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**ARE WE PREPARING
STUDENTS FOR 2075?**



Nikolas Hegedus

Age 10



He will retire in 2075

What will his world look like then?

Are we educating him for that future?

SOME BASIC ASSUMPTIONS

- ▶ Representationally Rich - do not make the base mathematical object more abstract
- ▶ Interactive - if we are not exploiting computational power, resolution, and color then why use such expensive systems?
- ▶ Cognitive with respect to accessibility
- ▶ Multimodal in terms of interface and appreciative of theories of learning, activity theory and pedagogy

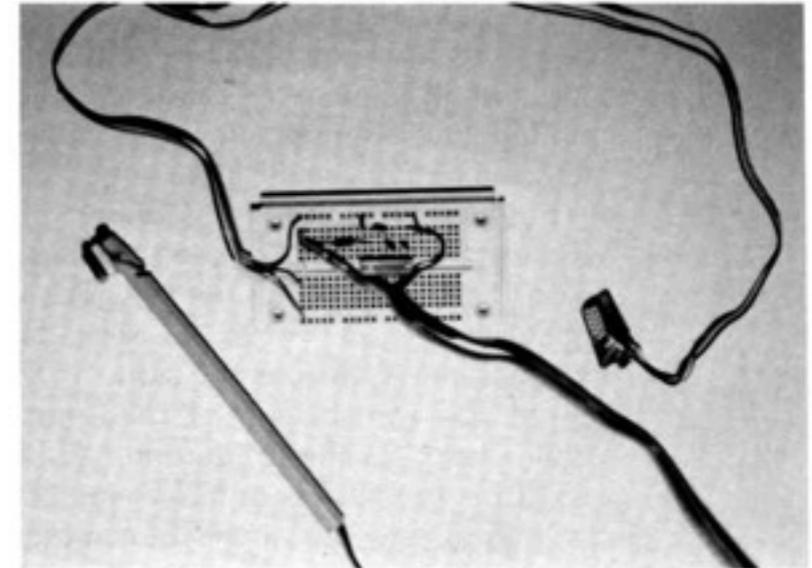
SOME BASIC ASSUMPTIONS

- ▶ Needs to co-evolve with software & curricular development and applications
- ▶ Not just peripheral but deeply embedded in the surrounding environment

EXAMPLE: SCALING HARDWARE

1. Light Pens - early pointing device where were used with a raster-display (electron beam scanned screen)
2. Elementary and Sophisticated Mice
3. Key-stroke devices (hand-helds, mobile phones)
4. Scribable Tablet based device (e.g. Palm/Table PC)
5. Interactive Whiteboard

Figure 6-4. The Completed Light Pen



The entire light pen system, comprised of the breadboard, the ink-pen-based light pen, and the 9-pin connector, can be used for a multitude of programming applications, from drawing programs to menu selection.



SCALES OF ECONOMY: CASE STUDY

- ▶ Scaling hardware leads to different marginal returns not just based upon immediate user feedback (click-click-do)
- ▶ Compare the use of a \$10 mouse with a \$2000 Interactive Whiteboard
- ▶ Things change as you get bigger but it does not mean more power per \$
- ▶ Think about what it buys you educationally, environmentally, communicatively

POINTING IS NOT NECESSARILY PERIPHERAL BUT CAN BE ENVIRONMENTAL WHICH HAS HUGE IMPACTS ON MATHEMATICAL EDUCATION:

- ▶ Pointer as an environmentally constrained device (ink pen)
- ▶ Pointer as operating system-constrained device (mouse)
- ▶ Pointer as a user-constrained device (tablet)
- ▶ Pointer as an embodied, gestural, public action (IW)

HEGEDUS, KAPUT & LESH (2007):

- ▶ “Even though technology has provided new tools to design and discover mathematical ideas, these have not evolved at the same rate as communication infrastructure and have not deeply interplayed with their evolutions. {...} not only does technology constitute the materials, which they operate within but also the social conditions with which such operations occur.
- ▶ As technology becomes more infrastructural and exploits communication power, the interaction of learners and educators will change and the shift in mathematical discovery and problem-solving skills will take on a more natural sociological feel.”

