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CONTS

Using TI symbolic calculator, Derive and DPGraph to ease comprehension.

Mathieu Guilbault
Mathieu Gouin
Ecole de technologie supérieure
1100, Notre-Dame street West, Montreal (Quebec) Canada
mathieu.guilbault.1@ens.etsmtl.ca
mathieu.gouin.1@ens.etsmtl.ca

ABSTRACT

We are both students in electrical engineering at ETS since 2 years. We had the opportunity to use various tools such as Texas Instruments symbolic calculator, Derive CAS and DPGraph in order to improve the comprehension of some mathematical concepts. During this presentation, we will show you our interest in using these tools from a student's point of view.

We will show the usefulness of these tools to solve concrete problems. Starting with the TI-92+, we will do a simple simulation for better understanding, then a graphical resolution of the problem. Moreover, this handheld calculator enables us to use mathematical software features without having to use a computer. Then we will present an example in which we use Derive as well as DPGraph. With Derive, we will show the clear organization and development of the problem, while DPGraph offers a good rendering of three-dimensional solids.

Introduction

During this conference, you will certainly hear quite often that the technology is changing the way we do mathematics. In the past, many mathematical tools helped doing calculations. We can recall some examples: abacus, trigonometric tables, slide rule, non-scientific calculator… While these tools increased the calculation time, the results were not accurate and did not help the user to better understand the concepts.

In the early 80's, Soft Warehouse developed a symbolic calculation software called muMATH-79 which evolved to become Derive in 1988. This tool revolutionized the mathematics world and started the interest to use CAS (Computer Algebra System). However, mathematic softwares have a main disadvantage of not being well suited for classroom use. To provide a solution to teachers and students, TI introduced in 1990 its first graphing calculator, the TI-81. Finally, TI released the first symbolic calculator, the TI-92 in 1995.

For high schools, colleges and universities, these tools are interesting because problem solving is much easier and they allow better comprehension of the concepts. This is mainly for these reasons that ÉTS strongly recommends the use of TI symbolic calculators throughout the engineering program. The exams and homeworks are made knowing that students have this calculator, so the level of difficulty can be higher and more concepts can be covered.

In this presentation, we will show you concrete examples demonstrating how theses powerful tools help students to increase their comprehension and for teachers to explain concepts in a more visual and interactive way.

Concrete problem solving using Derive and DPGraph

Derive is a good complement of the TI-92+, because this program uses some similar functions like solving symbolic equations, substitution of variables and variable assignment. Moreover, Derive is easier to use than TI-92+ for solving problem because we can see all developments in the same window. When the time is to make a report of a problem like homework, it is more interesting to do this with Derive than TI-92+, because it is easier for the teacher to understand the procedure of problem solving. Derive allows to rapidly solve equations system and make two or three-dimensional graphs. It is easy for a TI-92+ user to use Derive because a lot of parameter functions are the same. But TI-92+ is always a useful tool, because the student can have a powerful symbolic calculator without computer.

When we have to work with three-dimensional figures, we can use DPGraph. This software is used to see a figure on different angles. Moreover it is possible to see the movement of a prism in the time. Some students can have problems to see figures with their equations, this software is very helpful for these students.

We will use a problem from a homework gave by Yves Landriault (teacher of ÉTS) to show you the interest to use Derive and DPGraph.

Problem description

Find the z component of geometric center of a homogeneous solid limited by two surfaces $z = 8 - 2x^2 - 2y^2$ and $z = 2\sqrt{x^2 + y^2} + 4$.

Solving the problem

First of all, we can illustrate these equations with DPGraph. To do this, we have just to insert the equations and the Cartesian limits in the DPGraph editor.

