CONTS

The application of CAS in instruction of calculus*

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Abstract: The article provides information on the experience in the application of computer algebra system (CAS) in instruction of calculus in teachers' training in the Czech Republic at the University of South Bohemia. Using technologies in a proper way could lead to a better understanding of mathematical problems, to a higher creativity and thus to a more efficient education of mathematics. On the other hand without knowing the issue, the application of mathematical software often results in a lot of errors as well as an erroneous interpretation of the results. Some experience in the project oriented problems, whose solution is supported by MAPLE, are described.

Key words: Computer algebra system, mathematics instruction, examples.

Introduction

Presently the development of computer science also considerably influences instruction of mathematics. The innovations of instruction methods are often just based on the utilisation of computers. Through computer technologies the student learns to understand better the basic mathematical terms. A suitable application of computer technologies not only results in a more thorough understanding of a taught issue, but also in animation of instruction process and in an increased interest in mathematics.
In spite of all of the above advantages, the role of computer in instruction of mathematics is continuously discussed. The opponents of the computer-aided instruction refer to a mindless application of mathematical programs due to which the students are convinced that algorithm can be applied to everything.
The article provides short information on teachers' training in the Czech Republic at the University of South Bohemia. The instruction proceeds with the application of mathematical software, specifically in Program Maple. MAPLE is a computer environment developed at the university in Waterloo, Canada, for an easier application of mathematics. It ranks among interactive programs that, contrary to standard programs for numerical calculations, model mathematical operations with symbolic expressions.

Using Maple in education of calculus

Using Maple in education of calculus can be divided into three stages, namely

- Introduction in Maple – familiarisation with basic statements, solution to simple tasks
- Solution to more complicated problems
- Solution to examples with the application of the programming language of Maple program

1) Introduction in Maple

In the first and second workshop lessons, the students are familiarised with basic statements in Maple program by means of the manual and Help menu. They improve their knowledge and skills step by step by solving given problems and experimenting. The advantages of using Maple in the lectures of mathematics such as modelling, experimentation, visualization, animation, all of these performed in a simple way, are well known features. After being familiarised with the basic statements the students get simple tasks. For examples

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Example 1

One dose of medicine was punctured into the patient's body. The quantity Q of medicine in the patient's body decreases each hour at a rate that equals to 4% of the quantity of medicine in the body so that it holds true

$$Q'(t) = -0.04Q(t),$$

where t is time in hours.
Initial dose of medicine was 3 millilitres, i.e. Q(0)=3. Find a model that describes the quantity of medicine in the patient's blood and find out how many millilitres of medicine in the patient's blood will be after 10 hours.

Solution

Solution without the application of CAS

In a solution to the given problem the student shall solve the partial tasks as follows

- To solve differential equations Q´(t) = -0.04Q(t) with initial condition Q(0)=3 (the differential equation is solved by the variables separation method)
- To determine functional values of function Q acquired by the student through the solution to the above differential equation, in point t = 10, i.e. Q(10)

Solution with the application of CAS

In a solution by means of Maple program it is sufficient for the student to know two orders, i.e.

- The order for calculation of differential equation with initial condition:

```
eq:=diff(Q(t),t)=-0.04*Q(t);
pp:=Q(0)=3:
dsolve({eq,pp},Q(t));
```

$$Q(t) = 3\,e^{\left(-\frac{t}{25}\right)}$$

- The order for calculation of functional value:

```
evalf(3*exp(-1/25*10));
```

$$2.010960138$$

Thus the model describing the quantity of medicine in the patient's blood is based on equation $Q(t) = 3\,e^{\left(-\frac{t}{25}\right)}$ and the quantity of medicine in the patient's blood after 10 hours is 2.01 millilitre. For a better illustration we can, by means of Maple program, plot the graph of function $Q(t) = 3\,e^{\left(-\frac{t}{25}\right)}$ by the order

```
> plot(3*exp(-1/25*t),t=-40..40,Q=-1..6);
```

