Mathematics lessons and classroom examples, inspired by articles in newspapers

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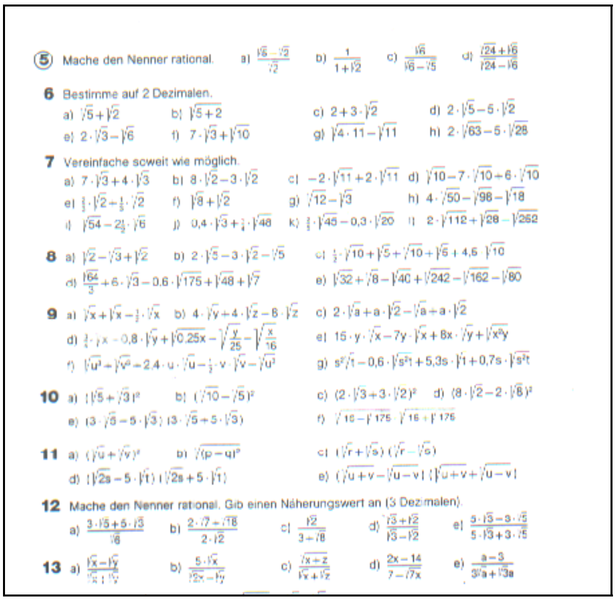
What is the connection between mathematics lessons and articles in newspapers like "Dresden Morning Post", an infamous newspaper in Saxony?
Students usually read such newspapers and so teachers can choice a topic or a problem which has any reference to mathematics. Starting out from the information of the "Morning Post" one should check the teaching curriculum and the teaching subjects. And so you can start to create an interesting problem for the lesson.

Why is it necessary to reform teaching of mathematics? We see tree reasons:

- „rigid" picture of mathematics
- availability of new media
- call for a new assignment culture

You see in this picture a typical side from a German mathematics book. You perhaps cannot recognize everything. This isn't so bad. Simply enjoy the Eroticism of these terms. Imagine a 14-year teenager now: He has so a "big interest" to experienced, what is the result of these terms and roots.



5 Mache den Nenner rational. a) $\frac{\sqrt{6}-\sqrt{2}}{\sqrt{2}}$ b) $\frac{1}{1+\sqrt{2}}$ c) $\frac{\sqrt{6}}{\sqrt{6}-\sqrt{5}}$ d) $\frac{\sqrt{24}+\sqrt{6}}{\sqrt{24}-\sqrt{6}}$

6 Bestimme auf 2 Dezimalen.
a) $\sqrt{5}+\sqrt{2}$ b) $\sqrt{5+2}$ c) $2+3\cdot\sqrt{2}$ d) $2\cdot\sqrt{5}-5\cdot\sqrt{2}$
e) $2\cdot\sqrt{3}-\sqrt{6}$ f) $7\cdot\sqrt{3}+\sqrt{10}$ g) $\sqrt{4\cdot 11}-\sqrt{11}$ h) $2\cdot\sqrt{63}-5\cdot\sqrt{28}$

7 Vereinfache soweit wie möglich.
a) $7\cdot\sqrt{3}+4\cdot\sqrt{3}$ b) $8\cdot\sqrt{2}-3\cdot\sqrt{2}$ c) $-2\cdot\sqrt{11}+2\cdot\sqrt{11}$ d) $\sqrt{10}-7\cdot\sqrt{10}+6\cdot\sqrt{10}$
e) $\frac{2}{3}\cdot\sqrt{2}+\frac{1}{3}\cdot\sqrt{2}$ f) $\sqrt{8}+\sqrt{2}$ g) $\sqrt{12}-\sqrt{3}$ h) $4\cdot\sqrt{50}-\sqrt{98}-\sqrt{18}$
i) $\sqrt{54}-2\frac{1}{3}\cdot\sqrt{6}$ j) $0{,}4\cdot\sqrt{3}+\frac{1}{4}\cdot\sqrt{48}$ k) $\frac{2}{3}\cdot\sqrt{45}-0{,}3\cdot\sqrt{20}$ l) $2\cdot\sqrt{112}+\sqrt{28}-\sqrt{252}$

