DGS and CAS as tools supplementing each other in an inquiry task "Locus curves"

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Abstract

In this paper we will present some results of the project ”Locus curves”, which was carried out as a part of the first university level geometry course for mathematics teacher students in the university of Tampere in Finland. The main goal of the project was to give teacher students an opportunity to apply both DGS- and CAS-environments for solving the same geometrical task and in this way give them a chance to compare the similarities and differences between learning processes in these two environments. The students were first asked to construct the locus curves geometrically with the help of the dynamic geometry software GEONExT and then to present the solutions as applets locally or in net. After that the features of curves were asked to be examined analytically with the CAS-software QuickMath (http://www.quickmath.com/), which can be run directly through the net. In this article I will report how students with these tools managed to find the locus curves which included in the tasks. The versatile family of locus curves included for example parabolas, ellipses, hyperbolas, straight lines, Cassinian curves, lemniscate of Bernoulli, a circle of Apollonius etc. In the article it is also discussed how the different points of view offered by DGS- and CAS-approach supplement each other in solving these kinds of inquiry tasks.

1. Introduction

Jones (2002) has recently emphasized that the research focusing on the use of DGS suggests that DGS alone cannot provide a sufficient environment to study geometry, but that other activities are needed for students to make progress in mathematics. Classroom experiments have shown that the software itself does not grant the transition from empirical to generic objects, from the perceptive to theoretical level. On one hand, the teacher plays a very import role in guiding students to theoretical thinking. On the other, hand the research has shown that also discussions in the classroom and group work are important components in this process. In the project "Locus curves" presented here studies in DG-environment were supplemented by studies in CAS in order to offer teacher students a chance to compare the similarities and differences between learning processes in these two environments.

2. The inquiry task *"Locus curves"*

This paper presents some preliminary results of the project ”Locus curves”, which was carried out in the university of Tampere in Finland as a part of the first university level geometry course for mathematics teacher students called Didactical geometry. The course forms one part in the basic studies of the so called didactical mathematics. Didactical mathematics can be described briefly to an approach in the university level mathematics teaching, where the contents of teaching and the methods of teaching have been especially designed to support the professional development of mathematics teachers and mathematics teacher students. Didactical mathematics resembles a branch of university level mathematics education called in USA sometimes as an educational mathematics. David Henderson from Cornell University has characterized educational mathematics in his web site (http://www.math.cornell.edu/People/ Faculty/ henderson.html) as *"...mathematics teaching where teachers (and thence their students) learn and experience ways of thinking that are as close as possible to the ways that mathematician's think, but yet simultaneously paying attention to the cognitive development of students and teachers and the underlying meanings and intuitions of the mathematics"*. The Didactical geometry course consisted totally of 32 hours lectures and 16 hours workshops in a computer class. The guidance of the project "Locus curves" was given during the computer class working but the tasks itself were expected to be solved as a homework.

When this project begun we saw that there are at least three kinds of problems of plausibility associated with technologically rich mathematics teaching in Finland. These were (and still are):

1. <u>The mathematics curriculum:</u> Although the technological tools, especially graphical calculators, are nowadays widely used in Finnish upper secondary schools, the new national framework curriculum for mathematics teaching on that level doesn't explicitly encourage teachers to utilize technological tools in their mathematics teaching. The almost only mention of the use of technological tools given in framework curriculum for mathematics says that student should *"learn to use appropriate mathematical methods, technical tools and sources of knowledge"*. The curriculum is also seen to be so full of contents that teachers and teacher students think that they are forced to restrict in their teaching almost totally onto the traditional content and apply the traditional teaching methods.

2. <u>The lack of resources:</u> Teacher students know or at least suppose that schools use to suffer from the lack of money and therefore they also use to suffer from the lack of mathematical software. Teacher students are also quite aware that departments of teacher education at university often have similar problems.

3. <u>The lack of skills:</u> The programs, which demand a lot of special knowledge from teachers and students, seem not to be plausible tools for taking in use.

