TIME 2004, Montréal, Québec. July 15th, 2004

Learning College Calculus in a CAS Environment: theory and practice

by

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Research Question

What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?

Research on Technology

- Moursund (2000): The difference between first-order and second-order use.

- Dugdale et al (1995): To what extent do our positive feelings toward "intuition-building" software arise because this genre of software expresses or enriches an algebra *that we already know*, but which students (and perhaps teachers) do not know? Can we turn the edifice of our understanding on its head, expecting students to build rich understandings of something that is understood by us via a set of cognitions that our students will never have?

Research on CAS

- Strand I: Use CAS to deliver traditional curriculum more effectively
- Strand II: Re-conceptualise and re-design curriculum in light of CAS (and other technologies).

What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?

- What is the role of experimentation in mathematical learning in a CAS environment?
- What does the process of learning look like in a CAS environment?
- How do students approach formalisation (in the sense explained in the Pirie-Kieren model below) of mathematics in a CAS environment?
- What are students’ perceptions of the role of technology in their learning?
- What is the students’ relationship to *Mathematica*?
- What is the effect on students’ conceptions of mathematics, as a subject, of learning in a CAS environment?
- What is the relationship between the pedagogy in the classroom and the use of technology?

Theoretical Framework

- The Rotman Model of Mathematical Reasoning (Rotman, 1993, 1995)
- The Pirie-Kieren Model of the Growth of Mathematical Understanding (Pirie & Kieren, 1990, 1994)