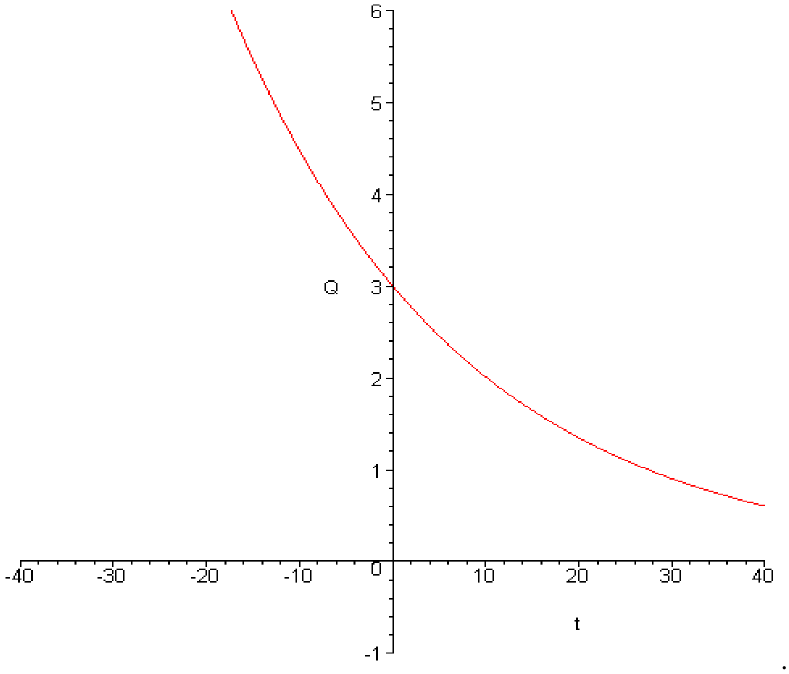

Fig. 1

The above example can be incorporated into the group of tasks solution to which without the computer application is not too difficult; a solution to tasks only requires easy learnt procedures. In the tasks of this type the role of computer consists in the fact that the student forms a better idea of the result, e.g. by means of geometric interpretation of the result. It is evident that the application of computer technologies results in a better understanding of a taught issue.

2) ***Solution to more complicated problems***

In the second phase of instruction the students solve the examples that already require more thorough knowledge of theory and its understanding. In solution they apply Maple program. The problem is solved "step by step" in the same procedure as in a solution by means of "pencil and paper". Maple is only used for partial calculations – the calculations that only require taught procedures. The aim of the computer-aided instruction of mathematics is to develop the feeling of the students for estimation of the solution, their ability to resolve a non-standard task, to develop their functional thinking. Everything is demonstrated on the example.

Example 2

Decide uniform convergence of functional series

$$\sum_{n=1}^{\infty} arctg\left(\frac{2x}{x^2+n^3}\right).$$

Solution

Solution without the application of CAS

In determination of uniform convergence the student shall solve partial tasks as follows:

- Determination of the uniform convergence criterion (Weirstrass criterion (see [3]))
- Detection of convergent numerical series $\sum_{n=1}^{\infty} a_n$ for which it holds true

$$(\forall \ n) \quad (\forall x \in R) \quad \left| arctg\left(\frac{2x}{x^2+n^3}\right)\right| \le a_n \quad .$$

In determination of convergent numerical series $\sum_{n=1}^{\infty} a_n$ the student shall

- Determine the maximum of function $f_n(x) = arctg\left(\frac{2x}{x^2+n^3}\right)$ due to parameter n
- If function $f_n(x) = arctg\left(\frac{2x}{x^2+n^3}\right)$ achieves the maximum in point x=a, then the student shall prove that series $\sum_{n=1}^{\infty} arctg\left(\frac{2a}{a^2+n^3}\right)$ converges.

From the above text it is evident that the application of technology in instruction needs necessarily very good knowledge of the relevant theory. No mathematical software facilitates a solution to the given task without this necessary precondition. Based on his/her knowledge, the student proposes solution, orientates himself/herself in the problem, experiments; mathematical software helps him to search for a solution. The student brings a creative thinking to the whole process.