By using freely available and easily learnable software we tried in this project to overcome the above mentioned plausibility problems. The main tools, which we used in our project, were the following free or inexpensive tools:

- GeoNext, which is dynamic geometry software freely downloadable from web (http://geonext.uni-bayreuth.de/),
- QuickMath (http:// www.quickmath.com) and Math.com (http://www.math.com/students/ solvers/online_solvers.htm), which are freely usable CAS-applications. Both are based on the software webMathematica™,
- Derive™, which is a commercial program, but due to its rather low price compared with its competitors, is commonly used in schools.

The other objectives, which we stated to the project, were

- comparing the advantages of DG and CAS,
- introducing students the concept of the locus both in a DG- and in a CAS-environment,
- making the different cognitive processes used in DG- and CAS-environments explicit to students.

3. Inquiry tasks given to students

We gave students the following tasks to be solved in groups of two to four students in each:

Select from the table below two cells, which do not lie either in the same column or in the same row. Use the dynamic geometric software and draw the locus curves defined by the cells, which you chose.

What kind of locus does the point P draw, if the distances d_1 and d_2 are interpreted in the following way			
and if the type of the constancy is the following one:	d_1 and d_2 are the Euclidean distances of the point P from two fixed points P_1 and P_2	d_1 is the Euclidean distance of the point P from a fixed point P_1 and d_2 is the Euclidean distance of the same point P from the line l	d_1 and d_2 are the Euclidean distances of the point P from two fixed intersecting lines l_1 and l_2
$d_1 + d_2 = a$, *a is constant*	L_{11}	L_{12}	L_{13}
$d_1 - d_2 = a$, *a is constant*	L_{21}	L_{22}	L_{23}
$d_1 \cdot d_2 = a$, *a is constant*	L_{31}	L_{32}	L_{33}
$d_1 \div d_2 = a$, *a is constant*	L_{41}	L_{42}	L_{43}

We also introduced the CAS-applications QuickMath and Math.com to students and suggested that they should utilize these sites as tools figuring out the form of the locus curve. The Scottish web-site *"Famous curves Index"* maintained by the University of St Andrews and the site "*A Visual Dictionary of Famous Plane Curves*" (http://www.xahlee.org/

SpecialPlaneCurves_dir/specialPlaneCurves.html) both would have been valuable tools in this inquiry process but this time we decided to restrict only on the tools mentioned above.

The family of locus curves, which was studied, is quite versatile. It includes parabolas, parts of parabolas, ellipses, hyperbolas, Cassinian curves, a lemniscate of Bernoulli, a circle of Apollonius, straight lines, line segments, rays and single points. All the locus curves, where either d_1 - d_2 = 0 or d_1 : d_2 = constant can be considered as the so called alpha-bisectors in two dimensional Euclidean space. Johnstone and Shene (1991) define the *alpha*-bisector $B^{\alpha}(a, b)$ of the objects *a* and *b* in their technical report (http://www.lems.brown.edu/vision/people/leymarie/Refs/CompGeom/Representations.html) to be the locus of points whose distance from *a* is a constant and non-vanishing ratio *alpha* of the distance from *b.* Except for the circle, a conic is the *alpha*-bisector of a point P and a line L, P not in L. When *alpha = 1*, the conic is the *parabola*; for *alpha < 1* it is the *ellipse*, and for *alpha > 1* the *hyperbola*. The *alpha*-bisector of 2 points, for a non-unitary *alpha* is the circle (of Apollonius).

