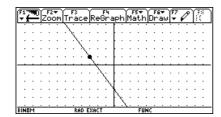
One group presentation of a selected chapter ("Facharbeit").

In the following I'll show how we did in this demanding year.

Additional Comment. Resulting from an earlier "Pedagogical and Didactical Project" I allow my students – with very few exceptions – the use of supporting materials during written tests (textbook, school- and home exercises, private notes, ...)

Basics Test (15 minutes, without TI, without any additional support)

1) Give the equation of the represented linear function?



Give the equation of the line which is perpendicular to the given line and passes the origin?

What is the equation of the linear function, if the distance between the grids in horizontal direction represents 10 units? (1 x 1 grid)

- 2) Given are two straight lines g: 6y 2x 12 = 0 and h: x + y = -2. Find their intersection point graphically.
- 3) How can you decide if two lines are parallel seeing their equations? Write down the equations of two parallel lines and underline the upper one.
- 4) Line g: 2x + 5y = -4 and line h: x y = 5 and three points: P(-2 / -1), Q(-5 / -10), R(3 / -2) Find out if the points are lying on g and/or h.

Basics Test - Power Rules (25 minutes, TI-92 allowed)

a) Write down the result as compact as possible and without denominator:

$$\left(\frac{a^2b^x}{c^n}\right)^3 \left(\frac{c^{n+1}}{a^{x-1}b}\right)^{-2} = \tag{4}$$

- b) Bring the expressions under one common root: $\frac{2x}{3y^2} \sqrt[3]{\frac{3y^2}{4x}}$ (3)
- c) $\sqrt[3]{3024 \, a^4 \, b^6 \, c^{20}}$; draw the root as far as possible.

 Describe the process how to reach the result. (4)
- d) Calculate with the TI and give reasons for the result: $\frac{(12a^2b^3c^3)^4}{(18a^3b^2c)^3}$ (4)

e)
$$\frac{x\sqrt{y} + y\sqrt{x}}{\sqrt{y} + \sqrt{x}} =$$
 Explain the calculator's result. (5)

In parenthesis are the points given which could be reached. As you can see the CAS-calculator is used as an assessing tool. The students need some competence in treating the calculator, but the device doesn't give the (full) answer in all cases. In contrary, it opens new and sometimes unusual questions. A sound knowledge of the power rules is necessary to pass the test

Spread over the year the groups had to prepare their presentations. I provided material (most of it in English) and they had the task to give a vivid and well organized demonstration of the subject including working on blackboard and projection device (ViewScreen, transparencies), preparing handouts and providing examples as home exercise. I underlined that I had not the intention to repeat their chapter in the following classes (not enough time to do it twice) and that their stuff will be assessed as usual in written and/or oral tests.

The 7 Presentations (Facharbeiten)

- (1) Application of a larger system of equations
- (2) Logistic growth
- (3) Complex numbers Theorem of Vieta Fundamental Theorem of Algebra
- (4) Repetition and extending the formulae for areas and volumes
- (5) Presentation of extended tasks from trigonometry
- (6) Introduction into Programming the TI-92
- (7) Compound interest Present value Future value

I include three sample pages of the presentations:

The first is an application of simultaneous linear equations. Two kinds of bikes must be assembled using several basic parts. The problem was not among the materials provided by me, the pupils invented it for their presentation. The home exercise which was to prepare for the colleagues was very nice: the task was to produce a combination of three different cakes and it was necessary to find out the right amount of ingredients (sugar, eggs, flour, jam,).

The second example is the transparency which should lead to logistic growth (other growth- and decay models had been part of earlier classes). They started with the discrete model (recursive sequence) and then presented the continuous model (as a Black Box), referring to the proof which would be given later with means of Calculus

The same group (4 boys) gave a second presentation: "Introduction into Programming the TI-92".

I made the experience that it is very hard – but necessary – for the teacher to remain as much as possible in the background keeping silent. The students are not very happy being interrupted and corrected at any occasion. It was a hard learning process for me, too. But it should be the students' hour(s) and not mine.

I recorded one presentation with a video camera and we had much fun later on, discussing and analysing this "performance".

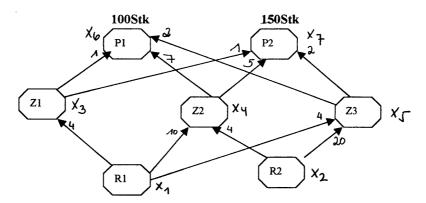
Another comment: The presentations were graded and one has to take care not make it too easy for the students to get too good marks. They grading must be fair but it is not sufficient to stand in front of the class and give any talk telling some stories.