8 a) $\sqrt{2}-\sqrt{3}+\sqrt{2}$ b) $2\cdot\sqrt{5}-3\cdot\sqrt{2}-\sqrt{5}$ c) $\frac{1}{2}\cdot\sqrt{10}+\sqrt{5}+\sqrt{10}+\sqrt{5}+4{,}5\cdot\sqrt{10}$
d) $\frac{\sqrt{64}}{3}+6\cdot\sqrt{3}-0{,}6\cdot\sqrt{175}+\sqrt{48}+\sqrt{7}$ e) $\sqrt{32}+\sqrt{8}-\sqrt{40}+\sqrt{242}-\sqrt{162}-\sqrt{80}$

9 a) $\sqrt{x}+\sqrt{x}-\frac{1}{2}\cdot\sqrt{x}$ b) $4\cdot\sqrt{y}+4\cdot\sqrt{z}-6\cdot\sqrt{z}$ c) $2\cdot\sqrt{a}+a\cdot\sqrt{2}-\sqrt{a}+a\cdot\sqrt{2}$
d) $\frac{3}{4}\cdot\sqrt{x}-0{,}8\cdot\sqrt{y}+\sqrt{0{,}25x}-\sqrt{\frac{y}{25}}-\sqrt{\frac{x}{16}}$ e) $15\cdot y\cdot\sqrt{x}-7y\cdot\sqrt{x}+8x\cdot\sqrt{y}+\sqrt{x^2y}$
f) $\sqrt{u^3}+\sqrt{v^3}-2{,}4\cdot u\cdot\sqrt{u}-\frac{1}{2}\cdot v\cdot\sqrt{v}-\sqrt{u^3}$ g) $s^2\sqrt{t}-0{,}6\cdot\sqrt{s^2t}+5{,}3s\cdot\sqrt{t}+0{,}7s\cdot\sqrt{s^2t}$

10 a) $(\sqrt{5}+\sqrt{3})^2$ b) $(\sqrt{10}-\sqrt{5})^2$ c) $(2\cdot\sqrt{3}+3\cdot\sqrt{2})^2$ d) $(8\cdot\sqrt{2}-2\cdot\sqrt{8})^2$
e) $(3\cdot\sqrt{5}-5\cdot\sqrt{3})(3\cdot\sqrt{5}+5\cdot\sqrt{3})$ f) $\sqrt{16-\sqrt{175}}\cdot\sqrt{16+\sqrt{175}}$

11 a) $(\sqrt{u}+\sqrt{v})^2$ b) $\sqrt{(p-q)^2}$ c) $(\sqrt{r}+\sqrt{s})(\sqrt{r}-\sqrt{s})$
d) $(\sqrt{2s}-5\cdot\sqrt{t})(\sqrt{2s}+5\cdot\sqrt{t})$ e) $(\sqrt{u+v}-\sqrt{u-v})(\sqrt{u+v}+\sqrt{u-v})$

12 Mache den Nenner rational. Gib einen Näherungswert an (3 Dezimalen).
a) $\frac{3\cdot\sqrt{5}+5\cdot\sqrt{3}}{\sqrt{6}}$ b) $\frac{2\cdot\sqrt{7}+\sqrt{18}}{2\cdot\sqrt{2}}$ c) $\frac{\sqrt{2}}{3+\sqrt{8}}$ d) $\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ e) $\frac{5\cdot\sqrt{3}-3\cdot\sqrt{5}}{5\cdot\sqrt{3}+3\cdot\sqrt{5}}$

13 a) $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ b) $\frac{5\cdot\sqrt{x}}{\sqrt{2x}-\sqrt{y}}$ c) $\frac{\sqrt{x+z}}{\sqrt{x}+\sqrt{z}}$ d) $\frac{2x-14}{7-\sqrt{7x}}$ e) $\frac{a-3}{3\sqrt{a}+\sqrt{3a}}$

We think about a gradual change of the assignment culture in Saxony at present. You should know: Saxony has central school leaving examinations and graphic calculators are obligatorily specified.