L_{11} An ellipse (a circle or a point if P_1=P_2).
L_{21} A hyperbola (the perpendicular bisector of the segment P_1P_2 if a = 0).
L_{31} Ovals of Cassini and in a special case the lemniscate of Bernoulli.
L_{41} The circle of Apollonius ($a \neq 1$), the perpendicular bisector of the segment P_1P_2 (a = 1),
L_{12} Parts of two intersecting parabolas, another opening upwards and the other downwards. The part is situated between the intersection points.
L_{22} A parabola (a = 1) or parts of two intersecting parabola, another opening upwards and the other downwards (a ≠ 1). The part is situated to the left from the left intersection point and left from the right of the right intersection point.
L_{32} A pair of graphs for two 4th degree polynomial functions.
L_{42} A point (a = 0), an ellipse (0< a <1), a parabola (a = 1) or a hyperbola (a > 1)
L_{13} A rectangle, if l_1 and l_2 are not parallel, otherwise a pair of lines parallel to l_1 and l_2.
L_{23} Lines, which are determined by the sides of a rectangle, excluding the sides.

The results to the tasks L_{11}, L_{21} and in the special case, when a = 1, also to the tasks L_{41}, L_{42} and L_{43} are well known for most of the students. However, the other tasks are can be quite challenging and surprising for students.

The figures 1 - 3 present examples of the locus curves which we think are in no way trivial. The graphs are plotted by the tool Advanced Plot from the site QuickMath (www.quickmath.com).

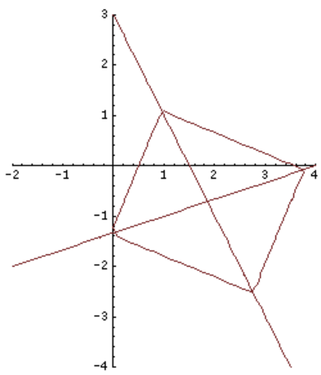
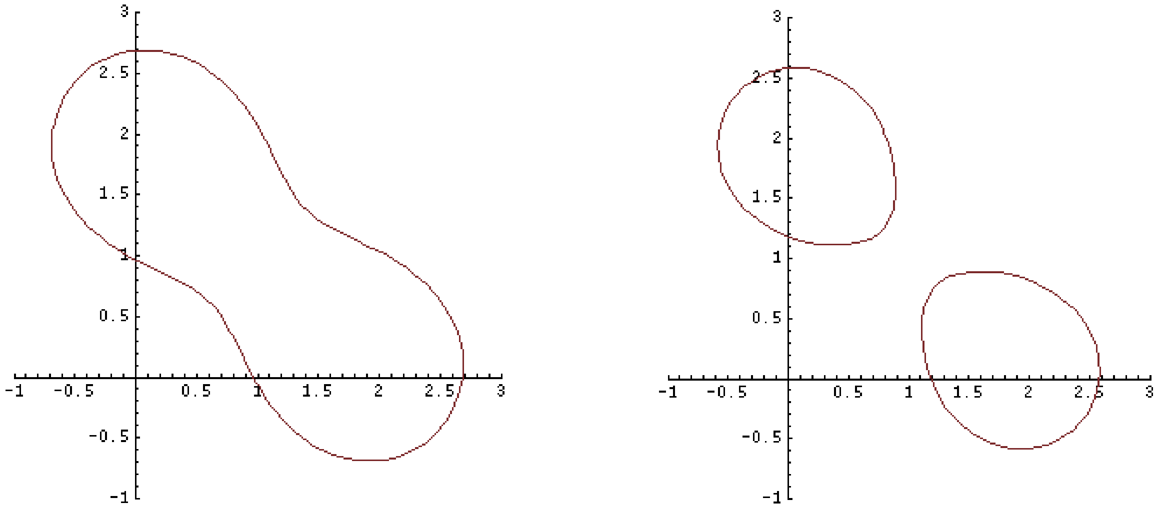


Figure 1. The sum of the distances measured from the point P to the line x - 3y - 4 = 0 and to the line 2x + y - 3 = 0 is constant (= 2). The locus curve should be a rectangle, but the inaccuracy of the graph makes it questionable.



The value of the product = 2.3

The value of the product = 1.9

Figure 2. The product of the distances measured from the point *P* to the fixed points (2,0) and (0,2) is constant.