The application of CAS in a solution to the given problem

After analysis of the given problem the student finds out that the solution consists in finding numerical series. For searching for this series the student utilises Maple program in which he/she sets the orders as follows:

```
> diff(arctan(2*x/(x^2+n^3)),x);
```

$$\frac{\dfrac{2}{x^2+n^3} - \dfrac{4\,x^2}{(x^2+n^3)^2}}{1 + \dfrac{4\,x^2}{(x^2+n^3)^2}}$$

```
> solve(2/(x^2+n^3)-4*x^2/(x^2+n^3)^2=0,x);
```

$$n^{(3/2)}, -n^{(3/2)}$$

```
> limit(arctan(2*x/(x^2+n^3)),x=infinity);
```

$$0$$

```
> limit(arctan(2*x/(x^2+n^3)),x=-infinity);
>
```

$$0$$

```
> evalf(arctan((2*n^(3/2))/((n^(3/2))^2+n^3)));
```

$$\arctan\left(\frac{1}{n^{(3/2)}}\right)$$

```
> evalf(arctan((-2*n^(3/2))/((n^(3/2))^2+n^3)));
```

$$-1.\ \arctan\left(\frac{1}{n^{(3/2)}}\right)$$

It is evident that numerical series is $\sum_{n=1}^{\infty} arctg\left(\frac{1}{n^{3/2}}\right)$. Now the student shall prove its convergence based on a limit comparative criterion, i.e. the student searches for convergent numerical series $\sum_{n=1}^{\infty} a_n$, for which it holds true $\lim_{n\to\infty} \frac{arctg\left(\frac{1}{n^{3/2}}\right)}{a_n} = A \in (0,\infty)$. In this case $\sum_{n=1}^{\infty} a_n = \sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$.

In searching for the limit the student again utilises Maple program and sets the order

```
> limit(arctan(1/(n^(3/2)))/(1/(n^(3/2))),n=infinity);
                                  1
```

Due to the fact that the limit is equal to one, functional series $\sum_{n=1}^{\infty} arctg\left(\frac{2x}{x^2+n^3}\right)$ uniformly converges $(\forall x \in R)$.

▪

Below you can find some examples whose idea consists in the fact that during their solution the students cannot search for a solution according to learnt algorithms incorporating standardised solutions, but they have to orientate themselves in the problem. The computer is only a guide for a more rapid receipt of results.

Examples

1. Determine the surface area limited with curves set by equations

$$y = \frac{x^2}{x^2-4}, \quad y = x^2 - 3$$

and lying between straight lines x=2 and x=-2.

2. The function *f* is given:

$$f : f(x) = \sqrt[3]{x^3 - 6x} \; .$$

a) Determine domain, odd arrangement, possibly even arrangement and limits in extreme points of the domain.
b) Detect the first derivative of function *f* in all points of its domain, monotony intervals and local extremes.
c) Draw up graph of function.
   Compare the results received in a) and b) with the graph of the given function received on computer

3. Prove inequality

a) $(\forall x > 0) \qquad x > \ln(1+x)$

b) $(\forall x) \qquad \left(e^x + e^{-x} \geq 2 + x^2\right)$

3) Solution to examples by means of programming language of Maple program

After the student was familiarised with basic orders in Maple program and went through the phase of instruction when he/she solved examples "step by step" with the utilisation of Maple program and brought creative thinking to the solution, he/she can pass to the third phase of the instruction process. In this phase he/she utilises programming language Maple, i.e. creates own procedures. This can be demonstrated on the easy example as follows.

Example 3

Draw the graph of function given by formula

$$
\begin{aligned}
f : f(x) &= \sqrt[3]{x^3 - 6x} & x &\in \left\langle -\sqrt{6}, 0 \right\rangle \cup \left\langle \sqrt{6}, \infty \right) \\
&= (x + \sqrt{6}) \cdot \ln(-x) & x &\in \left(-\infty, -\sqrt{6} \right) \\
&= (\sqrt{6} - x) \cdot x & x &\in \left(0, \sqrt{6} \right)
\end{aligned}
$$

Solution

Solution without the application of CAS

For plotting of graph of the given function the student shall execute a complete course of function. This means to determine domain of the function, even arrangement, possibly odd arrangement, limits in extreme points of the domain, monotony intervals, local and global extremes, convexity and concavity intervals, inflection points. In terms of calculation, this procedure is very demanding, thus it is advantageous to apply Program Maple.

