

## REPRESENTATIONS AND GRAPHIC CALCULATOR IN MATHEMATICAL TEACHING. A STUDY WITH CALCULUS TUTORS<sup>1</sup>

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*A group of mathematical tutor's productions, who participated in a formation program, based on the use of representation systems and graphic calculator in the teaching and apprenticeship process of differential calculus, are analyzed. The mentioned program was implemented through a course- workshop. The participants in the study were mathematical tutors from the basic cycle of the Economical and Social Sciences Faculty at the University of Carabobo, La Morita Nucleus in Maracay, Venezuela. The participants productions, withdrawal from their note books and reflections emitted by the tutors, at the end of every working session, are analyzed from a qualitative perspective. These productions analysis, concluded that the tutors showed up changes in the Didactic Knowledge, expressed in their reflections and productions during the activities carried on. It likewise, manage the participants to apply the representation systems, systematically, in the resolution of problems with the graphic calculator assistance, and also identified options for the differential calculus teaching.*

**Key Words:** Graphic Calculators, Representation Systems, Mathematical Tutors, Differential Calculus, Didactic Knowledge.

### INTRODUCTION

The development of this research wants to analyze the didactic knowledge derived from the implementation of a formation program that integrates, through the differential calculus, the use of the graphic calculator and the representation systems in the mathematical tutor's formation, at the economics and social science Faculty (FACES), at the University of Carabobo (UC). In order to the total analysis, we started from a theoretical structure supported on the following four dimensions: conceptual, cognitive, formative and social (Rico, 1997a, 1997b). This theory considers that the didactic knowledge of the mathematics topics must be based on the representation systems (Janvier, 1987, Duval, 1995), the modeling (Niss, Blum & Huntley, 1991; Houston, Blum, Huntley & Neill, 1997; Ortiz, 2002), errors and difficulties (Borassi, 1987), phenomenology (Freudenthal, 1983), the history of mathematics (Fauvel, 1991) and materials and resources. The study was conducted at the University of Carabobo, Venezuela, as part of exposed activities by the Mathematics I Cathedra for the academic year 2003-2004.

In the course of this research, we choose to use the TI-92 plus graphic calculator (GC), which is included as a didactic resource in the process of teaching and learning mathematics. The mathematical content involved was differential calculus, which offers possibilities for the use of several representation systems on the introduced concepts and properties like problem solving too (Tall, 1991; Hitt, 2000; Bedoya 2002).

The selection of the differential calculus is based on the actual curriculum at FACES, where is founded that in the subject of Mathematic 1, of the second academic semester, which belongs to accountant and administration careers, the limit's topics must be integrated, continuity and derivation of real functions of real variable, just like the problem resolutions used on the economics and social sciences.

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The relevancy of this research comes from the actual tutor's regulations at the University of Carabobo (1994), on consideration that in this mentioned regulation is given to the tutors, the teaching personal condition in formation, which main function is cooperate on the teaching and research works. In our specific case to cooperate with the mathematics' teaching at FACES, in which they don't have any didactical formation. These tutors are selected students at FACES through a public competition where is considered their good academic expedient and so they must take a written test and an oral test, both of them designed and applied by a court integrated by three teachers of the mathematics cathedra. In order to make the research was created a program which contains the representation systems, the graphic calculator and the differential calculus in the elaboration of the didactics activities.

The formulated questions were the followings:

Which didactic knowledge do the mathematical tutors develop during the handling and incorporation of the graphic calculator on didactic activities, and in which manner do they integrate it in their professional knowledge?.

Which is the way used by the mathematical tutors for the use of the representation systems and in which way do the work on it?

What didactic potentialities does the differential calculus give in order to establish entails and relations between the calculator and the representation systems during the mathematical tutor's formation?

## METHODOLOGY

This research was developed with eight subjects, which followed during 18 hours (6 set of 3 hours each one) the implantation on a formation program, using a course-workshop and the design is based on the representation systems and the graphic calculator as a resource on a mathematical context of differential calculus. The subjects in this research were mathematical tutors, and they were participating voluntarily.

This work is made inside the methodology in the case study (Yin, 1987). The focus of the research is based on the subject's production who worked during the implantation of this program.

In this research are considered the mathematical tutors' productions on relation with the graphic calculator's use and the representation on the differential calculus's teaching, the instrumental handling of the graphic calculator and its articulation with the representations, as well as, the employment of those persons in order to work on match teaching and learning home works, we can say: the didactic knowledge of the mathematical tutor.

