The assessment of mathematics skills in end of secondary school 'high stakes' CAS assumed examinations.

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Until recently much of the focus regarding the introduction of CAS into assessment at the end of high school has been on whether to, or not to, use CAS. However, it is now generally accepted that whilst there is still a long way to go, the danger presented by the introduction of CAS into examinations is not as significant as first thought. Of increasing interest is whether the introduction of CAS will lead to a change in the skills assessed, and if so in what way.

Introduction

There has been a growing interest and use of computer algebra systems (CAS) in end of high school 'high stakes' examinations. Within this context concerns have been expressed regarding the types of skills that are required to undertake CAS assumed examinations. A study was undertaken on the use of CAS in two national examination systems: the Danish Gymnasium, Denmark (DME), and the Victorian Certificate of Education in Victoria (VCE), Australia along with the Advanced Placement (AP) Calculus examinations in the USA. Within this context the forms of examination questions used will be described followed by a summary of the role of the CAS in these questions. A typology is developed for the analysis of the skills tested within CAS assumed mathematics examination questions. This typology is then used to investigate the skills assessed within the three examination boards CAS assumed examinations.

Examination Boards and their examinations

Three examination boards were used in this study two of which offered high stakes examinations for end of high school certification whilst the third, AP, is used for advanced credit for college entry in the USA. Mathematics examinations are made up of a combination of types of questions, these types are:

Multiple-choice questions: are common where computer marking is used.
Short Response questions: questions requiring two or three steps to solve them.
Extended Response questions: questions in which students are required to tackle a number of connected sub questions in the context of the overall question.
Oral Examinations: Whilst not the focus of this paper the DME also includes Oral examinations as part of it assessment process at the end of the gymnasium (secondary school).

The subjects included in this paper are those for which the examination boards has assumed the use of CAS in at least part of the examination system, these subjects along with the role of CAS within the examinations is summarised in Table 1.

Table 1 The examination boards and their respective subjects examinations used in this study.

Examination	**DME A and B-level courses**	**VCE Mathematical Methods Pilot CAS Project**	**AP AB Calculus**
Multiple Choice	Not Applicable	CAS assumed	Part of which assumes Ti89
Short Response	CAS excluded	CAS assumed	Free Response
Extended Response	CAS assumed	CAS assumed	Part of which assumes Ti 89

Potential role of the use of a computer algebra system in examination questions

To investigate the style of questions that are being used in CAS assumed examinations a categorisation scheme developed by Brown (In Press) has been used for this purpose, and is summarised in Table 1 (see Brown (2003) for examples of applicatiuon of the scheme).

Table 1 Categorisation scheme used for the analysis of the potential role that the graphics calculator or computer algebra system plays in the solution process of examination questions

Potential Role of a computer algebra system in examination questions	
Required	The question cannot be reasonably solved without the use of a computer algebra system (on a computer algebra system assumed examination).
Optional	A computer algebra system could contribute to the solution of the question but its use is not necessary it could be done as easily using a graphics calculator or pencil and paper techniques.
Neutral	The computer algebra system has no potential to contribute to the solution of the question e.g. 'show' that questions or conceptual graphical questions.
Excluded	The question is deliberately structured (or worded) in such a way that a computer algebra system cannot be used directly in its solution. This category is problematic in CAS assumed examiantions.

The categorisation scheme was applied to the 2003 CAS assumed examinations for the three examination boards and the results have been summarised in the form of parallel-segmented bar charts in Figure 1