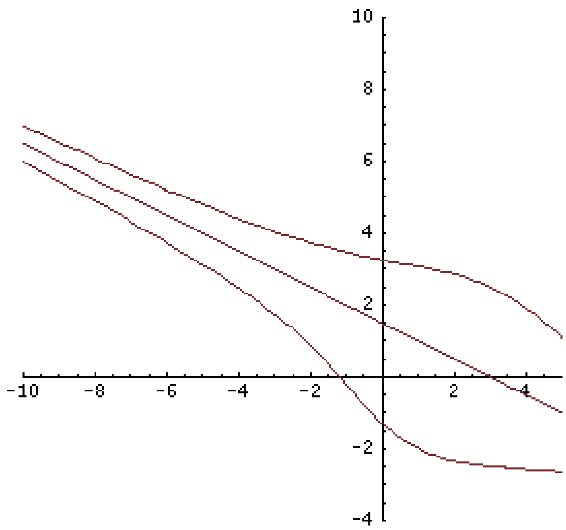


Figure 3. The product of the distances measured from the point *P* to the fixed point (2,0) and to the line $x + 2y - 3 = 0$ is constant (= 6).

4 Some observations about students' working in a DG-environment

When one tries to construct the locus curves like those described in our tasks one first ought to discover a way to present relations $d_1 + d_2 =$ constant, $d_1 - d_2 =$ constant, d_1 x $d_2 =$ constant and $d_1 : d_2 =$ constant geometrically so that line segments represents each variable and constant. The first two of the relations are easy to be modeled geometrically but the latter two are more demanding. In a way, we can say that students in this situation have to give up of the algebraic way of thinking and transfer to use "the arithmetic of magnitudes". The arithmetic of magnitudes refers here to the methods, which ancient Greek mathematicians and later for example Al-Khwaritzmi in the 9th century and Cardano in the 16th century used in order to solve algebraic problems geometrically before algebraic methods were invented (Charbonneau 1996).

Actually due to the short of time we had for this project we were forced to give the students examples of the solutions of these modeling tasks. Figure 4 presents one possible geometrical model for the relation d_1 x $d_2 =$ constant which gave as an example.

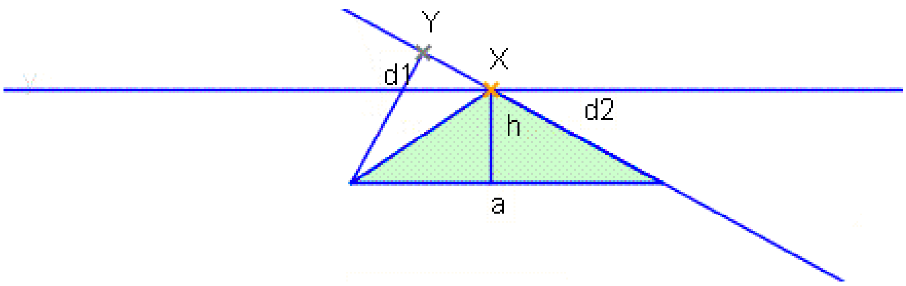


Figure 3. One possible DG-model for the relation d_1 x $d_2 =$ constant. Point X slides on the line parallel to the base a of the triangle. Because the product of the base and height of the triangle is the same for all bases, d_1 x $d_2 = ah$ =constant.

The DG-modeling of the relation d_1 x $d_2 =$ constant would have been much easier to implement, if the program GeoNext have allowed to put a slider point also to the curves, which are drawn analytically by giving the equation of the graph to the program. If this would be possible, we could first have drawn the graph $y = a/x$ and then put a slider point P to this graph. After that d_1 and d_2 could have been interpreted as being the coordinates of the slider point P. However, the version of the program GeoNext, which we used, did not allow this type of constructions. The reader is asked to think about the respective possibilities to make a DG-model for example for the relation $d_1 : d_2 =$ constant.