. Schrifts unterlegen for du Praturalie

Austraty 1

mehrstufiger Brosess - Nerfechtungsmattur Anwendungen des Gleichungssystems (GLS)

Ein Produktionsbetrieb fertigt aus den Rohstoffen R1 (Schrauben) und R2 (Draht) die beiden Endprodukte P1 (Mountainbike) und P2 (Citybike). Bei der Herstellung entstehen die drei Zwischenprodukte Z1, Z2 und Z3 (Sattel, Gestänge und Räder). Die folgende Zeichnung zeigt in Pfeilrichtung gelesen, wieviele Einheiten eines jeden Produktes für die einzelne Herstellung notwendig sind.



Die Zeichnung muss nun so gelesen werden:

Man benötigt 4 Schrauben, also 4R1 um einen Sattel (Z1) herstellen zu können. Das gleiche, man benötigt 10 Schrauben (R1) um ein Gestänge (Z2) herstellen zu können,.....

Wenn man dann für R1, R2, Z1, Z2, die gewünschten Einheiten einsetzt, entsteht ein lineares Gleichungssystem:

Wie hoch ist der Gesamtbedarf von R1 und R2?

Stelle dazu die passende Gleichung auf und versuche sie zu lösen!

Bringe nun das Gleichungssystem in eine ordentliche Form und löse es! alle unbekannen auf eni Seile

$$24$$
 $-4 \times 3 - 10 \times 4 - 4 \times 5 = 0$ $20 \times 4 = 17 \times 50$ $20 \times 5 = 17 \times$

Sample page 2



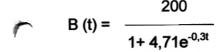
Das kontinuierliche Modell

Für das kontinuierliche Modell gibt es eine explizite Funktionsvorschrift, die sich wiederum aus der Analysis begründet:

B (x=t) =
$$\frac{K}{1+ (K/B_0 - 1) e^{-cKt}}$$

Die Proportionalitätskonstante p für kleine Zeitintervalle kann als Näherungswert für die Wachstumskonstante c verwendet werden.

Damit ergibt sich mit K = 200, B_0 = 35 und c = 0,0015 die Rehformel:



Vergleiche die Tabellenwerte der beiden Modelle!

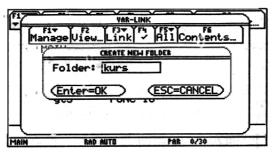
n/t	diskret	kontinuierlich
0	35	35
1	43,66	44,52
2	53,9	55,75
5	93,29	97,57
6	108,22	112,41
10	161,44	161,98





Einstieg ins Programmieten

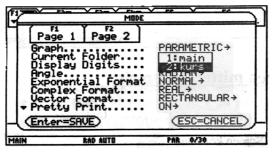
ANLEGEN EINES ORDNERS



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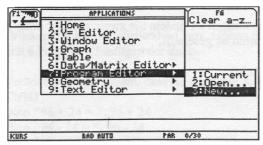
2nd – VAR-LINK
F1 (Manage)
5 Create Folder
Benennen des Ordners
Zweimal Enter

IN EINEN ORDNER WECHSELN



Drücken Sie: Mode Gehen Sie auf Current Folder Wechseln Sie auf Kurs Zweimal Enter

STARTEN DES PROGRAMMEDITORS



Drücken Sie:

APPS

7 Program Editor

3 New

Geben Sie einen Namen für Ihr Programm ein

17.05.00

Im Bereich zwischen Prgm und EndPrgm müssen die von dem Programm durchzuführenden Befehle eingetragen werden.

Stefan Rosenwirth, Bernd Sonnleithner, Michael Reiter, Joachim Vorstandlechner, Christoph Manseder Seite 1 **Short test 25 minutes** (including the stuff of one presentation!!)

- 1) z1 = 3 + 4i, z2 = 1 2i, z3 = 4 + iCalculate: $z_1 \cdot z_2 - z_3^2 =$
- 2) Given is the set of solutions $L = \left\{3, -1, \frac{1}{2}\right\}$.

Give two equations of degree 5 having L as set of solutions.

3) For which k have the parabolas a tangent in common:

$$y = k x^2 + x - 1$$
 und $y = -\frac{x^2}{2} + k x + 1$?

Perform the task without and with the TI-92.