The development of portability of information and knowledge

Clay



Papyrus



Paper



Screen



Biological mini-Screen

As media became more malleable man's efforts to communicate evolved in more expressive ways



MULTIMODALITY

**TO TOUCH AND
FEEL MATHEMATICS**

MODALITIES

- ▶ Input: Humans provide input (to the computer) through movement, action, device, sensors
- ▶ Output: Humans experience an output (from a computer) through a sense (e.g. visual, force feedback)

5 PRIMARY MODALITIES

- ▶ See
- ▶ Hear
- ▶ Touch
- ▶ Smell
- ▶ Taste

HAPTIC MODALITIES

- ▶ Touch, tactile
- ▶ Proprioception: acknowledging where our limbs and body is in space in relation to each other, essential in moving and effort needed

SECONDARY MODALITIES

- ▶ Thermoception: the sense of hot or cold
- ▶ Nociception: sense of pain
- ▶ Equilibrioception: vestibular sense; to sense balance and acceleration

TECHNOLOGY IN MATHEMATICS EDUCATION

- ▶ Primarily Visual
- ▶ [S]tudents can interpret visual, auditory and haptic displays to gather information, while using their proprioceptive system to navigate and control objects in their synthetic environment (Dede, Salzman, Loftin, & Sprague, 1999)

CENTRAL HYPOTHESIS

Learners of all ages can improve their mathematical knowledge and skills by learning within a multimodal environment that is structured by activities that utilize the technological affordances in mathematically meaningful ways.

BACKDROP

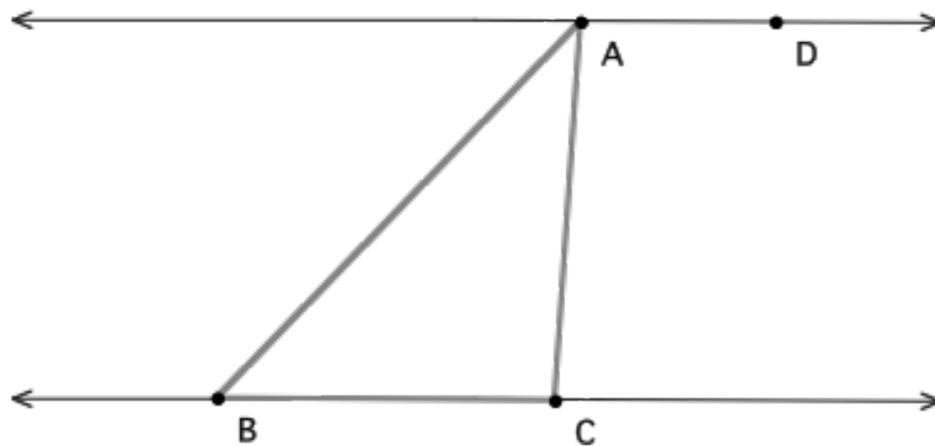
Multimodal interaction has evolved in various research areas and applications including computer vision/visualization, psychology and artificial intelligence with increasing use in education particularly in early learning and developmental psychology

HANS FREUDENTHAL

WE SHOULD NEVER THINK ABOUT WHAT KIND OF MATHEMATICS A CHILD CAN LEARN, BUT ABOUT THE KIND THAT CAN CONTRIBUTE TO THE DEVELOPMENT OF HER/HIS HUMAN DIGNITY.

EXAMPLES: REFLECT ON MATHEMATICAL AFFORDANCES

- ▶ Haptic Force feedback - Assigning mathematical attributes to a modality (e.g. area vs texture as an aesthetic)
- ▶ Sketchpad Explorer on the iPad - Multi-touch or multiuser input





SENSABLE'S OMNI HAPTIC DEVICE IS A DESKTOP HAPTIC DEVICE

VIDEO



Erase Traces Race Tools New object

Run Reset

time = 0.00



Point 2

Point 1

SKETCHPAD EXPLORER - COMBINING VISUAL WITH MULTI-TOUCH (MULTI-INPUT)

HOW SHOULD YOUNG CHILDREN BE DOING MATHEMATICAL PROBLEM-SOLVING IN THE FUTURE?

