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Teaching Mathematics by Math-XP

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Abstract

Math-XP [1] is a new and free dynamic geometry system (DGS) with a prover and solver. The prover is a geometry information searching system (GISS). The solver is a computer algebra system (CAS). Unlike the previous software, it can produce the traditional readable proof automatically for a geometry problem or give out the readable solution process for an algebraic problem. It is definite that the readable process is very helpful for the mathematics education in the meaning of pedagogy. This paper introduces the main functions of Math-XP and demonstrates how to use it in teaching mathematics.

Key Words: Math-XP, DGS, CAS, GISS.

1 Introduction

Lately, along with the rapid development of information technology, the dynamic geometry system (DGS) and the computer algebra system (CAS) are used popularly in teaching mathematics. The common DGS includes the *Geometer's Sketchpad*, *Cinderella* and *Cabri*. The famous CAS is *Derive, Maple* and *Mathematica.* As it is clear that the main functions of DGS facilitate users to define basic objects such as points, lines and circles and to construct geometric objects depending on them by classical constructions (straight edge and compass), linear transformation (translation, rotation, dilation, reflection) and/or algebraic relationships using coordinates (Cartesian and polar). Moreover, by the functions of dynamic measurements and drawing loci of moving points and envelopes of moving lines some very interesting geometric facts can be observed. But, if the users want a traditional proof process produced by computer automatically for a geometry problem, the previous DGS can not do it. Math-XP not only covers all functions of DGS, but also has a geometry information searching system (prover), GISS for short, to prove a geometry problem.

In fact, the CAS is a symbolic computation platform, which is a powerful tool of solving algebraic problems like factoring and solving equations, but solving geometry problems is a drawback of theirs. Generally, the answers given out by CAS have no readable solution process. Math-XP has a primitive computer algebra system (solver), which is designed for teaching mathematics especially, so it is distinctive that the readable solution processes can be generated by computer automatically for some algebraic problems. It is definite that the readable process is very helpful for the mathematics education in the meaning of pedagogy.

Math-XP in English Version was introduced in Visit-Me2002 [1] and CASUSA2003 [2], after the two conferences more and more researchers and teachers pay attention to it. The interface and DGS part of Math-XP is developed by Java, and the GISS and CAS parts are developed by Lisp. The Fig.1 is the main interface of Math-XP, which include 11 menu bars *File, Edit, Construct, Condition, Conclusion, Solution, Transform, Animation, Parameter, Algebra, Help* and 3 tool bars not listed in the menu *Formula, View, Greek*. The running environment of Math-XP is Windows 9X/2000/XP; the interested people can download it and its manual from following web site: *http://www.acailab.com/english/download.htm.*