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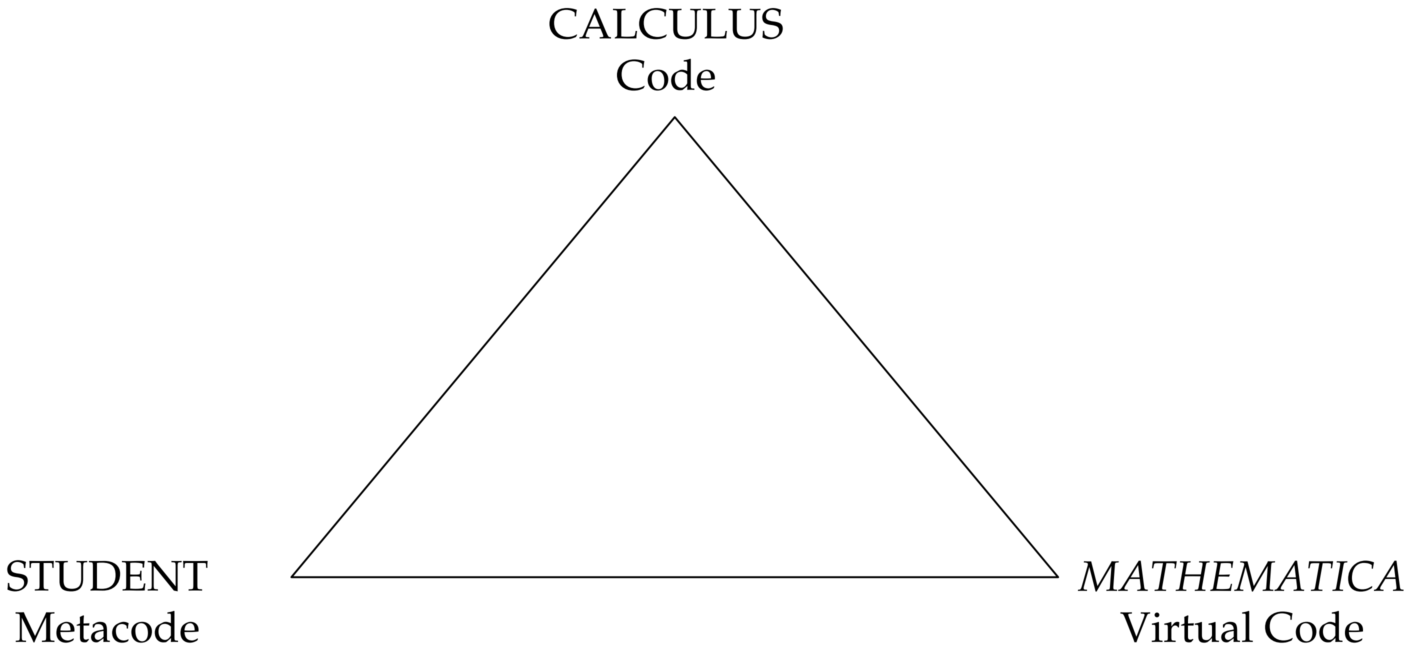
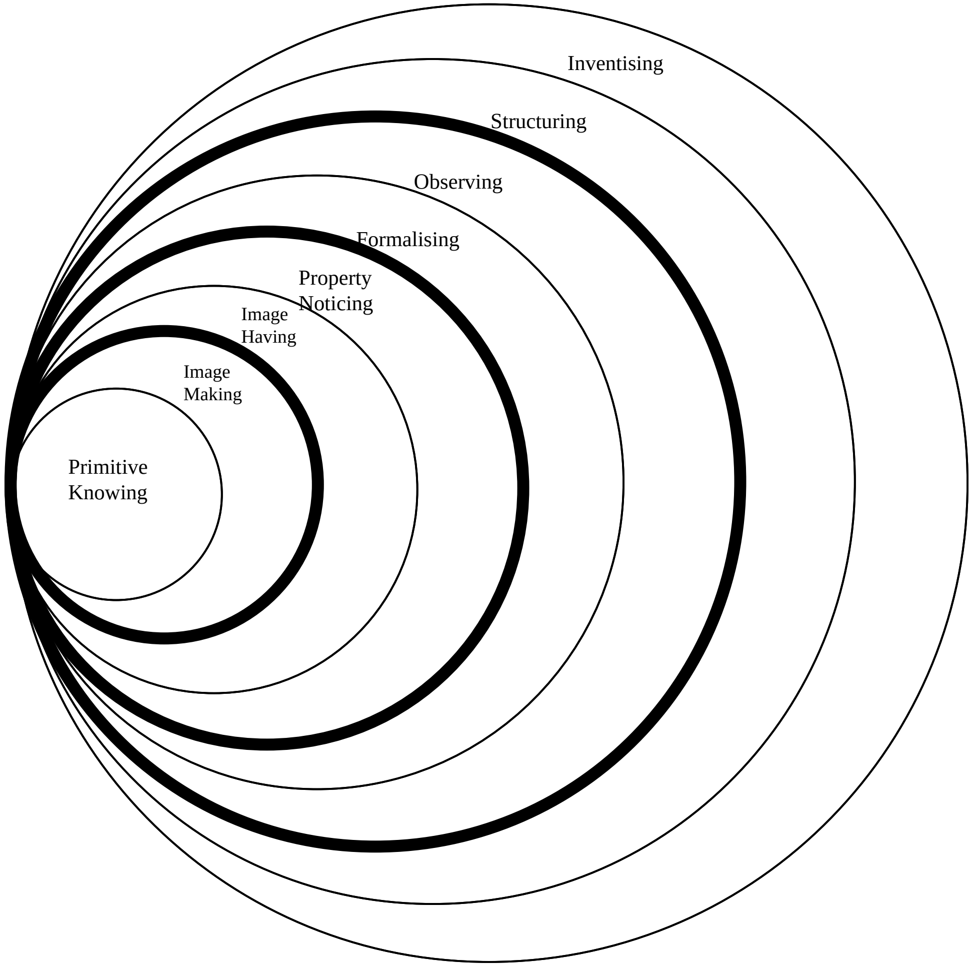
The Modified Rotman Model of Mathematical Reasoning