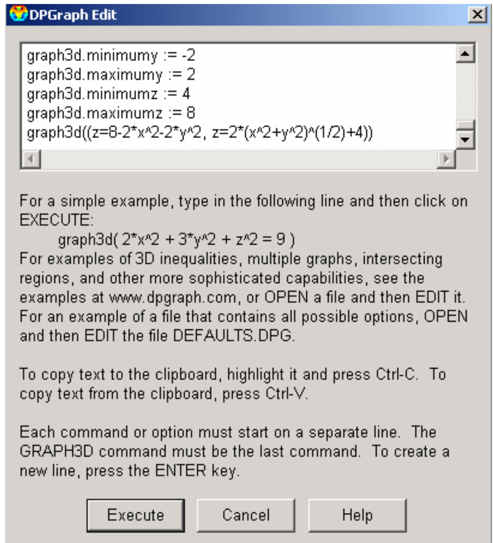

Figure 1:DPGraph editor

After the execute button is pressed, it is easy for a visual student to see the three-dimensional solids associated to the equations.

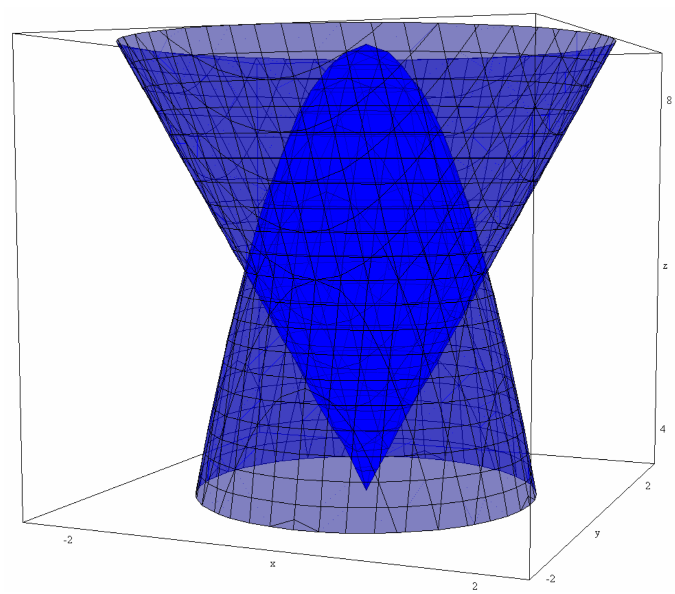

Figure 2:Three-dimensional solids associated to the equations

With this illustration, we can see a cone and a reverse paraboloid. Moreover, this figure shows a ring made by the junction of these two solids. Mathematically we can prove it and we can find the radius and the centre position of the ring. First of all, to simplify the calculations we will convert the rectangular equations to cylindrical coordinates. To resolve this exercise, we will use Derive.

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Firstly, we enter the rectangular equations in the window to convert them in cylindrical coordinates.

2 equations are:

Reversed paraboloid and shifted:

#1: $z = 8 - 2\cdot x^2 - 2\cdot y^2$

We substitute the x and y to transform the equation in cylindrical coordinates

#2: $\text{SUBST}(z = 8 - 2\cdot x^2 - 2\cdot y^2, [x, y], [r\cdot\text{COS}(\theta), r\cdot\text{SIN}(\theta)])$

#3: $z = 8 - 2\cdot r^2$

Cone:

#4: $z = 2\cdot\sqrt{(x^2 + y^2)} + 4$

We substitute the x and y to transform the equation in cylindrical coordinates

#5: $\text{SUBST}(z = 2\cdot\sqrt{(x^2 + y^2)} + 4, [x, y], [r\cdot\text{COS}(\theta), r\cdot\text{SIN}(\theta)])$

#6: $z = 2\cdot|r| + 4$

"r" is positive then:

#7: $z = 2\cdot r + 4$

The variable substitution function is a tool that saves time and space in the resolution of mathematical problem. If we do not use symbolic calculator software, we lose time and we have a chance to make an error. Now, we have the two equations in cylindrical coordinates, we can find the intersection of these solids.

We find the intersection of these 2 solids

#8: $\text{SOLVE}(8 - 2\cdot r^2 = 2\cdot r + 4, r)$

#9: $r = -2 \lor r = 1$

"r" cannot be negative then:

#10: $r = 1$

This answer informs that the intersection is made by a circle centered in 0 with a radius of 1. Moreover θ goes from 0 to 2π.

