Development of the variational thought in secondary students

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In the curricula of basic education, development of variational thinking of high school students is not formulated explicitly. So our project plans involve its development through various content addressed at this level and look for alternatives to get students started the study of variable quantities at an early age, in order to lay the basis for the calculation. The research project is collaborative, which consists of two members. It also proposes a series of didactic sequences, in which activities will relate to the three years of secondary education.

1. INTRODUCTION

Throughout history, human beings have been interested in the study of the natural processes that occur in the universe, which has allowed a broad development in different branches of science, because of the questions and answers that arise in order to understand, quantify and predict phenomena that occur in nature.

Mathematical situations presented to us persons, both in everyday life and in the school context, are not static, they change, they are dynamic, they have relationships and regularities; i.e., such situations are related to variational thought, which has to do with the recognition, perception, identification and characterization of the variation and change in different contexts, as well as its description, modeling and representation in different systems or symbolic registers, whether verbal, iconic, graphic or algebraic.

However, focusing on the phenomena of change, first of all it is substantial to define the concept of "physical quantity" which is defined as a property or quantifiable quality of any object, process or phenomenon. Then, as the project is focused on the development of the variational thought it is important to add the adjective "variable" to the concept of magnitude, which is complemented with the analysis of variational properties of these magnitudes. I.e., we must focus on the measurable attributes of the magnitudes, which present changing numeric values.

Several studies clarify that there are difficulties in learning calculus in middle and higher level institutions, reasons why intensive stuff, same that do not have greatly benefited students, since it does not solve the problem of learning have been implemented. Therefore, it is convenient to direct the focus of attention to the causes giving rise to the problem.

2. PROBLEM

Is of utmost importance that is conducive to development of variational thought, throughout daily life there are processes, phenomena and objects which presents changes or modifications, for example in basic education: the variation of the ambient temperature, the movement of a person from their home to work, increase or decrease of the volume of water in a dam in times of rain or drought, the growth of a plant for one month, etc. However, these variations sometimes is difficult to detect them at first glance, since are implicit in our daily life.

For his part, methodology suggesting plans and programs of study omitted important stages in the didactic approach to the development of the variational thought, which has impact on the learning process of the students, as they mention in their research Imaz and Moreno (2010) when considering a "pedagogical conclusion", referring to the source of the problem is the current curriculum.

Because of this, it is appropriate to attend such problematic from the beginning, starting the construction of scaffolds of previous knowledge required to understand the calculation, and the answer is, basic education, since this is the moment where they begin to build the variational ideas at an early age in learners.

3. OBJECTIVES OF THE PROJECT

The objective of the research is to develop ideas and basic concepts in students related to variational thought. For this reason, this project arises the need to carry out a series of actions, within which the elaboration of the thesis work is to develop. Specifically formulated the need to perform the following actions.

1. Joint delimitation by the participants of the project from a body of elements theoretical explanatory variational thought in their early stage development.
2. Based on the theoretical conceptions, three blocks of situations will be designed teaching for the purpose of promoting the development of the variational thought (one for each grade of secondary education), duly coordinated their temporal sequencing, both in terms of the mathematical content involved.
3. Test pilot in the classroom, with students of secondary education, of each of such teaching blocks, in order to refine and perfect your design, your driving in the class methodology, and the mechanisms and instruments to assess their effects on the learning of students.
4. Analysis and reporting of results . Elaboration of the thesis and the grade test reports.

4. THEORETICAL ELEMENTS

4.1. THE THINKING VARIATIONAL ON THE CURRICULUM OF BASIC EDUCATION

The comprehensive reform of the basic education (RIEB) aims to integrate the three levels of basic education, contemplating the life skills, competences disciplinary, thematic areas, curriculum standards, and expected learnings.

Now, focusing on the purposes of mathematical education in secondary school which are related to the development of the variational thought refer to that expected that the students: undertake processes of search, organization, analysis and interpretation of data contained in tables or graphs and identify sets of quantities which vary or not proportionately, i.e., aspects related to the covariance of two variables magnitudes.