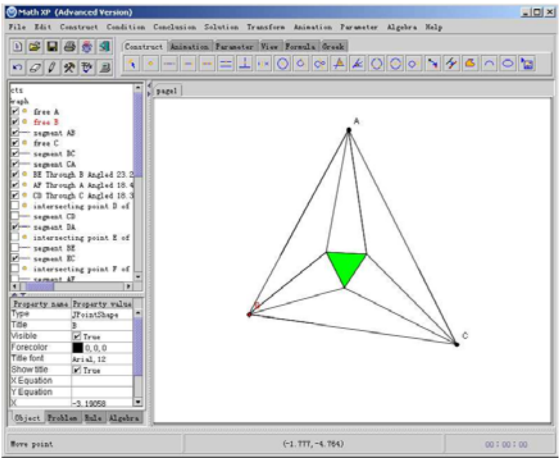

Figure 1: The Interface of Math-XP

2 Dynamic Geometry Systems

Similarly, Math-XP enables users to define objects (sometimes called primitives - i.e. building blocks) such as points, straight objects (segments and lines) and circles (or arcs). From these one can construct further geometric objects (which are dependent on them) by classical constructions of the"straight edge and compass" type (midpoints, bisectors, parallels, perpendiculars etc.), and/or linear"affine" transformations (translation, rotation, dilation, reflection) and/or algebraic constrains. Measurements may be made of objects, such as the coordinates of a point, the length of a segment, the size of an angle and the value of an arbitrary expression. At last, the finished courseware can be published on web also. Compared with previous DGS, two distinctive features can be figured out in Math-XP. First, the algebraic constrains can be automatically constructed by Math-XP. For example, if the user want to let the length of a segment constantly equal the plus of other two segments or the ratio of the lengths of two segments, without drawing, the user can directly add the algebraic constrains on the free points by the condition in Math-XP so that the resulting geometry construction satisfies the constrains automatically. Second, there is a very convenient tool to input math formula and the dynamic parameters can embedded into a math formula. The tool is similar to the math tools of MS Word, but it is different, the formula can be changing according to the changing of parameters. With the additional functions, one can design some more complicated loci easily. Here it does not describe how to use the DGS of Math XP in detail. Instead, it leaves an interesting example; the reader can use different DGS to draw out the loci, and will find Math-XP is very exciting.

Example 2.1 *what are the loci of the circle centers, which are tangent with two fixed circles.*

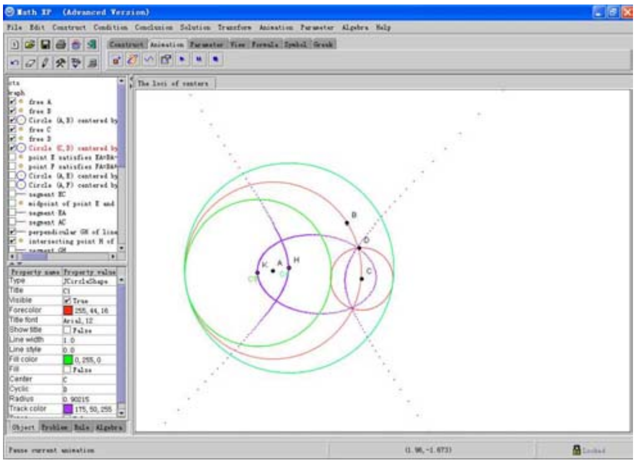

Figure 2: The Loci of Example 2.1

For the different locations of the two fixed circles, it seems that the loci may be very different, when the relation of the two circles is inclusion, the loci are two branches ellipses; when the relation of the two circles is intersection, the loci are one branch of ellipse and one branch of hyperbola; when the relation of the two circles is separation, the loci are two branches of hyperbole.

3 Geometry Information Searching System

Firstly, here is a simple geometry problem *"imagining an arbitrary quadrilateral with its four midpoints connected to form a quadrilateral inside it. The users wonder whether this inner quadrilateral possesses any special properties, and if these properties continue to hold when the vertices of the outer quadrilateral assume different locations?"*

By any dynamic geometry software, with a few clicks and movements of the mouse, the construction is complete: there on the screen is a model of the quadrilaterals that can be freely manipulated. Some dragging of the outer quadrilateral's vertices convinces the user there is a behavior worth noticing: *"the inner quadrilateral appears to stay a parallelogram for seemingly every outer quadrilateral"*.

But saying *"seemingly every"* is not enough, of course. Mathematics requires strict logic proof. Having used the software to convince ourselves that a theorem seems plausible, one may then turn to a traditional paper-and-pencil medium to ponder a proof. At the meanwhile students should be aware that experimental evidence alone doesn't constitute proof, the scenario described above submits such a problem: *if a dynamic geometry could contain an automated reasoning engine to produce a traditional readable proof automatically for substituting parts of our brain's work?*

In the end of 70's last century, Wu [3] proposed a mechanical algebraic method to prove geometry theorems automatically by a computer. It is a complete method by which mathematicians can proof almost all constructive geometry theorems. But the proof process is a series of algebraic eliminating variables, the distance from traditional Euclid proof is too far. Till the middle of 90's last century, Zhang [4] proposed a new readable method based on area invariants, the proof process is shorter and more readable than Wu's method, but it is not the traditional proof taught in schools also. Symbolic computation software like *Mathematica* and *Maple* provide powerful tools to solve mathematical problem, but solving geometric problem is a weak point of theirs.