To chance the public "picture of mathematics" we defined in our new curricula the following general goals of teaching mathematics:

- Advancement of problem solving competence
- Critical use of reason
- Advancement of the competence of using mathematical language appropriately.
- Advancement of the ability to visualise things
- Advancement of the competence of using basic mathematical objects appropriately

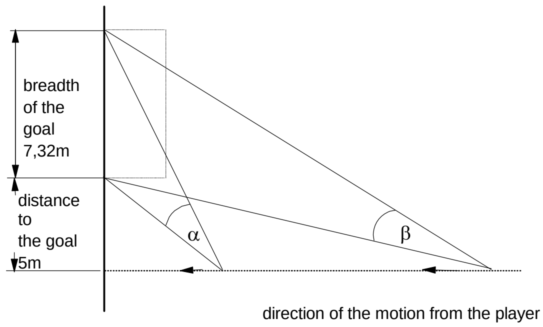
Graphic calculators are obligatorily specified in Saxony. The reasons for using CAS and graphic calculators are the following didactic reasons:

- explorative learning - experimentation
- visualisation
- motivation
- use as a Calculator
- change of assignment culture
- cross-curricular teaching and learning

I would like to illustrate it with some old and some new examples. **Example 1: Football player**

The goal of the example: visualizing, motivating
In October 2001 was a match between Germany and Finland. The result was 0:0 and the press was disappointed and enraged. That was not realy funny.
Especially the German player Oliver Bierhoff was in the centre of the criticism, because he did not hit the goal from a distance of 8 meters.
A student asked "Why 8 meters? Is it more terrible to miss the goal from a distance of 6 meters or 10 meters?"

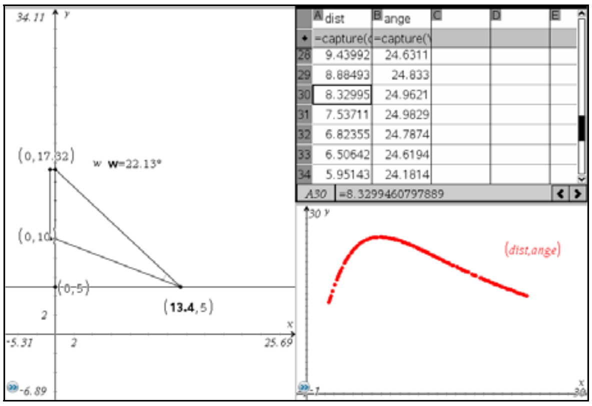
The forward goes along a imaginary line, parallel border of the field in the direction of the opposing goal.
We estimate the distance from this line to the goal about 5m.
The breadth of the goal is 7,32m.



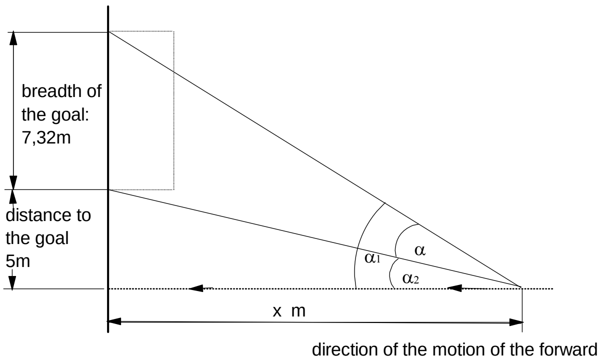
Look for the function: "distance of the forward from the demarcation of the field in line with the goal → angle of the shoot into goal".
short: **distance → angle**

Do exist a maximum for this angle for the shoot into the goal?