The didactic approach mentioned that the development of activities that involve problem situations deemed the previous knowledge of the student in order to develop expected learning outcomes, and also to awaken the interest of students creating the reflection from resolution, argumentation and validation methods is relevant to.

Also, an important component is the development of mathematical skills in students, which possess characteristics linked to the development of the variational thought, following: solve problems independently, communicate mathematical information, validate procedures and results, efficiently manage techniques.

4.2 CONCEPTIONS OF THE VARIATIONAL THOUGHT

Research in mathematical education within the line of the calculation has created different concepts, closely related to the learning of the mathematics of change and variation; These are being used as tools in the implementation of educational reforms and theoretical tools in various investigations. One of variational thought has had different interpretations and applications in some educational research.

to) *the mathematics of change*, emerged at the heart of the reform movement of the calculation in the USA in the 1980s.

(b) the concept of *qualitative calculation*, developed within the Group of work led by Kaput for the Organization TERC in United States, and in the center of Shell research in England.

(c) the concept of *variational thought*, developed by Vasco to the curriculum reform of Colombia at the end of the 1990s.

(d) the concept of *functional thinking* of caves, Baka and collaborators. Departamento de Matemática Educativa, CINVESTAV.

(e) the concept of *thinking and variational language*, developed by Cantoral, Farfan and his team of collaborators, with influence to some Latin American groups. Departamento de Matemática Educativa, CINVESTAV.

(f) the concept of *reasoning covariacional*, developed by Carlson and Thompson.

Now, the different concepts that cover the term in question, although in their statements they give to understand that they speak of the same idea, in its practical concretions show that they are not trying to the same topic. In this case, we can locate two groups:

In the first, we should be to those who focus their educational accomplishments to the study of the behavior of variable quantities. The abstract mathematical model for them is the notion of variable. Once the behavior of the different variable quantities has been more or less studied (solo; each variable magnitude separately), the study of the joint variation of two or more variable quantities, are transits which leads to the notion of covariance. The abstract mathematical model for the covariance is the notion of function, but it is contingent on the modeling of natural phenomena.

In the second group we put together scholars who assume that the center of educational achievements for the development of the variational thought lies in the study of functions and their properties, from the very first moment. Under this approach, the natural phenomena (and with them, the variable quantities) are relegated to a second or third level; What matters are the functions and properties. Features overshadow the varying magnitudes.

Taking into account the foregoing, the conceptualization of "Variational thinking" refers to the analysis, interpretation and characterization of quantities that vary and their behavior within natural phenomena and situations in context, which may represent changes (variations) or present an interrelation of varying magnitudes (covariance). So our definition is mostly related to the conception of the first case of those mentioned previously.

4.3 REASONING COVARIACIONAL

This approach called Covariacional reasoning is presented by Carlson et al. (2002) and by Thompson (1994) models by concatenating a series of complex mental actions that reflect different levels of development of this way of thinking

This framework classified in five levels ranging from level 1 (the most basic) to level 5 (the more developed), and they are described by the manifestation of certain behaviors of analysis and their reasoning about phenomena or situations of covariance which are faced students, these behaviors correspond to what in this context is called *mental action* also classified into five types (AM1 - AM5).

Each mental action can be described through a combination of covariance that student go forming, and

images of the arguments that speak for the activity. These behaviors reflect or shocked the type of

coordination that the student is able to do on the varying magnitudes involved in the phenomenon and/or its reason for change.

It is appropriate to mention that the description of each level of reasoning covariacional, is not only based on the mental action associated to that level, but this and all those preceding it; Therefore, to locate a student at level 4, you will have to show behaviors that show the domain of mental actions of type 1 to type 4 (AM1 - AM4).