The analysis of the productions were made taking as basement; the identification of three important moments during the course-workshop: the first set, from second to fifth, and the sixth set.

Beginning from the analysis, the abilities' development was identified in order to solve problems; attending to several systems of representations and the connections between them, the most important aspects arose on discussions and reflections about the problems, the critical evaluation of every part on the developed activity, oral and written communication abilities and abilities to work in groups. Concerning the support of the GC, as a didactical resource, its use was analyzed for the comprehension of the mathematic concepts and properties of those given situations in the design of didactic activities. We take care too on the utilization of calculus possibilities, experimentation, visualization and contrast of possible results we can get using the GC according with Kutzler (2000).

## RESULTS

### *Mathematical Tutors' situation at the initial moment*

The tutor's initial situation can be enclosed in the following general aspects:

1. They are opened to the employment of the GC, but they keep a modest position about the GC use by the students.
2. They have relative ability to propose important situations for the students. They prefer to state exercises and similar problems to those which appear in the study book of the cathedra.
3. They keep the conduction's scheme of the class dominated by the teachers.
4. Limited initiative when they propose assessment's activities.

### *Mathematical Tutors' situation at intermediate moments*

The target of the second set was to use basic commands for the use, and the graphic calculator handling, through practical exercises related with functions and representations systems.

One of the proposed situations in the second implementation set of the program was the following, oriented on using the representations capacities of the GC:

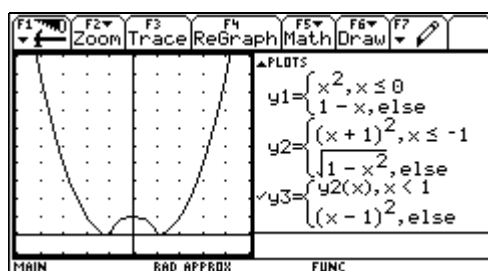
*Represent each one of the following functions:*

$$\text{a) } f(x) = \begin{cases} x^2 & \text{si } x \leq 0 \\ 1-x & \text{si } x > 0 \end{cases} \quad \text{b) } g(x) = \begin{cases} (x+1)^2 & \text{si } x \leq -1 \\ \sqrt{1-x^2} & \text{si } -1 < x < 1 \\ (x-1)^2 & \text{si } x \geq 1 \end{cases}$$

### *Direct resolution using GC*

In this case we have the use of the GC but without previous introduction to its visualization; in fact, we let it "explain" by itself an example of this case is represented by the tutor's productions P2 and P3. This last one shows his answer, with the view screen's support to the others tutors, showing finally the figure 1.

Figure 1



In the answer of the figure 1, are showed the symbolic representations of  $f$  and  $g$  (at the right side) like this graphic of the function  $g$  (at the left side). The graphic calculator dominion can be appreciated by P3, to answer the formulated questions with its use. First of all you see functions editor's know ledges (¥ [W]). Then the scales handling for the screen's configuration (¥ [E]) and the used of graphic commands (¥ [R], O y ). We can see too the ability to show functions in parts in the GC, as in its symbolic representation as its graphic representation. Generally the tutors in this activity just attended to the symbolic and graphic representation. In this last production was determined the GC technical dominion on graphic functions but it was not used to make conclusion about the used representations, this point could has been conducted to make new conditions and new functions.

### Detailed resolution using GC

In this case, mathematical tutors were trying to explain their reason's details, and so they incorporated the GC in their productions and enter upon the use of graphic representations but the used several functions in the editor, this means that, three functions were defined and showed simultaneously. This strategy was used by the tutors P6 and P7 as it is appreciated in the graphic figure 2. We could see too, some of the tutors, before the work's beginning on the GC, they made some drawings on paper.

Figure 2

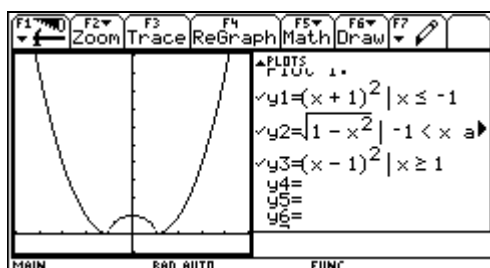
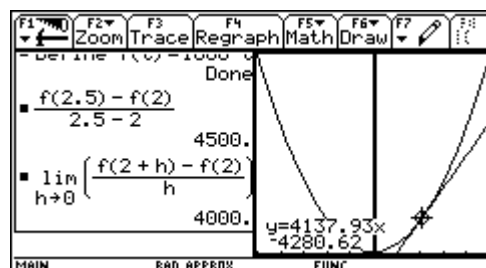


Figure 3



The target of the third set was to solve problems related with the derivative of a function using commands of the calculator with the help of algebraic methods, tables and graphics.