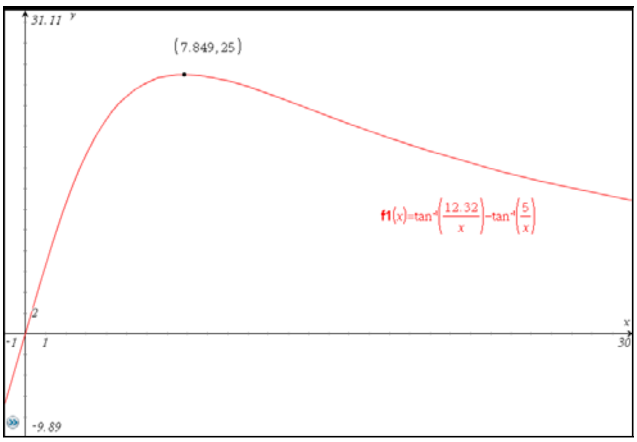
What is the optimal distance for shooting?



We solve it with the TI-Nspire with geometry, spreadsheet and graphic and get:
(Distance for shooting: x-axis, angle: y-axis)



$$\alpha = \alpha_1 - \alpha_2 = \tan^{-1}\left(\frac{12{,}32m}{xm}\right) - \tan^{-1}\left(\frac{5m}{xm}\right)$$



The optimal shooting distance is 8 metres, “Dresden Morning Post” was on the right track!!
The optimal angle is about 25°.

What a surprise!

Example 2: Attention Toads

The goals of the example: open tasks, experimenting, visualizing

If I drive home with my car, I see daily a traffic sign on my way: Speed limitation on 30 km/h and an additional sign on which a frog is shown.
After traffic order in Germany the additional sign indicates those traffic participants to whom the speed description applies.
If a bus is for example shown, then it applies to busses. If a motorcycle is shown, then it applies to motorcycles.
But in this case there is a frog on this sign!

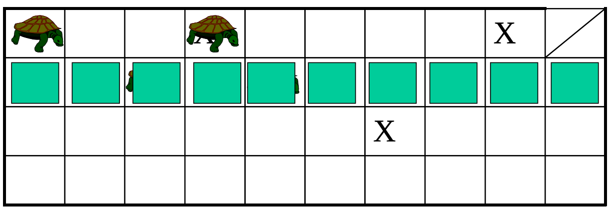
If the police there had stopped me, I would have been very astonished.

However, the cause for the classroom-example was an article in the "Dresden morning post". The danger for the toads crossing the street was described.
A toad needs up to 20 minutes to cross a road that is 7 m wide.

Dresden Morning Post

With students of a 10th class we wanted to discuss the danger for the toads and the use of the sign.
We worked into groups with 4 students each. At first an idea was prepared in the class together:

Underlay e. g. 200m street with a decimeter raster.
How often (in seconds) does the scheme of bouncing change?



The first group calculated:

$$\frac{70\text{dm}}{20\,\text{min}} = \frac{1}{\text{x}\,\text{min}} \Rightarrow \text{x} = 0{,}285\,\text{min} = 17{,}1\text{s}$$

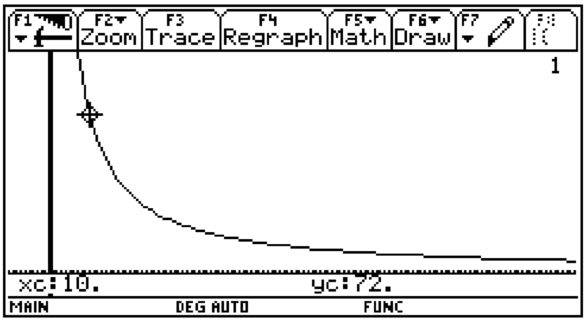
The scheme of bouncing changes every 17 seconds.