4) Find the solution without using solve():

$$36 x^6 + 132 x^5 - 263 x^4 - 638 x^3 + 926 x^2 - 770 x + 1225 = 0$$

We had discussed very deep the solutions of equations of higher degree. But I had not posed a question like question (2) before. The students knew about the existence of multiple solutions, and how they appear by factoring the equation polynomial, but I had not given the reverse question. Questions of this kind are important to really test, if - at least some of them - understood the "big ideas" behind.

Then the first "great day" was here: the Problem Solving Test. I was asked over and over: "Sir, is this true that we will have not seen the problems before. Is this really absolutely new for us? How can we manage this?".

When I set the problems I had a bit pity with them and started with an introductory example which was not unknown to them, because we had worked through a large range of examples covering exponential growth and decay.

The two following problems were really hard. We had talked a lot about recursive models, but this one was new to them and the third problem was really a problem, because they first had to understand the text and transfer it into a sketch (we are on a business school and not on a technical one!!). Then the sketch forms the base of a GOZINTO-graph which shows the interdependence of the singular parts and finally this graph must be described by a system of linear equations, which then easily can be solved by any device, followed by the final interpretation of the result.

I promised to mark the parts of example (3) separately, i.e. if they misunderstood the text and produced a wrong sketch but if then the graph follows this sketch, then the graph would be regarded as right, and if the system of equations follows from their (wrong) graph correctly, then this step would be regarded as correct, etc.

As you might assume, grading this test was not an easy task!! It turned out to be "Problem Solving" for the teacher.

The Problem Solving Test!!!

- A1) The "Warming Up"
 - A Radium isotope has a half life of 11.7 days.
 - a) What will remain of 250g after 3 weeks?
 - b) What will remain of any mass after three times half life?
 - c) How long do you have to wait until 20g will remain of the given 250g?
- A2) You take a loan about 150 000ATS at an interest rate of 5.75%. Try to find a way for finding the repayment after *n* years. You may assume that

there are no extra payments between.

(The interest rate is the growth rate of the loan for one year.)

What is the debt after 8 years?

Set up a (recursive) model for the case that we have a payment of

20000 ATS at the end of each year. How many payments are necessary?

Describe your way to find an answer.

What is the recursive equation?

Write down parts of the table or sketch the graph.

(Set the [WINDOW]-values nmin und xmin = 0!)

Try to find payments such that you will have paid back the whole exactly after 12 years.

Give a report about your tries. (it is not sufficient to only write down a result!!)

A3) A threepod is assembled: each of the three legs consists of two halflegs, which are put together by 6 bolts. Each of the halflegs consists of two rods, screwed together by 4 bolts. The three complete legs are fixed on a plate using 18 bolts giving the threepod.

Produce a sketch of the threepod. And then set up the Gozinto-Graph.

There is one order: 15 threepods + replacement parts (additional 8 halflegs, 4 plates and 400 bolts). Find the system of equations to fix the production plan.

It was interesting that the very easy "Warm Up" was only solved by three students. The excuse of the class: "We didn't expect well known problems, so we didn't prepare for them!!".

Interestingly for me was the fact, that they performed rather well tackling the "problems". So finally the marks were not too bad and didn't differ from the marks given earlier on traditional written tests.

The first term ended and we (students and I) agreed on 4 questions which should be answered by them

Questions to the students after the first term:

- (1) Are my expectations fulfilled?
- (2) What did I like / dislike until now?
- (3) Has my attitude towards maths changed?
- (4) Are there any changes in success in learning?
- (1) 15 yes / 8 no (because most of them had expected to have more group work, but this was not the aim of the project!)
- (2) They liked best the presentations, liked the preparation of the presentations working in teams, no single dislike!!!
- (3) Yes and No, special comments on the group presentations and their positive effects.
- (4) half/half

2nd term: 2 short basics assessments, one very short test on formulae

Formulae Test: Volume of a Cone

Surface of a Cylinder

Give a sketch of a segment of a sphere with two bases

Area of a equilateral triangle

etc.

Basics Test (30 minutes with TI)

Triangle with given area A = 153, altitude h_a = 17.3 and angle β = 27.75°. Find the missing sides and angles.

- 2) Two masts are standing on a horizontal plane in a distance of 37m. From the pedal points of the two masts the tops of the respectively other mast can be seen under elevation angles 26.03° and 42.87°.
 - a) What is the height of the two masts?
 - b) A rope is stretched tautly from one top to the other. What is the minimum length of this rope?
- 3) In a quadrangle with a, b, c and d, sides a and b form a right angle: a = 2.82; b = 3.17; c = 4.28; d = 5.12. What is the area of the figure? What is the size of the angle opposite to the right angle?

