MEANINGS ASOCIATED TO THE GRAPHIC OF A SINUSOIDAL FUNCTION

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Through the trigonometric functions is possible to model periodic phenomena. Most of high school students misunderstand the concepts involved with trigonometric functions, situation that can be solved if we begin to identify and analyze the meanings that students construct about them; in particular, about the sinusoidal functions. This research was designed under this hypothesis, and with the aim of identifying the meanings that students of this educational level construct about sinusoidal functions from interaction with some periodic physical phenomena and mediation of digital tools, such as the motion sensor and a graphing program, because this way it is possible for students to achieve associate meanings to parameters such as period, amplitude, horizontal dilation, among others, of the sinusoidal functions

INTRODUCTION

For many subjects of physics, engineering and even advanced mathematics is essential to have a good understanding of trigonometric functions; however, research on students learning trigonometry is rather sparce (Moore, 2010). In this research, we investigated how students from high school (15-16 years old), construct meanings of trigonometric functions, in particular the sinusoidal function, associating them to the study of some periodic phenomena, in that way the following questions were set:

- Which meanings construct the students establishing connections between periodic physical phenomena and sinusoidal functions?
- How the use of technological tools supports the construction of meanings of sinusoidal functions?

To try to convert the movement into a conceptual object we lean on the production and analysis of Cartesian graphs, however, this research took into account the difficulties which the graphics themselves represent for students (Radford, 2009); for example, they do not reflect iconically the modeled phenomena and, most of the time, the process that leads to them is unknown; so in order to minimize the difficulties mentioned above, we will have as main mediator for generating such graphics a motion sensor and a graphing software. For that reason, we define the following objectives:

- Analyze, describe and systematize the way students relate the periodic physical phenomena with the sinusoidal functions.
- Describe and analyze the arguments used by students to justify and prove conjectures when technological tools are used in the construction of meanings of sinusoidal functions.

THEORETICAL FRAMEWORK

One of the aspects in which the educative mathematic is interested is to determine meanings associated to mathematical terms and symbols. Balacheff (1990), mentions that the keyword in the problematic related to educative mathematics is the meaning, and if we get into the field of meanings, is indispensable to talk about the semiotic, and related aspects to this area; for example, potential that some artefacts may have in terms of help to build mathematical and personal meaning; in order to convert those artefacts into tools which will allow the semiotic mediation (Mariotti, 2010; Moreno, 2014), and with that, the possibility of build meaning of one mathematic object, like the sinusoidal function.

METHODOLOGY

The research design is based on the characteristics of qualitative research. We rely on case studies. Five sessions of two hours each took place with a group of ten students who had just started the upper secondary level, and none of them had any knowledge of the components and characteristics of trigonometric functions.

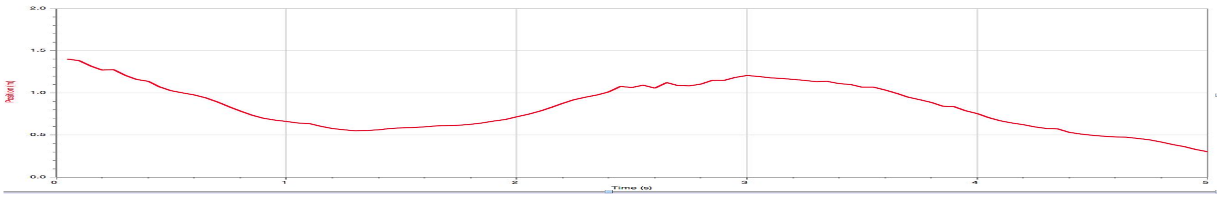
The main activity was to show students a graph of a sinusoidal function, any student did not know the graphic belonged to such functions.

The students were asked to do the necessary to make a similar graph using for a motion sensor.

RESULTS AND CONCLUSIONS

In the introductory activity, with the purpose of the students got their own graphics and understand the meaning of them, students were shown a graphic of a sinusoidal function (without letting them know it belonged to that kind of functions), they were asked to perform the necessary trajectory to reproduce the graphic in front of a movement sensor. On the first session, students walk iconically, following broken lines, or associated each kind of graphic with the walking velocity. After that, one of the students (Rafael) mentioned that the graphic was like “parabolas joint together but inverted”, as a result the discussion by observation and through brain storming, they came to the conclusion that were “repeated parabolas” alternating their concavity, which led to the assumption that what was shown in the graphic was a repetitive or periodic movement.

In the second session, Rafael was able to identify whether " the parables repeat themselves", then the movement in question had to be the " same ". Based on these ideas and continue the discussion after twenty-five minutes (approximately), another student (Carlos) said it could be as dancing, he mentioned that if he "had to take 2 steps forward then more 2 steps back, " Students verified that initially requested graph can be generated this way



(Graphic 1). They identified the movements that had faced in front of the sensor with graph made in the software.

Graphic 1: Graphic generated by Carlos

Interviewer: Why did you think that the movement should be in that way?

Carlos: Because if I got closer (towards the sensor), the graphic went toward 'down' (\ ) ; if I got away the sensor , the grapich went 'up' (/).

In a second activity a pendulum and a spring mass system were shown to the students. For reasons of space, only the pendulum results are reported.

First it was requested to predict, with pencil and paper, the graphic representation of the position versus time of the pendulum movement (Figure 1). Later they observed and discussed the different representations that were generated with the graphing software when the sensor perceived the pendulum, and when the parameters of it were modified.

Figure 1: Prediction of Samantha.

Representing graphically periodic physical phenomena such as the pendulum, the students were able to associate the meanings to the parameters involved in it; although, the meaning is not the institutional, intuitively and according to the student's visualization, they generated an acceptable meaning of the periodicity and amplitude. Regarding to periodicity, students managed to discard or check certain intuitive reasoning such as the mass of the pendulum and the angle of oscillation influences the periodicity, and they verify that the length does affect the periodicity, a bigger length of the pendulum, represented bigger period and viceversa. They mainly associated that the period is the time that pendulum takes in one oscillation and that the periodicity is related, in equal conditions, to the succession of oscillations.

An aspect to highlight is that students establish benchmarks, which means they are comparing parameters.

With regard to amplitude, they associated with the elongation and they understood that although the oscillation is observed physically “from a horizontal perspective, in the graph it is shown in a vertical way. (Samantha, 15 years old).”

They also determined that the vertical offset from the x-axis occurs " graphics that escaped from the x axis " (Diana, 15 years old). Although not associate any meaning to the horizontal displacement, it was necessary that the teacher explained it, it did not register if the students understood this parameter.

Except horizontal displacement, these students are able to determine the value of other parameters involved in the sinusoidal function.

As for horizontal displacement , even though the sensor is the reference point or the origin of the graph is determined by the sensor , students unassociated this produces a time delay relative to the instant when the pendulum passes through the equilibrium position ; however, the use of technological tools such as motion sensor and the computer program facilitates the generation , reproduction and analysis of graphical representations of the periodic phenomena , such as the students found that the graphics are not iconic, or the sinusoidal shape of the graph does not depend on the speed with which the pendulum swings.

In general, the motion sensor and the model of periodic phenomena, the design and guidance of the activity were essential mediators to eliminate some of the difficulties and complexities that representations of the sinusoidal functions have by themselves; as for example the way to generate the graphical representation, allowed to identify and assign meaning to signs or features that represent the parameters of the sinusoidal functions, and when it was necessary to modify these parameters , the students did it, in a relatively simple manner, since they only had to walk in front of the sensor or repeat the pendulum experiment, and check if the predictions or guesses that they established were true or had to be rethought.

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