In order to get the readable proof, researchers of this paper have to introduce the GISS based on the logical method. Though it is not a complete method, indeed it is very practical approach to serve the mathematics education. In following part, here are two examples to illustrate how to use the Math-XP in proving the geometry problem.

Example 3.1 *(Simson Theorem) C, D, and E are the points on circle A, B is the other point on the same circle. F is the perpendicular foot from point B to line CE; G is the perpendicular foot from point B to line CD; H is the perpendicular foot from point B to line DE. The question is to show that Points F, G and H are collinear.*

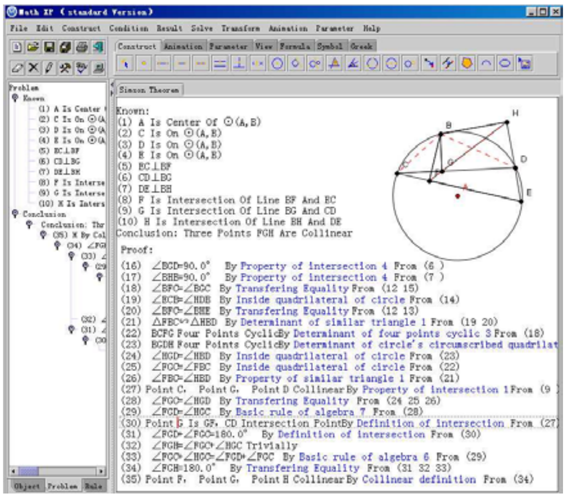

Figure 3: Simson Theorem proven by Math-XP.

Example 3.2 *(The Five Circle Theorem) $P_0P_1P_2P_3P_4$ is a pentagon. Point Q_j is the intersection of lines $P_{j-1}P_j$, $P_{j+1}P_{j+2}$, and M_j (differ from P_j) is the intersection of circle $(Q_{j-1}P_{j-1}P_j)$ and circle $(Q_jP_jP_{j+1})$ (the subscripts are understand to be mod 5). The question is to proof that points M_0, M_1, M_2, M_3, and M_4 are on one cyclic.*

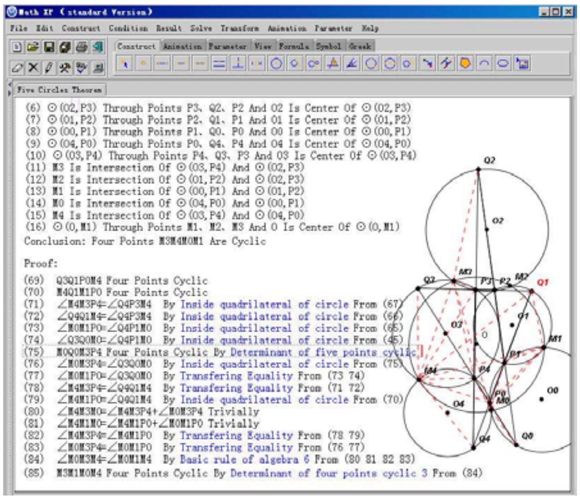

Figure 4: The Five Circle Theorem proven by Math-XP.

4 Computer Algebra Systems

Though the computer algebra system of Math-XP is simple, it has a distinctive character, that is, the CAS part not only gives out the answers, but also produces a readable solution process. This is different from *Maple* and *Mathematica*.