After, we can find the moment of xy plan:

$$\#11:\quad mxy := \int_0^{2\cdot\pi}\int_0^{1}\int_{2\cdot r + 4}^{8 - 2\cdot r^2} 1\cdot z\cdot r \; dz \; dr \; d\theta$$

$$\#12:\quad \frac{31\cdot\pi}{3}$$

And the mass of the solid:

$$\#13:\quad m := \int_0^{2\cdot\pi}\int_0^{1}\int_{2\cdot r + 4}^{8 - 2\cdot r^2} 1\cdot r \; dz \; dr \; d\theta$$

$$\#14:\quad \frac{5\cdot\pi}{3}$$

The last step its find the geometric centre of the figure, for to this we just have to divide m_{xy} by m:

$$\#15:\quad z = \frac{mxy}{m}$$

$$\#16:\quad z = \frac{31}{5}$$

$$\#17:\quad z = 6.2$$

With the variable assignations, it is easy to use them for another operation like this one. Without this software or another symbolic calculator, it is tedious and slow to recopy and calculate the results.

Concrete problem solving using TI-92+

In this part of the presentation, we will show you some important features of the TI-92+ that will help solving a concrete problem. It is important to note that when we refer to the TI-92+, it can also be applicable to the TI-89 and the Voyage-200, which have similar features. The problem we will present was given in part of a homework assignment in the first course in mathematics at ÉTS. This course has also an objective to get us accustomed to the TI calculator.

Problem description

A boat sails North at a speed of 12 knots and sees a giant oil tanker at 3 km North-West that travels East at 15 knots. For security reasons, boats must keep a minimum distance of 100 m between them. Find the closest distance of the two boats if they keep their respective direction and speed constant. Then, determine if they must change their path.

Solving the problem

To begin solving this problem, we can sketch a diagram with the information we have.

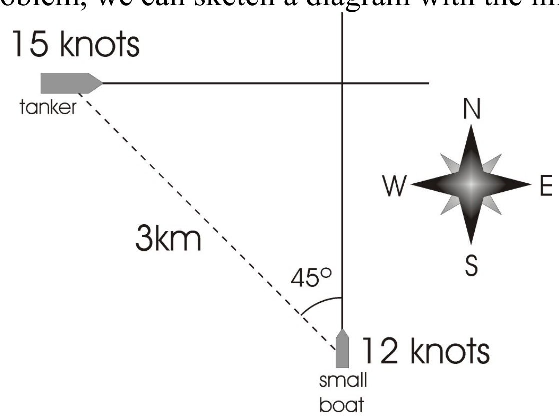

Figure 3: Diagram of the problem

We now have a general idea of the problem. The two boats will sail at perpendicular direction toward each other. Because the tanker travels faster than the small boat, it is logical to assume that they will not come too close. As this is just hypothesis, we must continue our reasoning to validate this assumption.

In this problem, we are given measurement units in two different systems: SI and Imperial. We have to convert them all to SI. For this part, the TI-92+ has many built-in units and can easily converts between them. This feature comes handy when solving physical, mechanical or electrical problems that often deal with units.

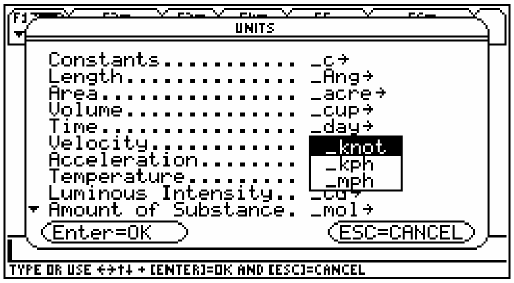

Figure 4: Selecting units

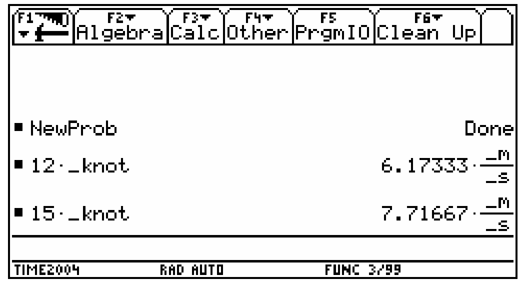

Figure 5: Converting units to SI

We can now move on further by doing a basic simulation of the motion of the two boats. This simulation will greatly help us to better visualize and understand the problem and confirm the numeric answer we will find later on. For this step, we will use the parametric graphic mode of the TI-92+.