How long does a car need to go 200m depending on speed?

$$s\ \ in\ \ s = \frac{200m}{\left(\dfrac{x\dfrac{km}{h}}{3{,}6\dfrac{m}{s}}\right)}$$

At the picture you see the way in dependence of the speed:

$$s = \frac{v^2}{100}\ ;\ \ v\ \ in\ \frac{km}{h}\ ;\ \ s\ \ in\ \ m$$



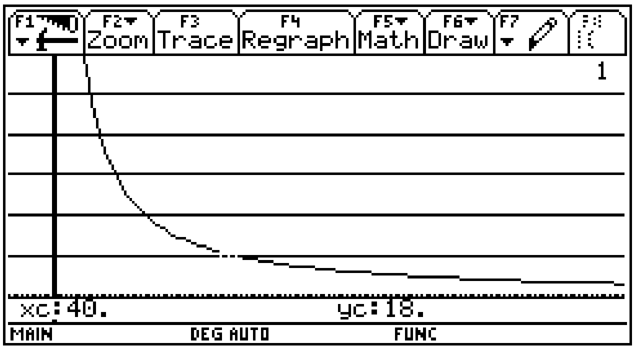
y(1)=200/(x/3.6) in steps of 10 km/h:

x	y1
0.	undef
10.	72.
20.	36.
30.	24.
40.	18.
50.	14.4
60.	12.
70.	10.286

x=0.

How many of the „bouncing schemes“ is the car coasting during this time?

Meter-reading of the intersection at the multiple of 17 says:



For example:
85sec. (5 schemes) at 8,5 km/h
68sec. (4 schemes) at 10,6km/h)
51sec. (3 schemes) at 14,11 km/h)
34sec. (2 schemes) at 21,2 km/h
17sec. (1 scheme) at 42,6 km/h)

The result: **„The more I slow down the more toads I hit.“**

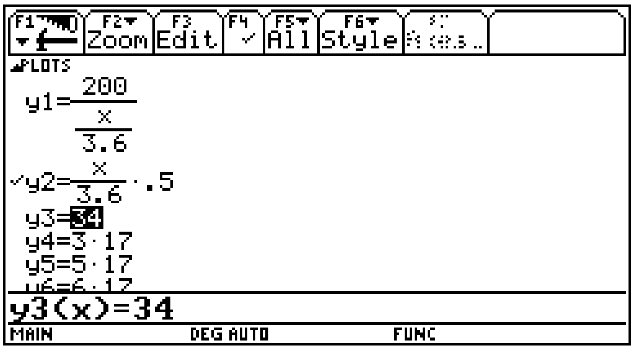
„At 42,6 km/h and faster, it does not matter, there is no difference anyway.“

An other group investigated the question: What influence does the reaction rate of the toad have?

Toads are able to see objects up to a range of 4 meters, to react on them within 0.5 sec. and to jump in case of emergency.

Distance of the car within 0.5 sec.:

$$\frac{\text{v}\left(\text{in}\ \dfrac{\text{km}}{\text{h}}\right)}{3{,}6} \cdot 0{,}5\text{s} \Rightarrow \text{s}\,(\text{in}\ \ \text{m})$$



x	y2
0.	0.
10.	1.3889
20.	2.7778
30.	4.1667
40.	5.5556
50.	6.9444
60.	8.3333
70.	9.7222

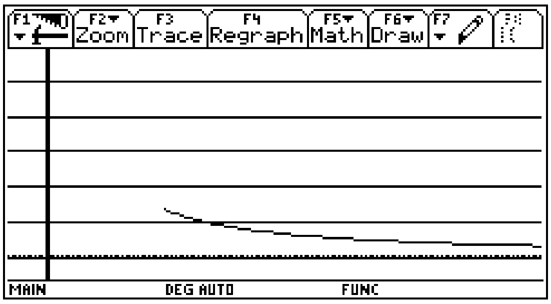
x=0.