Example 4.1 Proving $n + \frac{4}{n^2} > 3$, *while $n > 0$.*

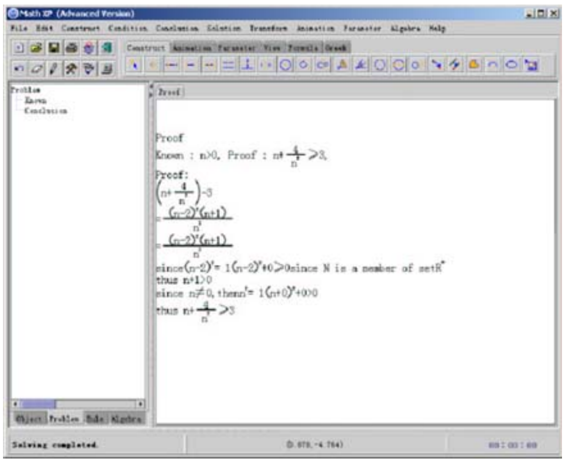

Figure 5: Inequality proving

Example 4.2 *Finding the definition domain of the function:* $y = \sqrt{\log_2(x^2 - x)}$.

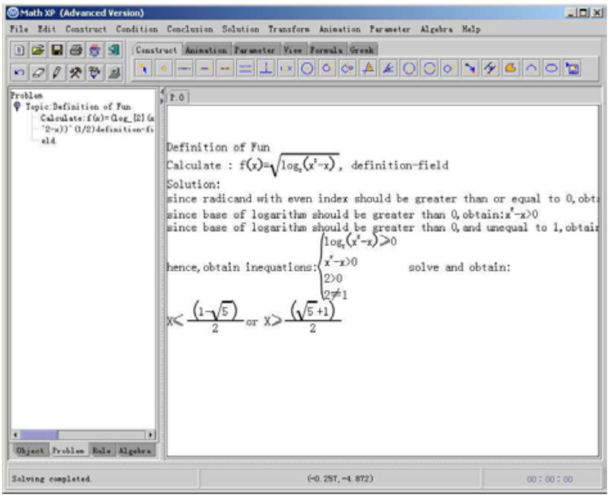

Figure 6: Definition of function.

Example 4.3 *Solving equations of* $\begin{cases} x^2 + y^2 = 4 \\ y = 2x \end{cases}$

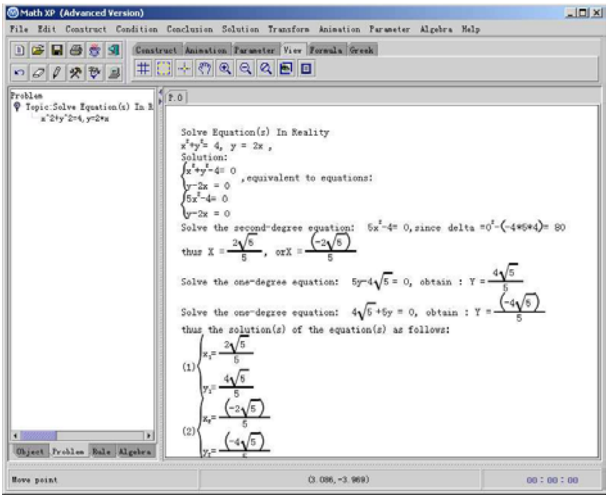

Figure 7: Solving equations in $\mathbb{R}$

5 Conclusions

Base on the basic frameworks mentioned above, researchers of this software keep on developing Math-XP. In the future, the technical trend of Math-XP should contain the following aspects:

1. Extending Math-XP to solid geometry problems.
2. Mechanizing other traditional skills such adding auxiliary points and congruence method to enhance the efficiency of the automatic reasoning engine.
3. Let users add rules in the rule database, so that the system has the learning ability.
4. Developing more convenient inputting methods including handwriting and nature language in mathematics.

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By permitting students to experience mathematics, Math XP will affect the learning process and, as a consequence, can put pressure on teaching process. In this way, so long as adequate conditions are given, the new technology can reinforce the change process that is taking place in the teaching and learning in some areas of mathematics. And the process of change will be consolidated through curricular innovation and teacher preparation.

References

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