The Pirie-Kieren Model of the Growth of Mathematical Understanding



Primitive Knowing

Image Making

Image Having

Property Noticing

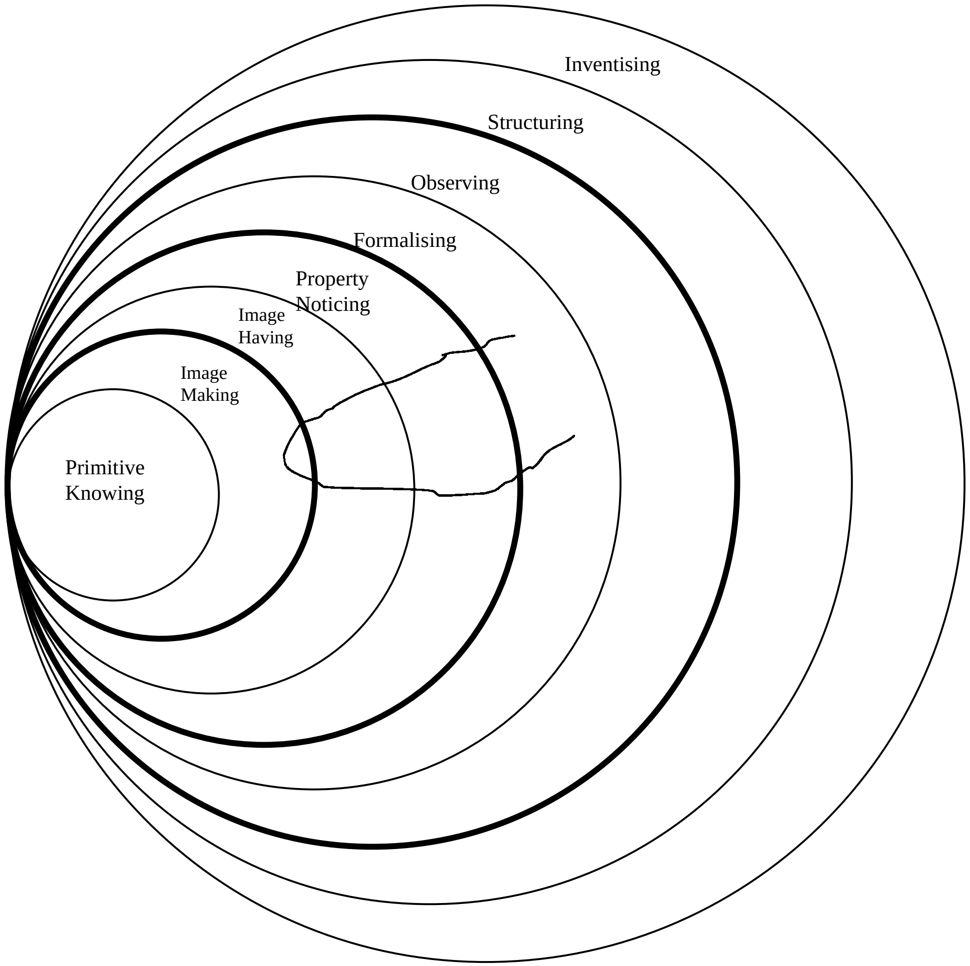
Formalising

Observing

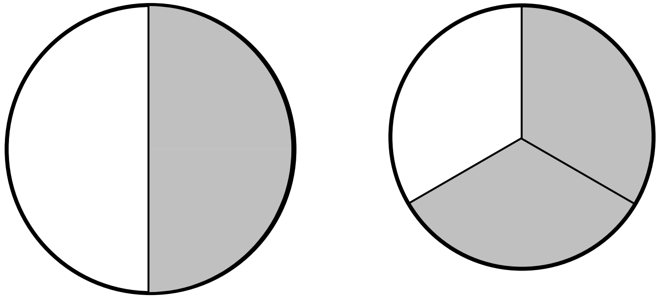
Structuring

Inventising

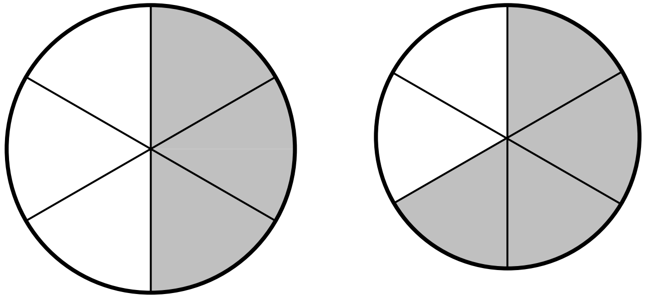
Example of the Pirie-Kieren Model



Compute $1/2 + 2/3$



Equivalent Form



Compute $3/6 + 4/6$

The Study

Case study of three students in a Calculus & *Mathematica* (C&*M*) class as individuals and in a group.

Primary Data: audio tape and video capture of computer screens of the group's discussions and collaborations representing learning episodes in the C&*M* classroom.

Supplementary Data: interviews with students at various stages in the quarter; survey administered to the entire class; interview with instructor; students' quizzes, homeworks, and assignments.