Quite many students gave only partial solutions to their tasks. The partial failure in the solution of the tasks may have caused by several reasons: Firstly, the DG-program GeoNext was new to students and students had to put much effort also to learn, how the program itself functioned, which might disturb the problem solving process. Secondly, the geometrical way of thinking "arithmetic of magnitudes", where every variable (number) and relation must be presented geometrically was strange to students. Thirdly, the students worked most of the time quite independently and perhaps got too little help for solving the problems, which they met. We cannot give examples of the students' solutions her, because the solutions were dynamic constructions. However, the reader can look at examples of solutions for instance from the web address http://www.uta.fi/~henri.saarivirta/geometria.html.

The most typical phases of constructing locus curves in dynamical geometry environment and in a computer algebra system seemed to be the following:

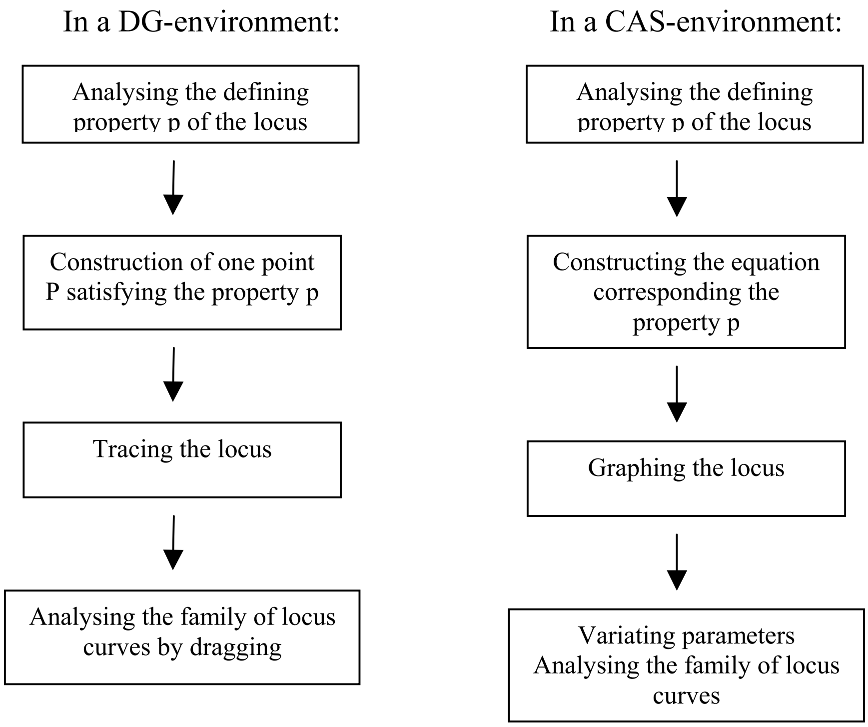


Figure 5. The typical processes of constructing the locus curves in a DG- and in a CAS - environment.

This was the first time we run the project *Locus curves* and the observations, which we have made, must be seen very preliminary. In the next time, when we will give the same inquiry task to students, the observing process can be made in a more controlled way.

However, after this experiment we are quite sure that there are considerable differences between the ways how students work in DG- and CAS-environments and that teacher students ought to have possibility to analyze these differences. The main differences between working in these environments seemed to be the following:

Table 1. The comparison of students' working methods in DG - and in CAS - environments.

The phase of the process	in DG-environment	in CAS-environment
Starting point	Every variable (number) and every relation must be presented geometrically, which often causes difficulties. Algebra → Geometry (back to the thinking via the method of "arithmetic of magnitudes")	Geometrical properties must be changed to equations. Geometry → Algebra Students are well acquainted with the methods of analytic geometry.
The drawing process of the curve	Constructive, creative and instructive. Often laborious and difficult for beginners.	Mechanistic and easy way. Easy to start.
Analysis of the results	Curves change dynamically. Students often get only a part of the whole curve. The parameters have clear geometrical meanings to the students.	New curves can be produced easily by changing the parameters in the corresponding equation. The parameters often have no geometrical meanings to the students.

References

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