Mental action	Description of the mental action	Behaviors
AM1	Coordination of the value of a variable changes in the other.	Designation of you shafts with verbal signs of coordination of the two variables) changes with changes in).
AM2	Coordination of the direction of change of a variable with another variable changes.	Construction of a line straight growing. Verbalization of the consciousness of the direction of change of the output value while they are considered changes in the input value.
AM3	Coordination of the amount of change of a variable with another variable changes.	Location of points/construction of straight cutting. Verbalization of the awareness of the amount of change of the output value while they are considered changes in the input value.
AM4	Coordination of the reason of average change of function with uniform increases in the change in the input variable.	Construction of straight contiguous blotters for the domain. Verbalization of the consciousness of the reason of change of the value of output (with respect to the input value) while deemed uniform increases in the input value.
AM5	Coordination of the instant reason for change of function with the continuous changes in the independent variable to the entire domain of the function.	Construction of a smooth curve with clear indications of changes in concavity. Verbalization of the consciousness of the instant changes in the reason for change for all the domain of the function (tipping points and the direction of the recesses are correct).

Figure 1. Carlson et al. (2003) table of mental actions in covariacional reasoning.

An adaptation, which is to include an action mental zero (AM0), since we consider relevant to begin the study of variation only analyzing the behavior of a variable magnitude, where the stages will be necessary will be proposed to this theoretical framework of: perception and identification of varying magnitudes, its quantification, domain, as well as its different representations, etc.

It is worth mentioning, that mental actions described above are complemented by another approach called qualitative calculation, which is a system of representation about the intensification of change presented by Walter Stroup, same as described below.

4.4. QUALITATIVE CALCULATION

The central idea of W. Stroup (2002) mentions that the qualitative calculation is a cognitive structure in its own right, and that develops or evolves in ways that appear to conform to the General characteristics of the development of operational thinking, according to the analysis by Piaget. In particular, the intensification of the reason for change, and two types of reversibility between what is called "how much" or "how much" (the amount) and "how fast" (the reason for change), are interactive, and as a whole, characterize and help define the understanding of the qualitative calculation. Although it shares a family resemblance with the traditional expectations which could mean learning the qualitative calculation is not built from ideas based on the reason (ratio) or the proportion, as the slope, since they are typically associated with the definition of reason for change.

Support, the approach of "delta blocks" is used for learning the mathematics of change (Stroup, 1996) which was developed as part of the SimCalc project. The project committed to express starting from the integral and then pass to the derivative. This approach of "delta blocks", students engage the reason for change, either in qualitative or comparative form. The

movement is described as "medium", "slow", and then "medium".

Instead of starting with a speed graphic, and directly cause difficulties related with the ratio and the extensive quantification of speed, the approach of "delta blocks" sought, first, personifying the area "under" a graph of speed.

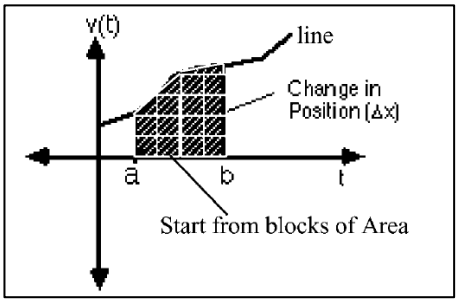

Figure 2. Stroup (2002) approach of "delta blocks" starts from blocks of area, not the own curve.

Then the research focuses on the ways in which the arrangement of the blocks is related to a simulated character's movement. The curve is considered the top edge of an arrangement of blocks. Under this approach, the "shape" of the curve is interpreted qualitatively, without having to start with the ratio, or without trying to insist on a formal measure of reason for change (for example, "second floor"). Students discuss the 'what' fast qualitatively, in terms of the meaning which has the arrangement of blocks.

Research on the learning of qualitative calculation can be defined as the study of the dynamic interaction of the qualitative understandings developed by students about the "how" and the "how fast". These can be used to define a certain level of what might be called competition with the qualitative calculation. To develop understandings of the qualitative calculation, students are increasingly able to differentiate and reintegrate their notions of the "how" and the "how fast".