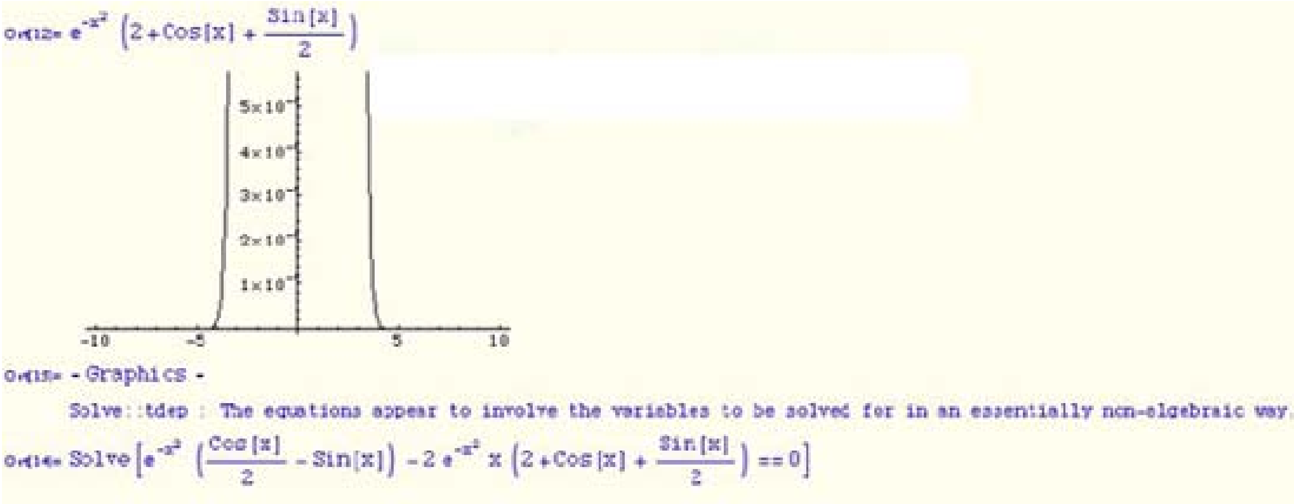
Timeline of the Study

- Mid to late September: identify students who will be willing to take part in the study.
- Late September: initial interviews with the participants.
- Late September to early December: daily recording of students working in the C&*M* class.
- Early November: mid-quarter interviews with participants.
- Early December: administration of grounded survey.
- Mid-December: Final interviews with participants.
- Spring quarter: Follow-up interviews with participants.

An Example

Find the highest point on the graph of f[x] = e^(-x^2) (2 + Cos [x] + (Sin[x])/2). Is there a lowest point on the graph?

Formalising (Algebraic); Image Making



A: OK. Is there a lowest point on the graph?

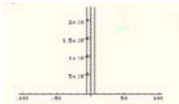
C: I don't know. Let's plot it and find out.

C: Let's solve for the maxs and mins here as well.
(silence)

A: Did you put the "Solve" function in wrong or is something that's too complex for it?

An Example (contd.)

Image Making



Formalising (Numerical)

Out[31]= - Graphics -

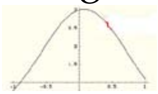
Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

Solve::tdep : The equations appear to involve the variables to be solved for in an essentially non-algebraic way.

Out[32]= $\text{Solve}\left[2.71828^{-1.\,x^2}\,(0.5\,\text{Cos}[x] - 1.\,\text{Sin}[x]) - 2.\;2.71828^{-1.\,x^2}\,x\,(2. + \text{Cos}[x] + 0.5\,\text{Sin}[x]) == 0.,\ x\right]$

An Example (contd.)

Image Making (*not* Image Having?)



Property Noticing

C: Try “f′(0)” and see if that works.

[Output: 1/2]

T: Go to the right and see if it’s still increasing.

C: f′(1). Wow.

T: Put a 1.

[Output –2.38879]

C: Yeah. So it’s decreasing at 1.

T: So it’s between 0 and 1.

C: Well, it’s between 1/2 and 1. You’re right 0 and 1. Closer (as A tries other values).