30 minutes seems to be very short, but it must be said that we developed a package of functions which allowed quick calculations with triangles and we could do without Sine- and Cosine Rule, because we had changed the "White Box" into a "Black Box" and from this moment, the student could focus on the problem and on the strategy how to solve the problem supported by a sketch. The results of this test was fine.

The second "Great Day" of our project approached. The students had been satisfied with the tests, their successes with the presentations and the good mood of us all, feeling a special class moving outside of the law, doing and trying things which nobody else in Austria was allowed to (except some other project classes). The Group Assessment was unique, because only one colleague from another school joined me with this experiment. Encouraged by a talk given by Marlene Torres-Skoumal at the ACDCA-Summer Academy in Goesing about group assessment I wanted to experience this. We always hear that our society and the companies expect group competence from the students when they leave school, but we very seldom give the chance to train it in school.

The first question of all colleagues (and students, of course) was and still is: How did you form the groups? I had the choice of two models:

- (1) Try to form the groups according to their maths performance (bright to bright and weak to weak). But then I had to set different problems and didn't know how to grade fair in this case.
- (2) Mix the groups. But then I had to face the danger that the weaker students in the group will "live" on the work of the others. How to evaluate the work of each single student?

Questions over questions

This is how I did:

I told the students that they should form groups of 3 or 4 students and try to distribute their mathematical resources (more gifted students) in the best possible way. I didn't want to have the two brightest mathematicians in one group. They should bring their proposal for approval one week before the Group Assessment. I was free to order changes. They did their best, I didn't accept one group. They had to separate two better students.

The second question was: "Which kind of tasks will you give?"

I answered very sincerely: "Believe it or not, I don't know at the moment. It is for me as challenging, demanding and thrilling as it is for you". And indeed, I didn't really know until last evening before the test. Then I set down and after a while it was ready

The Group Assessment.

26 pupils formed 8 groups (2 \times 4 and 6 \times 3). 50 minutes working time were extended to 100 minutes.

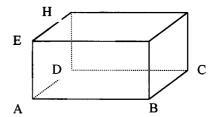
1) Given is a rectangular prism with

AB = a = 7cm

BC = b = 5 cm

AE = c = 4 cm.

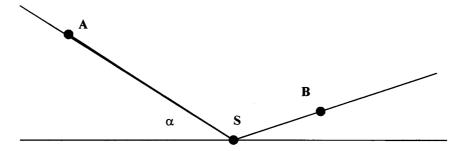
Find the distance from point H to the space diagonal EC.



From a point A lying on an incline falling under $\alpha = 32,1567^{\circ}$ one can see a point B lying on the opposite incline under the depression angle of 22,3462°.

Having moved from A 150m downhill to a point A', you can see B from A' under a depression angle of 14,5883°. A' has a distance of 125m from the bottom of the valley.

In a map you can read off B's height above sea level with 982m.



- a) Neat drawing of the situation in measure 1:2500.
- b) What is the height above sea level of points A and A'?
- c) What is the inclination angle β of the opposite slope (containing location B)?
- d) What is B's distance of the bottom of the valley?
- 3) Give a proof using an appropriate sketch:

In each quadrangle with diagonals perpendicular to each other the sums of the squares of two respective opposite sides are equal.

It was very fascinating observing the students organizing the work in their groups. First they read an discussed the problems. Then they tried to distribute the work according to their abilities and preferences. This took much more time than I had expected, but I saw that this was a very necessary and positive aspect. So I decided immediately to extend the working time from one to two hours. No one single student did only copy the other's work or was not really busy on his part.

For the second hour I asked a colleague – teaching commercial subjects – to supervise the class. In the next break she handed in the assessment papers and she was very excited about the working style and attitude the students showed during the test. From this day we had one important voice more in our school concerning our new way in teaching mathematics.

Each group had to deliver one paper. All members of a group received the same marks. Finally only one group had a "5" (= "Not Sufficient", marks in Austria from 1-5), which was accepted by the students without any claiming about grading too hard.

At the end of the year I asked for the students' opinion to 8 items:

(1) Separation basic knowledge - problem solving

positive, problem solving was for some too difficult, "problem solving was easier within the group", "fine, because of the applications in problem solving test", "good, I like to face problems!"

(2) Project - Presentation

very motivating, the students appreciated to work on one item very concentrated using provided materials, "when we received the materials we never thought to be able to manage the task, we asked older students and friends from other schools, nobody could help us. Then we spent one afternoon together and step by step we worked through the materials. Now we are sure that we have a very sound knowledge about that stuff, because we have learnt that by ourselves".

