CAS AND EXAMINATIONS

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The main concerns put forth for not allowing symbolic calculators in examinations are:

- What will there be left to examine if calculators can do symbolic manipulation?
- Won't examinations become much harder if we need to devise ways where students do not profit from the power of such calculators?
- Are we not signing the "death warrant" of our own subject by permitting such devices which "dumb down" algebraic and analytical techniques in a similar way that scientific calculators have robbed students of numeracy facility, and some teachers claim, graphics calculators are slowly diminishing students' understanding of functions?
- Don't people setting examinations need to be thoroughly trained (at whose expense) to be familiar with the capabilities of all machines on the market so as to be able to set questions that do not favor students with one machine over students with a different make?

All of the concerns mentioned above are certainly valid, and we need to ensure that none of the scenarios occur. We need to take particular care that the examinations do not become more difficult, thus making success even more inaccessible as here-to-fore. This would certainly serve as a key factor in de-motivating students to pursue careers in mathematically related fields, and making our subject more elitist than the popular conception.

An example taken from the International Baccalaureate Higher Level May 2000 paper one final examination reveals that many of the concerns above can be allayed without sacrificing the pedagogical principles most mathematics educators hold dear. The IB requires a graphics display calculator for both papers one and two of the final examination in Higher Level and does not permit a calculator with symbolic capabilities.

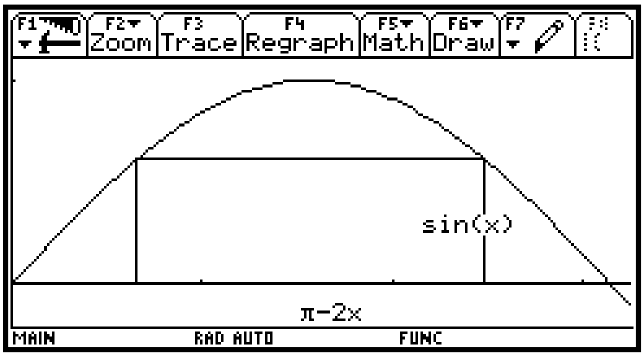
IB Higher Level May 2000 Paper 1 No. 17:

A rectangle is drawn so that its lower vertices are on the x-axis and its upper vertices are on the curve $y = \sin x$, where $0 \le x \le \pi$.

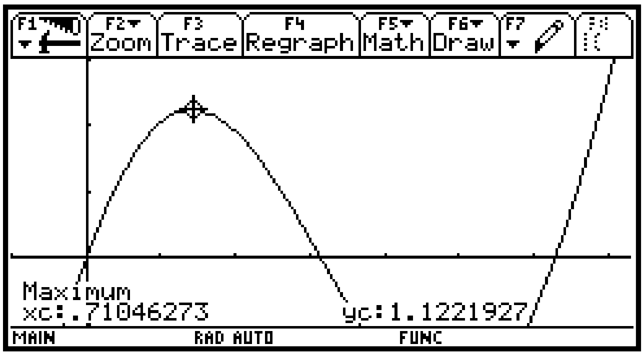
(a) Write down an expression for the area of the rectangle.
(b) Find the maximum area of the rectangle.

The first question one must ask oneself is: *what is this problem assessing*? The student must first translate the words into a mathematical model. The model is geometric, i.e., the graph of the sine function between 0 and π. The student must then sketch a rectangle with the given conditions and deduce the base and height both from the model and the given information.

Once the problem has been correctly translated into a mathematical model, the student can easily now see that A = sin(x)(π-2x).



In order to answer (b) it is essential that the student arrive at the correct model. With a graphics calculator, there is no understanding of calculus necessary to successfully answer this question. The student need only enter the formula for the area obtained in (a) and using the calculate menu, read off the maximum value of the function within the given domain 0≤x≤π, A= 1.12 square units.