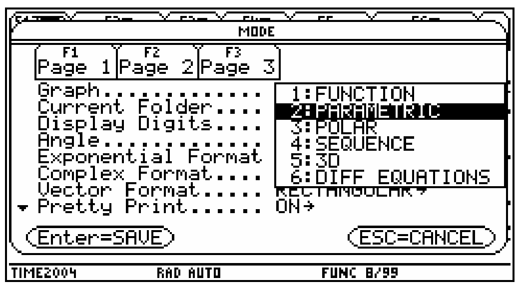

Figure 6: Setting the graphic mode to "parametric"

The parametric mode allows us to define two separate expressions that depend of the variable t for the x-position and y-position of the curve in the Cartesian plan. In our case, we can suppose that the independent variable t will represent the time from the start of the simulation.

As explained above in the diagram of the figure 3, the small boat that we will note #1, is moving straight North, so its x-coordinate will be 0 and its y-coordinate will be changing over time according to its speed. For the big tanker (boat #2), it is sailing straight East, so only its x-coordinate will be changing. If we "*translate*" this using the TI-92+, we can define the following parametric functions. In addition, we compute the vertical offset of the boats at time t = 0. Furthermore, when we store the functions in the home screen, they become automatically active in the graphic functions and are ready to graph.

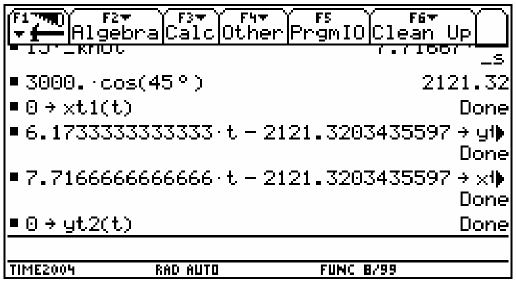

Figure 7: Calculating parametric functions

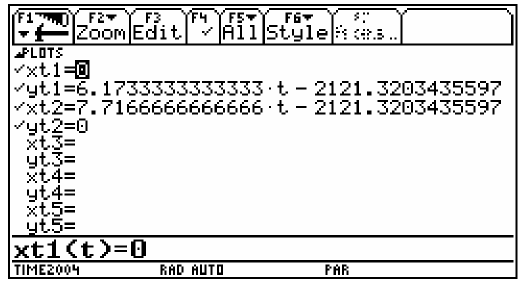

Figure 8: Parametric functions list

We then select the proper windows setting. The trick here is to choose a small `tstep` in proportion of the x-range, so the graphing time will be slow enough to allow us to see the simulation. Another important setting is the `simul` mode. This mode draws the curves simultaneously instead of sequentially.

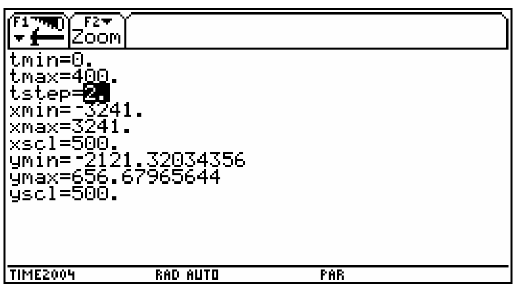

Figure 9: Setting the proper window parameters

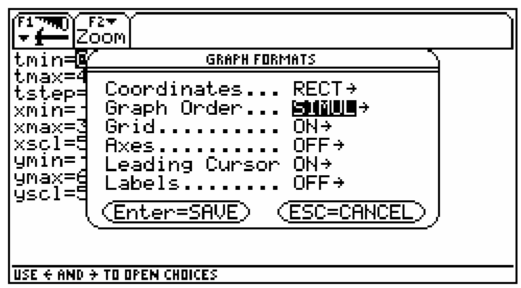

Figure 10: Configuring the graphic format

When we graph, we see the curves being slowly traced and we can make pauses during the graphing process. We can guess the minimum distance of the boats, knowing that we set a grid spacing of 500 m.