This means: The covered distance of the car is 1.38 m at a speed of 10 km/h and so on.
Is this distance smaller than 4 meters, the toad is able to react.

solve (v/3.6*0.5=4,v):

$$v = 28{,}8\frac{km}{h} \approx 30\frac{km}{h}$$

Linking to the former model:



The graph only exists from 30km/h up. There I coast appr. 1.4 „bouncing schemes“ and therefore **hit the highest possible numbe**r of toads.“

An other group investigated the question: What influence does the stopping distance of the driver have?
Stopping distance according to driving school rules:

Speed	Stopping distance in m
10	1
20	4
30	9
40	16
50	25

But there is the question: In which distance is it the driver possible, to see a toad on the street.

Now the pupils called to the Dresden traffic police and get the following answer:

“You must not exclusively see the danger for the toads.
Because of the slobber that is produced by driving over the toads the wheel grip is reduced to an extent that is comparable to aquaplaning. As a result you could lose control over the car. The signs are for your own safety.”

Example 3: Streetcar company

Goals: Visualizing, motivating, experimenting


Schon 5 000 Ermittlungsverfahren in diesem Ja

DVB gnadenlos! Jeder Schwarzfahrer wird angezeigt

“The streetcar company in Dresden is merciless! institute 5000 court proceedings against fare doger!”
In the message the Morning Post called a Passenger without ticket: fare dodger and postulate: “Every passenger should be checked average at least once in three month.”Some facts: Everyday 480 000 passengers use the streetcar. With a honest passenger the company earn 0,40€, with a fare dodger 40€ fine (punishment).

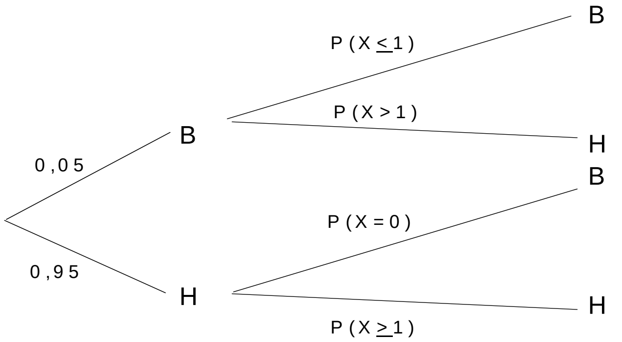
The 456 000 honest passengers pay every day 182 400€, the 24000 fare dodger had to pay 960 000€, if a streetcar-company guards would catch them all.

But:
If all 480 000 passengers would pay honestly, the streetcar-company would only earned 480 000 • 0,40€ = 192 000€.

The Problem is:
The company would have to check the tickets exact so often, that the company get sufficient money and the fare dodger would be preserved.

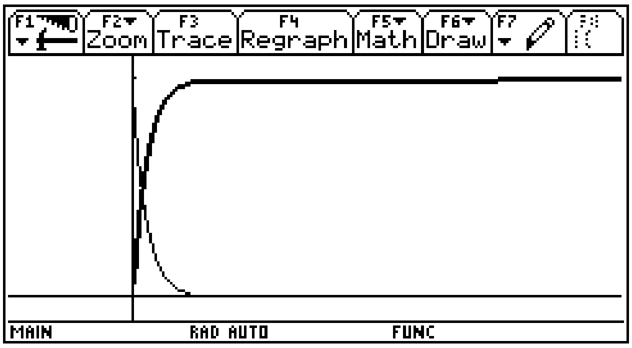
Which number of inspectors is necessary for the biggest profit of the company?

We look at a period of time of three month, nearly 180 trips for a student. Assumptions:At the beginning there are 5% fare dodgers.If a fare dodger would be caught at least two times in this period, he will change into a honest passenger.If a honest passenger would never be controlled in this period, he will change into a black driver in the next time.
X is the number of checks in the period for any passenger. The distribution of X is binomial with n=180 and unknown probability p. p is the probability, that a passenger would be controlled.