The proposed activities on the third implementation's set of the program were related with: function's behavior around a given point, 2) derivative numeric study, 3) problems resolution of marginal utility and population's growing. In these activities, the tutors attended to graphics and tables in the GC to embrace and answer the proposed questions. specifically on the marginal utility problem, the tutors used the tangent straight's command at the graphics application ( $\dagger$  N) and they show it on divided screen (see figure 3).

The fourth and fifth sessions were related to maxima and minima and to the problems resolutions of differential calculus applications. In each sessions, in addition to the group and individual work, the tutors showed the results to the group, supported with the view screen, and this stimulated several discussions and reflections about this subject and its didactics.

### Mathematical tutors' performance at the final moment

The target of the last session was design a didactic activity related with the differential calculus, to be developed with students at the mathematic school, according to the following demonstration:

*Lets suppose a mathematic tutor who needs to make a didactic activity in order to achieve the comprehension on some differential calculus topic. To satisfy this proposal we ask you to describe (or propose) some exercises or problems with that assignation. Assuming that the tutor knows the use of the representation systems and he will use the graphic calculator with their students:*

- State two questions at least, which answer should be the representations systems and the graphic calculator;*
- Order the activities sequence (script) to follow by the tutor in order to achieve the target;*
- Propose two aspects at least to be evaluated( on the students) and indicate how would you do them.*

Its very important to show that the statement in this activity was the same to the proposed one at the part B in the initial session.

In the first question, the mathematical tutors, mentioned classes standards exercises except the participant P7 who proposed a growing population situation and the tutors P1,P2 and P3 who

mentioned a commercial area situation. Generally the tutors went to different interesting subjects to suggest the problem situations. The proposed situation by P1, P2 and P3 was the following:

*For the first semester of the year 2003 at the supermarket Domy was registered a monthly offer and corn flour demand corresponding to the following table:*

$x$	Supply	1020	1680	2240	2880	3420	4020
$f(x)$	Demand	1020	860	820	980	2520	1470

To continue we show you some of the formulated works by the tutors:

- Look for the mathematical model which approach to the data on the table.*
- The general Manager wants to know which was the instant variation rate for the prime trimester final 2003?*
- Based on the mathematical model, which is the offer that produce the minimal and maximal demand for the first semester on 2003?*
- Between those two points which is the required offer in order to the demand begins to decrease?*

This activity helps the abstraction on the mathematical model's obtainment process moreover, the model construction for its mathematic treatment and after interpretation in the original situation aspect. Data was introduced trough the commands O and Data/Matrix/Editor. The tutors gave their answers from the showed part on figure 4 and 5.

Figure 4

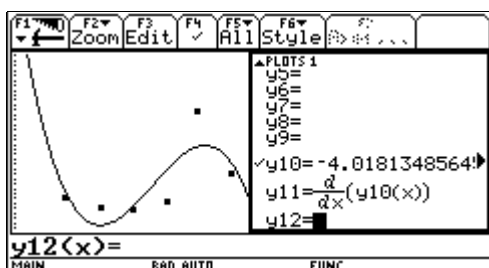
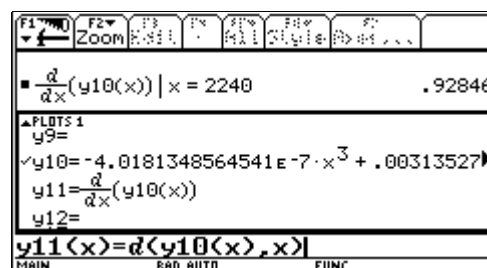


figure 5



This kind of problems helps the technical dominion of the graphic calculator and allowed the tutors to get more skills on the commands handling and the best thinking way for the mathematical model selection and its different representations; which let them to recognize the didactic and experimental capacity in the graphic calculator and apply it for the physical and social world problem resolutions which the students could suggest at the mathematics school.

About the second question of the formulated didactic activity in the last session, the formulated questions by the participants suggest the comprehension level about the course-workshop content in according to the didactic use of the representation systems with graphic calculator in differential calculus context. Opened and no traditional questions have been present which could create a questionable and searching atmosphere in the students; for example, consider the mathematical model construction whit questions which increase the discussion. This characteristic of questions was not watched in the first sessions, and it indicates changes on didactics competitions between participants about the model process and its respective application as well as the representation systems. By the other hand, the proposed statement by participants, let an open possibility of the graphic calculator's use (GC), it means that, they don't ask its use directly. This last topic is another achievement because we wish that students don't be obligated to use the GC but must be themselves who decide to use it or not.