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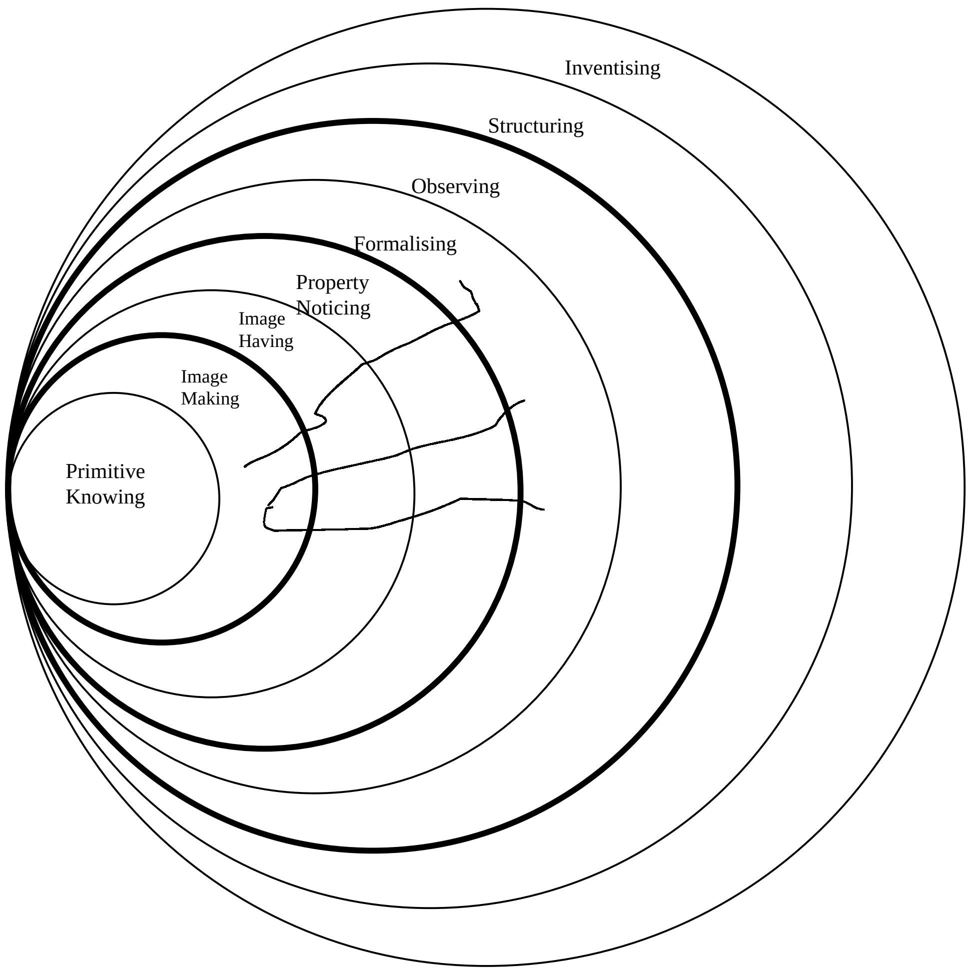
Pirie-Kieren map of the example



Themes

- Framing of Technology
- Experimentation
- Sense of Achievement
- Attitude to Technology

Framing of Technology

- Introduction by Instructor
- Structure of Materials
- Logistics of using Technology
- Type of CAS

Experimentation

- Awareness of Strategies
- Use of Strategies
- Willingness to Experiment

Achievement

- Emphasis on Meaning
- Self-Efficacy about “Mainstream”

Attitude to Technology

- Calculators vs Computers
- Macs vs PCs
- Capability of the Technology

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References

- Moursund, David (2002) Getting to the second order: Moving beyond amplification uses of information and communication technology in education. Available: http://www.uoregon.edu/~moursund/dave/second_order.htm [April 14, 2003].
- Pirie, S. E. B., & Kieren, T. E. (1990, April). *A recursive theory for mathematical understanding – some elements and implications.* Paper presented at the American Educational Research Association Annual Meeting, Boston, MA.
- Pirie, S. E. B., & Kieren, T. E. (1994). Beyond metaphor: Formalising in mathematical understanding within constructivist environments. *For the Learning of Mathematics, 14*, 39-43.
- Rotman, B. (1993). *Ad infinitum: The ghost in Turing's machine.* CA: Stanford University Press.
- Rotman, B. (1995). Thinking dia-grams: Mathematics, writing, and virtual reality. In B. Herrnstein Smith & A. Plotnitsky (Eds.), *Mathematics, Science and Postclassical Theory* (pp. 380-416). Durham, NC: Duke University Press.