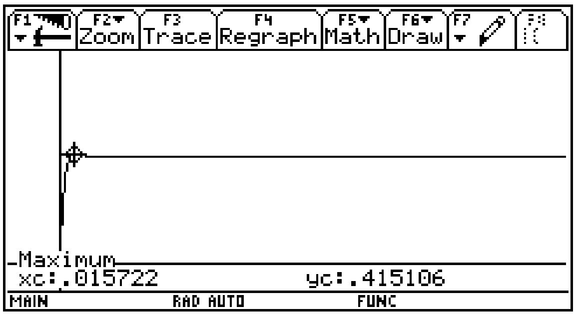
$y1 = 0.05 \cdot binomcdf(180, x, 1) + 0.95 \cdot binomcdf(180, x, 0)$ *and*
$y2 = 0.05 \cdot (1 - binomcdf(180, x, 1)) + 0.95 \cdot (1 - binomcdf(180, x, 0)).$

We examine the functions into dependence of the probability P.



Z: profit of the streetcar-company in the period for any passenger in dependence of the probability p

z_i	0,40€	40,00€	0,00€
$P(Z=z_i)$	y2(p)	$p \cdot y1(p)$	$1-(p \cdot y1(p)) - y2(p)$



For p = 1,6% earn the company the highest profit.

Compare the model and the reality:

	model	reality
number of inspectors	13	30
number of checks (600 per inspector per day)	7800	18000
check-probability p (480 000 passengers)	0,01625	0,0375
average earn of the company per passenger (Z)	0,415€	0,40€
daily earn of the company (480 000 passenger)	199 200€	192000€

Result: If the streetcar-company would dismiss 17 inspectors, they would earn per day 7200€ more.

Example 4: Traffic Jam

Goals: Visualizing, motivating, experimenting

Questions of the “Dresden Morning Post”:

- Why is everyone dawdling?
- Couldn't feel faster if wasn't the truck in the street?

distance = length of the car a + safety distance

Distance = length of the car + $a+\frac{1}{2}\cdot\left(\frac{v}{10}\right)^2$

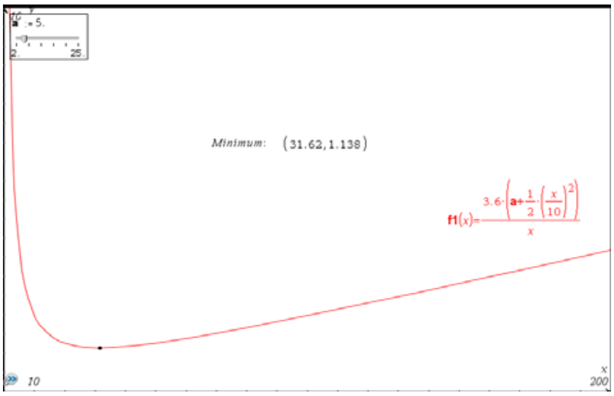
(This is the formula for the safety used on drivers school.)

We examine the connection:

Speed (km/h)	necessary distance of the car	Place is up on 200 m of street
0	5	40.0
10	5.5	36.3
20	7.0	28.5
30	9.5	21.0
50	17.5	11.4
70	29.5	6.8
100	55.0	3.6
130	89.5	2.2
180	167.0	1.2

In which time passes the car the necessary distance of

it´s own ? $t(v,L)=\frac{L}{v}=\frac{3{,}6\cdot\left(a+\frac{1}{2}\cdot\left(\frac{v}{10}\right)^2\right)}{v}$



We examine it with the TI-Npire and find, the optimal speed of 5m-cars is nearly 30km/h. If you observe trucks with a lengths of 20m, the optimal speed is nearly 60 km/h.

REFERENCES

Christoph Drösser: Der Mathematik-Verführer,:- Rowohlt Taschenbuch Verlag,.- Hamburg 2008.

BIOGRAPHICAL NOTES

Dr. Rainer Heinrich is an head of Division in the Saxony State Ministry of Education. He managed the higher Schools in Saxony with about 10.000 teachers. He also is responsible for the central school leaving examinations.
Till 2006 he was teacher for mathematics and geography and was also trainer in the teachers training. Her researched in the field of use of technological tools on students' problem solving experiences and the influence the use of these tools has on their perceptions of mathematics.