About the third question, in general lines the tutors consider the following sequence:

1. Organize the students in groups and motivate them for the future activity,
2. Suggest the problems or exercises,
3. Suggest works which help to understand the exercise or proposed

problem (the GC can be used), 4. Take a look to the problem using several representation systems (with the GC's support), 5. Problem's resolution, 6. Solution's interpretation, 7. New question's formulation and 8. Suggest similar situations. This sequence shows a non traditional boarding manner in the sense that everybody see the motivation as a very important thing, suggesting and discuss of the problem situation. This seems to be taken as a very good strategy for the tutor. Moreover, they give participation to the student through an heuristic strategy when they propose questions, which is good to generate the use of different systems of representation. Finally, when they talk about the calculator's use, seems clear the didactic handling of it by the tutor and its use by the students because they don't mention anything about the kind of operation and commands to be used in the graphic calculator. In the same way, the participants take the context of differential calculus where is suggested the didactic activity which they answered.

The suggest of the fourth question, is about the assessment, tutors suggest to consider the assessment as an information searching for the tutor. Because he could make a general sight of the students and take decisions about the teaching and learning strategies. We haven't mentioned the assessment yet as a way to make stronger students in their intellectual capacities and use possible advantages which is offered by school context. Specifically tutors mention some aspects like: 1) using several representations in the assessment, 2) give value to the different forms to make problems, 3) consider a process evaluation more than results and, 4) with some exceptions, the tutors consider to assessment the productions related with mathematical dominion and problem resolutions, but they don't count with the calculator dominion, in the same way as they don't assessment the paper and pencil dominion.

### ACHIEVEMENTS AND FINDINGS

Tutors suggested exercises and situations of the real world involved to the levels on administration and accountant careers and closed to the students approach and dominion which they attend as teachers. About to organize materials and resources is evident the capacity in technical and didactic handling of the GC, and the options that it offers, making it very important for the tutor and for the student. A new sight was showed about the mathematics teaching which place the student as an active person, where he could experiment, make questions, solve explain and contrast with others partners and with the tutor. Tutors attended to different systems of representation and their interconnections, which showed the alternative searching to make the students comprehension easier. They explore ways to explain the differential calculus to the students as several ways to increase the comprehension of exercises and problem situations. The integration of representations were showed and the GC on the problems resolutions for the presented didactic activity of design.

The participant's productions were mentioned to: 1-the systematic application of the representation systems on the problems and exercises resolution, 2-the use of the experimentation with the GC to solve problems, 3-The use of the graphic calculator in the comprehension and problems resolution and 4-The use of the graphic calculator potentialities with didactic goals. It means that the tutors showed their capacity to incorporate new didactic competitions in the representation system's use and the graphic calculator for the mathematic teaching and learning process. Tutors put their dominion clearly on applications to integrate the representation systems and the graphic calculator use in the dynamic of teaching.

Tutors proposed real problems which could help in the development of the students intellectual autonomy and foment the use of the mathematical model as a very rich strategy of their didactic competitions. All this was proposed having as main nucleus the integration of the representation systems and the graphic calculator, because during all the work-course the participants showed productions in this direction.

By the other hand respect to the sequence, at the initial moment we just want to suggest the problem situation, show the mathematical model, solve, give others similar examples and confirm

results with GC; meanwhile the final moment is considered the group work by the students, in the statement of situations and the selection and formulation of problems, the multiple representation and construction of the mathematical model (with the GC support), interpretation of the solutions and new question's formulation. This shows advantages in the didactic knowledge of the mathematical tutors, created in the program implementation. In this way, the given statements to the different problem situations, were focused to the student's development, through the representation systems, the smarting and comprehension about concepts of the differential calculus as well as create attraction on new situations and achieve a mathematics more integral vision.

About the differential calculus's importance for the teaching, tutors suggested contexts which let us use the calculus concepts for the use and application of different representation systems and the recognition of the different teaching options. Such importance was showed in the productions and discussions during this proposed program implementation.

Taking care about what we have exposed before, we could affirm that the course-workshop helps to the development of wished conducts in mathematics' teaching and learning, applying to the use representation systems and the graphic calculator, in the context of differential calculus for the design and survey on didactic activities. In this way we answer the questions of the present research.

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