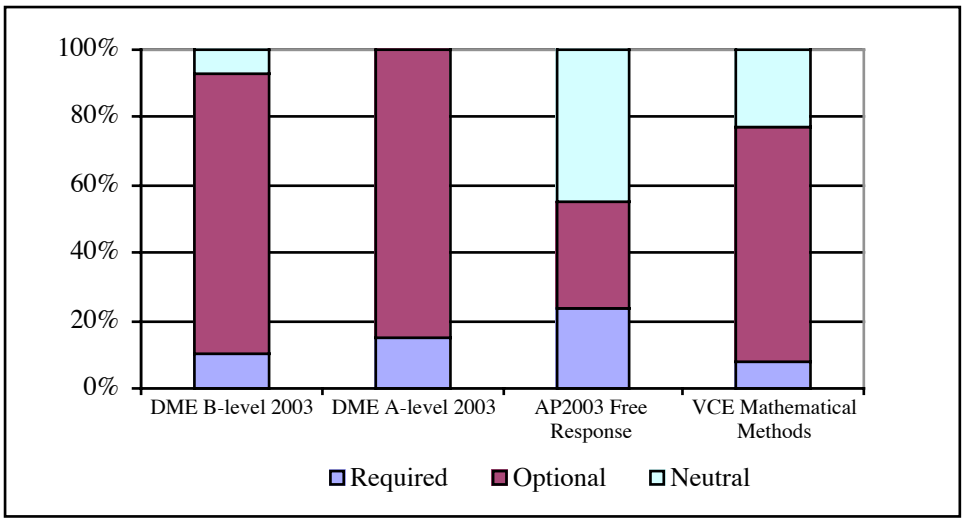

Figure 1 The overall profile for CAS usage in the CAS assumed examinations for 2003.

It is clear that all examination boards make use of 'required', 'optional' and 'neutral' questions whilst there is no evidence of the use of the 'excluded' category, perhaps unsurprisingly. It is also evident that with the exception of the AP Calculus examination the highest proportion of questions are CAS 'optional' implying that students are free to choose to use the CAS, or may undertake tasks using other techniques. Of interest is the high proportion of 'neutral' questions for the AP implying that many questions are written such that the CAS has no role to play.

In summary, the examination boards only use a small proportion of CAS 'required' questions whilst for the VCE and the DME the highest proportion of questions are CAS 'optional' whereas for the AP the highest proportion of questions are CAS 'neutral'.

Determining mathematics skills assessed in a CAS environment

A number of authors such as (Galbraith & Haines, 2000; Leinbach, Pountney, & Etchells, 2002; Poutney, Leinbach, & Etchells, 2002; Sangwin, 2003; Smith, Wood, Coupland, & Stephenson, 1996) have developed a taxonomies for the analysis of examinations in terms of the mathematical skills required to answer examination questions especially, for university mathematics courses. The schemes of (Galbraith & Haines, 2000 and Sangwin, 2003) are summarized in Table 2.

Table 2 Schemes developed for the categorisation of mathematics skills assessed in examination questions.

Galbraith and Haines	**Sangwin**
Mechanical: require the performance of some standard procedure that is cued in the wording of the question.	Factual Recall Routine calculation Classify a mathematical object Substitute into a formula
Interpretive: require the retrieval of conceptual knowledge and its application, do not involve mathematical procedures.	Interpret situation
Constructive: involve the use of both conceptual and procedural knowledge in which necessary procedures have to be introduced by the student. Responses involve the construction of a solution …	Prove show justify Extend a concept Construct an example Criticize a fallacy

Using the work of (Galbraith & Haines, 2000; Sangwin, 2003; Smith et al., 1996), with appropriate modifications, has led to the development of a categorisation scheme outlined in Table 3. There are three primary categorisations of mathematical skills, which are labelled, after Galbraith & Haines (2000), as mechanical, interpretive and constructive. Within each of these categories there are descriptors (or subcategories) based on Smith et al. (1996) and Sangwin (2003). The scheme as elucidated serves two purposes. Firstly, the hierarchical nature of the general skill level categories, mechanical, interpretive and constructive, as demonstrated by Galbraith & Haines (2000), enable the scheme to be used to monitor changes in difficulty levels of examinations that have been suggested as a possible outcome of allowing the use of CAS. Secondly, the sub categorisation of each of the general skill level categories after Smith et al. (1996) and Sangwin (2003) enables the scheme to be used to monitor any changes in types of questions used within skill levels.

Table 3 Mathematical skills categorisation scheme used for this study.