5. PROPOSAL OF SITUATIONORN DIDTOPRACTICE

The proposal of didactic situation consists in the design of consistent didactic sequences for the three levels of secondary education, which have the aim of developing the variational thinking in students. For its design, ideas linked to the theoretical elements mentioned previously will be supported in technological resources for their development.

Activities for sequences focus on problematic on emptying and filling of containers, and some cases (shooting vertical and oscillatory) motion in order to first of all address the ideas of variation (of a single variable magnitude) and accordingly the covariance (two variable quantities). Such situations shall be the digital video using the software AviMeca, which is characterized by a program designed to matematizar the information that suits the user, within which you can insert a drawing of coordinate axes, and rescue by the points relevant to study is dispersed in the frames of the video data corresponding to the variable magnitudes which are immersed in the phenomenon.

As a complement, the data extracted from the video is kept as a list, which will be used to bring them to the graphic form in the digital environment of GeoGebra, other software with many resources advantageous to work math, in which will be studied and analysed the behaviour of varying magnitudes using applets, which allow to visualize the behavior of variable quantities of a dynamically in the graphics.

Now, described some features that will take into account the design of the sequence, in which we have two phases exemplified: variation and covariation.

For the first phase (variation), will be presented videos of filling and emptying of containers, and some cases of movement, in order for students to develop skills related to the level 0 and the Mental action 0 (AM0, N0) where students will study the following points:

-Perception and identification of varying magnitudes of the phenomenon.
-Quantification of a variable magnitude of the phenomenon.
-Representation of a variable magnitude (numerical, algebraic, graphic).
-Classification of variable magnitude (increasing, decreasing, oscillating).

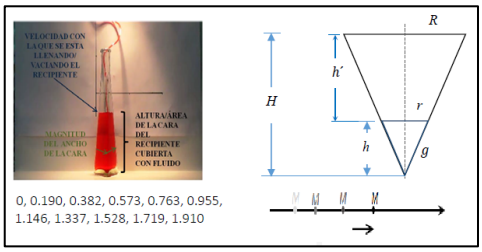

Figure 3. (Video), numerical, algebraic and graphic variation, digital representation of a variable magnitude.

For the second phase (covariance), will also work with videos of filling and emptying of

containers, and some cases of movement, where the students should develop skills related to levels of 1 to 5, using the mental actions 1 to 5 (AM1-5, N1-5) where students will need to develop the following points:

-Perception and identification of the covariates magnitudes in the phenomenon.

-Elaboration of a tabular record showing the coordination of changes in two variable quantities.

-Log construction graphic showing the coordination of changes in two variable quantities.

-Algebraic representation of different magnitudes, as well as its relationship of dependency.

-Coordination of the absolute changes in the graph.

-Tabular representation of the values of varying magnitudes, and their absolute changes, with their treatment to obtain the average exchange ratio.

-Graphical representation of the average changes, identification of concavities and notable points.

Below is an example of the graphical form in the case study of a variable magnitude (mobile point on a number line) and the correlation of two variables magnitudes (Cartesian plane).

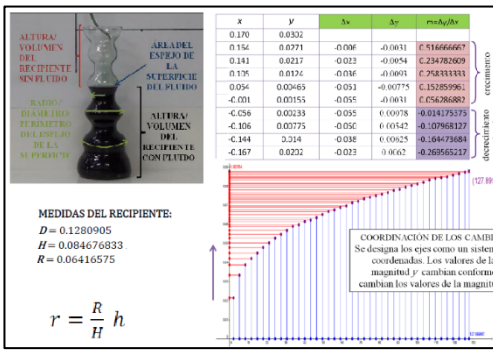

Figure 4. Representing digital (video), numeric, tabular, algebraic and graphic of the covariance of two variables magnitudes.

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