Solution with the application of CAS

For plotting of graph of the given function we use Maple program. The student shall utilise programming language of this program and create the procedure illustrating respective graph of function. The procedure can look e.g. as follows

```
> w:=proc(x) if (x^3-6*x)>0 then (x^3-6*x)^(1/3) elif (x<-sqrt(6)) then
((x+sqrt(6))*ln(-x))else ((sqrt(6)-x)*x) fi end;
 plot(w,-5..5);
```

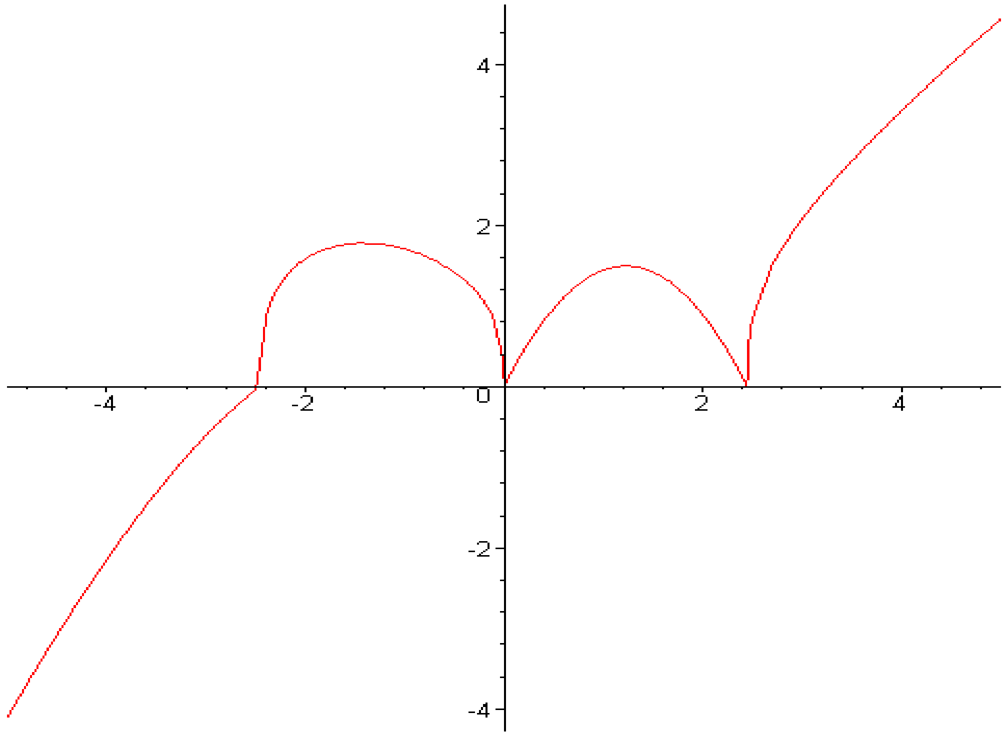

Fig. 2

After completion of all of the above phases of mathematics instruction by means of Maple program the student is ready to work with this mathematical software. Now it depends on him/her how he/she utilizes own knowledge in a solution to the given problems.

Conclusion

New software products and information technologies bring quite new problems, but also new challenges especially in didactics and teachers' training. The aim of the computer-aided instruction of mathematics is to develop the feeling of the students for estimation of the solution, their ability to resolve a non-standard task, to develop their functional thinking. It is necessary to focus preparation of the students on the fact that they should not search for the solution to the problems according to learnt algorithms, into which they have incorporated standardized solutions, but that they should orientate themselves in the problem, model, express their opinions and defend them.

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