- ▶ Early Introduction
 - ▶ Exposure
- ▶ Technology Use
 - ▶ Participatory/Collaborative role
 - ▶ Interaction of teachers/students/technology

- ▶ Engagement (immersion)
 - ▶ Mathematical affordances
 - ▶ Open-ended tasks
 - ▶ Role of non-scholastic language

Hegedus, S. (2013). *Seeing little children do new mathematical problem-solving with new technologies: Future perspectives*. In L. Moreno-Armella & M. Santos (Eds.) (2013). Special Issue: International Perspectives on Problem Solving Research in Mathematics Education. *The Montana Mathematics Enthusiast*, Vol. 10 (1&2)

MEDIATION

- ▶ Semiotic Mediation
- ▶ Human interaction (if man is a sign then it could be a form of semiotic mediation as well - debate)

Güçler, B., Hegedus, S., Robidoux, R., & Jackiw, N. (2012). *Investigating the mathematical discourse of young learners involved in multi-modal mathematical investigations: The case of haptic technologies*. In D. Martinovic, V. Freiman, & Z. Karadag (Eds.), *Visual Mathematics and Cyberlearning* (pp. 97-118). New York: Springer

HOLLAND ET AL. (2004) OUTLINES THE ROLE OF A MEDIATING DEVICE:

- ▶ A typical mediating device is constructed by the assigning of meaning to an object or a behavior. This symbolic object or behavior is then placed in the environment so as to affect mental actions. (p.36)

VYGOTSKY (1980)

- ▶ Activity structures the social environment of interaction and the very behavioral routines of members of that environment
- ▶ Such embodied actions of pointing, clicking, grabbing and dragging parts of the geometric construction also allows a semiotic mediation between the object and the user who is trying to make sense of, or induce some particular attribute of the diagram or prove some theorem.

HEGEDUS & MORENO-ARMELLA (2010)

- ▶ "...Co-action (in this case with the artefact) as the main characteristic of the mode of existence of human activity. This type of co-action generates a zone of proximal development (ZPDA) for the artifact that will be realized (in human activity) as long as the artifacts are stable and full of visibility in the cultural/technological medium.
- ▶ Mediated activity eventually leads to reflecting on the medium itself, the artifact. The stable presence of the artifact within an activity medium, its continued visibility, works as a guarantee that that technology will have the opportunity to evolve."

10 DESIGN PRINCIPLES

- ▶ For teachers, educators, faculty, researchers, curriculum and software designers

10 DESIGN PRINCIPLES

1. Executable Representations
2. Co-action
3. Navigation
4. Manipulation and Interaction

#5 VARIANCE/INVARIANCE

- ▶ Understanding how quantities vary or not under certain interactions allows a large wealth of mathematics to be explored. In addition to annotations such as measurement, linking variation to force feedback allows meaningful feedback to help guide the learner to make sense of important features, co-varying relationships or invariance.

10 DESIGN PRINCIPLES

- ▶ #6 Mathematically Meaningful Shape & Attributes
- ▶ #7 Magnetism
- ▶ #8 Pulse/Vibration
- ▶ #9 Construction
- ▶ #10 Aggregation

HEGEDUS & TALL (2015):

- ▶ Such technologies can be thought of as cognitive extensions of the biological self (i.e. thinking through our fingertips), or the drawing closer of our natural biological senses and the abstraction of mathematical thinking through the mediating effect of multimodal technologies.

Hegedus, S. & Tall, D. O. (2015). *Foundations for the Future: The Potential of Multimodal Technologies for Learning Mathematics*. The Third edition of the Handbook of International Research in Mathematics Education, L. English & D. Kirshner (Eds.). Taylor & Francis

HEGEDUS & TALL (2015):

- ▶ “This notion of simplification is both mathematical (where mathematicians and teachers make sense of the mathematical structures by formulating new concepts that can be manipulated as mental entities) and also personal (where learners make sense of the compressed entities in flexible ways).”
- ▶ “This might not be possible for the whole spectrum of learners and future works needs to address this critically as more curriculum is developed that integrate such technological affordances in meaningful ways.”

HEGEDUS & TALL (2015):

- ▶ "...we also see the need to work with multimodal technologies for specific categories of learners, such as those with special educational needs"
- ▶ "Depending on the stage of their careers, teachers might be faced with re-thinking how such technologies can enhance the learning environment or even transform the very nature of the classroom in terms of how mathematics is re-described."
- ▶ Representational re-description (Tomasello, 1999) means rethinking teacher preparation and development (long-term)

CLOSING STATEMENTS FOR TEACHING

- ▶ Where is the sole agent for learning?
- ▶ Who is learning?
- ▶ The teacher or the student?
- ▶ Is a teacher at the critical juncture or rubicon of the technological civilization in mathematics education?

Please remember Nikolas and 2075





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