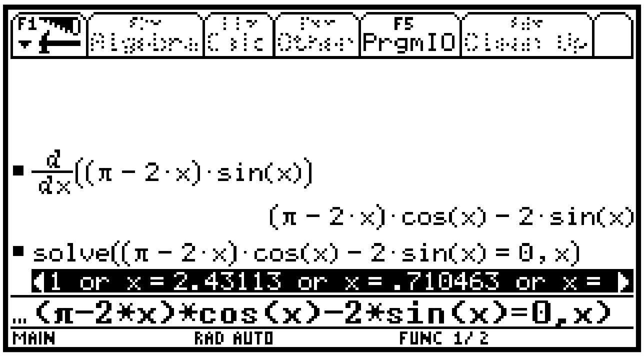


(We need only ascertain that the area at the endpoints of the domain are less than this value, and we can easily see from the formula that when x=0 and x=π, A=0.)

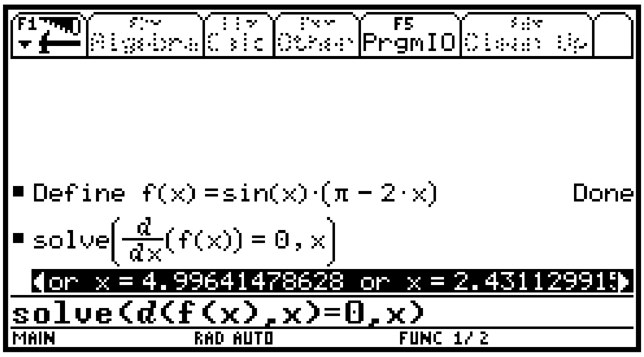
Indeed there is no pedagogical value in the process involved in finding the maximum – the only skills being tested here are calculator ones. The student does not need to understand any calculus concepts to obtain the correct answer.

If we, however, would like to test some calculus concepts in obtaining the solution to this question, we could easily require an ***analytical*** solution to this question where the students would use a CAS calculator and document all **essential** steps.

To answer part (b) analytically, the student must know that the maximum of the function occurs when the first derivative is zero. Hence, using a CAS:

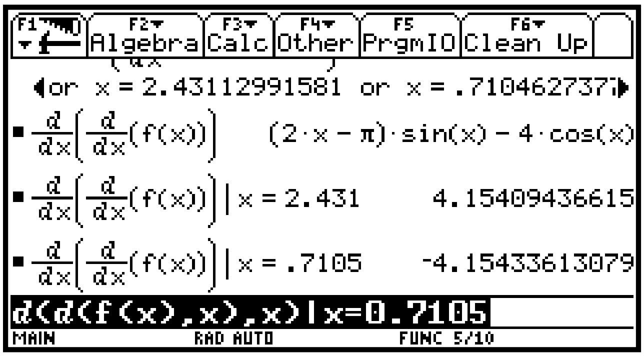


Or in one step:

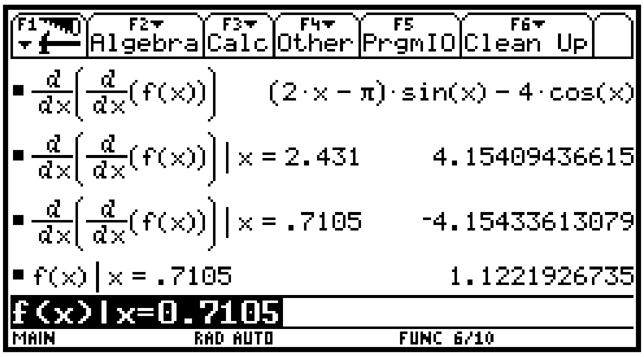


The student must now interpret these results within the given domain and select x=2.431 or x=0.7105 (which is visible when the highlighted line is scrolled to the right).

Using now the 2nd derivative test:



The student must interpret what this means, i.e., a maximum occurs at a point when the 2nd derivative at this point is negative. Now, the student must evaluate the area function at this value, and obtains that the area of the rectangle is 1.12 square units!



An analytic solution in this case is only possible with technology!

Teaching tip: Throughout teaching with technology the teacher must be very clear as to the nature and amount of documentation the student is required to show in assignments and assessments. The student learns this best through the classwork the teacher does by way of examples and model questions. Throughout classroom instruction the teacher must distinguish between sufficient, important, and essential documentation.

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