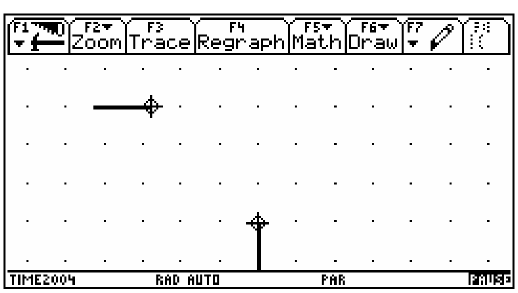

Figure 11: Start of the simulation

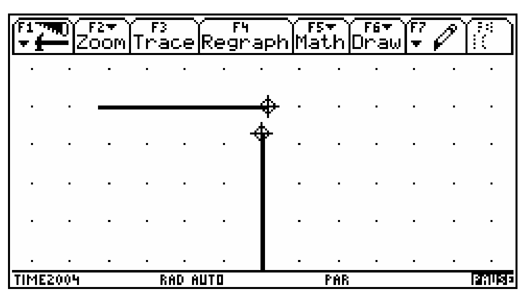

Figure 12: Approximate minimum distance

It is now time to find a way of calculating the numerical minimum distance that the two boat will be separated in order to determine if they are under the security limit. There are many ways of performing this calculation, but they all consist of calculating the minimum values of an expression that gives the distance between the two boats at any given time. We will show you here a graphical method.

We start by recalling the parametric functions previously defined and use them in a 1-row matrix that will represent the (x, y) position of each boat. Then, we calculate the distance between these two Cartesian points using the `norm( )` function. This gives us the expression of the distance as a function of time. We then graph this function and find its minimum.

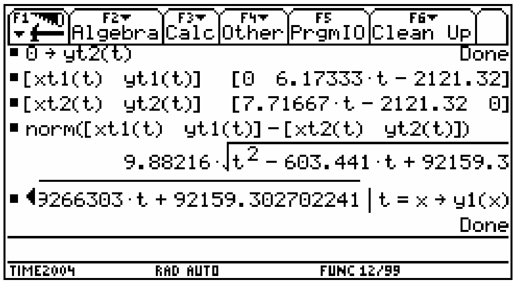

Figure 13: Calculation of the distance equation

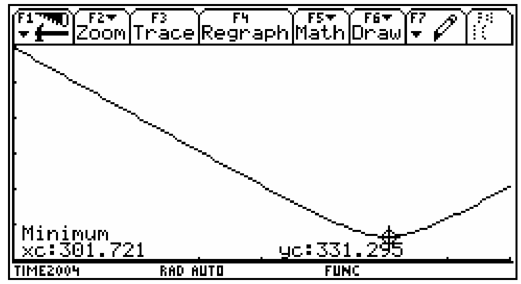

Figure 14: Finding the minimum distance

The minimum distance will be 331.295 m. We can conclude that the boats will remain within the security distance of 100 m, so they do not have to change their direction or speed. Furthermore, we can roughly validate this answer from the simulation we did earlier. We had a grid of 500 m and the two points were closer than a grid space but obviously not greater than 100 m.

Conclusion

As we have seen in this presentation, DPGraph is a good software that students can use to see, understand and explore various figures that come from equations. This tool is really user-friendly. Also, Derive is a helpful software for solving mathematical problem and keeping a good organization of the development. Moreover, Derive functions and parameters are similar to those of the TI-92+. For this reason, a user of one system will be accustomed to the other one easily. For educators and students in a classroom, we have shown the advantages of using TI. We can state for examples: to view the graph of functions and find their properties, to validate calculations and hypothesis and to do simple simulations.

References

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