(3) Total Year's time for assessment

was accepted by all students and very welcome

(4) Group test

the students liked it, many of them wrote down, that they had talked a lot during the test and had a good working atmosphere and that they so were able to find at least partial solutions. They also expressed that they split the work according to the group members' abilities, i.e. that even in a very short time they were able to organize themselves within the groups.

(5) Change in maths teaching

the students recognized correctly that maths teaching didn't change very much - that was not the aim of the project. But there were some notes that the lessons became funnier and more interesting.

(6) Change of attitude towards maths

8 wrote that their attitude has improved, "although I am not a good mathematician, the many various ways to tackle a problem were very very interesting, great and motivating for me"

(7) Estimation of maths knowledge

No remarkable comments

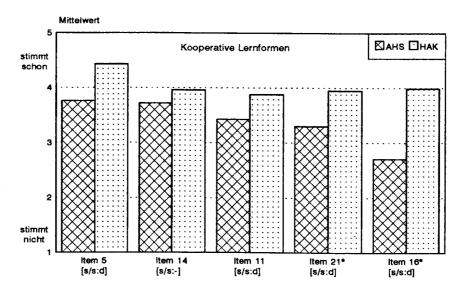
(8) Global impression of the project

The overall impression was excellent. All of them agreed in their wish to renew this project in the next year.

Two results from the report given by the ZSE (Zentrum für Schulentwicklung = Centre for School Development) are following.

The ZSE collected questionnaires from all participating students and teachers and gave a very extended report.

Abbildung 8: Angaben der Schüler hinsichtlich kooperativer Lernformen getrennt nach Schulart (N_{AHS}=204, N_{HAK}=101)16



- 5 Ich arbeite gerne mit anderen Schülern gemeinsam an einer Aufgabe
- 14 Bei umfangreichen Aufgaben führt das Arbeiten in einer Gruppe schneller zu besseren Ergebnissen
- 11 Ich wünsche mir mehr Gruppen- oder Partnerarbeit im Mathematikunterricht
- 21 Am liebsten behandle ich umfangreiche Aufgaben < nicht > alleine
- 16 Bei Gruppenarbeit arbeitet meist < nicht > nur der beste Schüler und die anderen schauen ihm zu

HAK = Business College, AHS = General Secondary School

- (5) I like to collaborate with other pupils solving a problem
- (14) Working on extended tasks group work leads quicker to better results
- (11) I'd like to have more group- or partner work in maths teaching
- (21) Treating extended problems I prefer working not alone
- (16) In group work in most cases not only the best student works and the others are watching

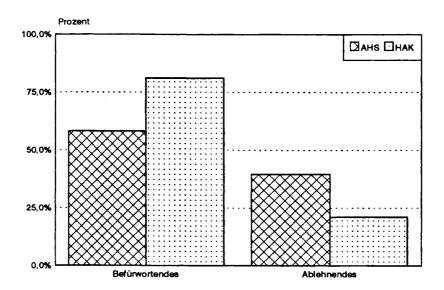
1 – I completely disagree, 5 – I completely agree

It was very interesting for all participating teachers that the vocational schools showed quite other results than the general secondary schools with respect to their attitude towards cooperative teaching (and assessment) forms.

One reason might be that our students (Business College) are more used to collaborate work. Another one could be that our students were one year older than the others.

The second diagram shows the acceptance of the new forms of assessment. The first two boxes are for "I'd like to do it once more", the second box represents the percentage of students who did not like these new forms. Here again the vocational schools show a significant preference for the new forms.

Abbildung 3: Befürwortende und ablehnende Stellungnahmen zum erprobten System der Leistungsfeststellung getrennt nach Schulart (N_{AHS}=182, N_{HAK}=90)



The last word to one of my students:

"Overall I found it a hard maths year, but it was very informative. If I had to choose between an "ordinary" math teaching and this one, undoubted I would take this one. And in my opinion this should be introduced at all schools and in other subjects, too, because one will learn how to work independent AND in a group, as well".

This affirmed my position and I was sure to be not only on an interesting but also on a right way.

There was a 4th ACDCA-project in 2001/2002 (CAS IV).

We defined four main research fields:

- Supervision (experienced teachers and beginners) / English as working language in maths education
- Teaching Materials under Special Consideration of Technology
- Quality Standards in Maths Education
- Self Responsible Technology Supported Learning

Extended reports on all ACDCA-projects (CAS I – CAS IV) can be found and downloaded at www.acdca.ac.at.

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