Mathematical Skills	Descriptor
Mechanical	*Factual knowledge*: These are recall of knowledge type problems. *Comprehension*: The ability to understand a problem and to select and use a standard technique. Significance of symbols in a formula, Ability to substitute into the formula. *Routine use of mathematical procedures*
Interpretive	*Information transfer*: Transforming the information given into a different form for use in a different procedure *Applications in new situations*
Constructive	Considering Implications, Comparing, Conjecturing, Justifying, Evaluating

The categorisation scheme was applied to the 2003 CAS assumed examinations for the three examination boards and the results have been summarised in the form of parallel-segmented bar charts in Figure 2.

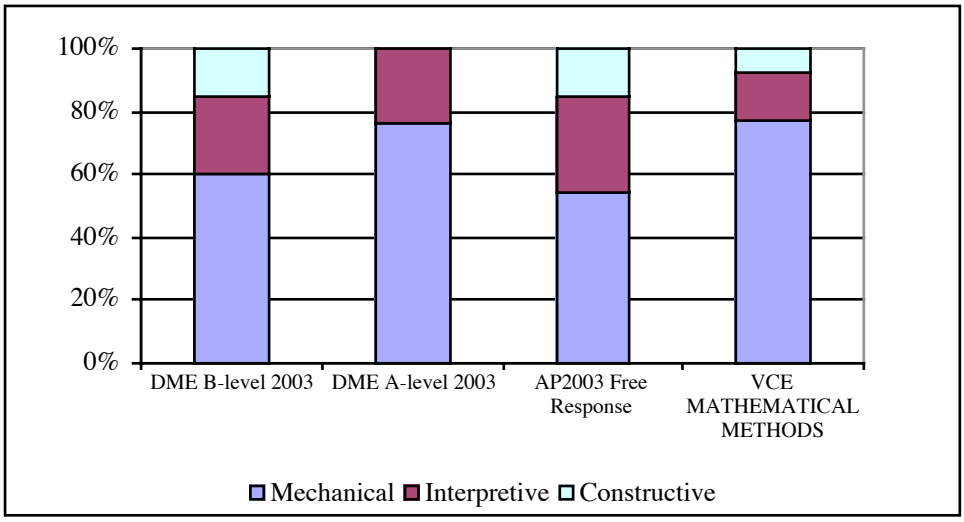

Figure 2 The overall profile for the percentage of marks awarded for the mathematics skills assessed in the CAS assumed examinations for 2003.

It is evident that there is a high proportion of marks awarded for the testing of mechanical skills, with the smallest proportion for the testing of constructive skills. The AP has used the least proportion of questions that are categorized as mechanical and the following statement supports this:

> *One of the primary goals of the changes that AP Calculus has undergone in the past several years is to move away from testing of rote manipulation and toward problems that probe an understanding of the fundamental concepts*. (College Board, 2003)

Indicating a general policy towards to the use of interpretive and constructive questions.

With the exception of the DME A-level, the examination boards have been using 'show that' questions which according to Flynn & Asp (2002) are intended to "assess students' ability to construct and present a logical well-reasoned mathematical argument using a sequence of intermediate steps … [and to] increase the accessibility by supplying a result required in later question parts." The increased use of such questions may lead to an increased difficulty in parts of the questions that require the students to 'show that' but allows the students to use a given result in a latter part of the questions, thereby simplifying the latter part of the problem for the student.

In summary, the examination boards are using a greater proportion of questions that test mechanical skills, than the other categories, supporting the work of Brown (In Press) who has reported similar findings for graphics calculator assumed examinations. Indicating that the introduction of the assumed use of the CAS has not led to an increase in the testing of interpretive or constructive skills, implying that the level of difficulty of the examination has not increased as a result of the introduction of the CAS.

Final comments

Whilst this study only reports on one year of CAS assumed 'high stakes' end of secondary school mathematics examinations some observations can be made. It is apparent that examiners are using a high proportion of CAS 'optional' questions and a lesser proportion of CAS 'required' questions. Similarly, there is a high proportion of questions testing mechanical skills and lesser proportions testing interpretive and constructive skills. Within the context of testing constructive skills there is evidence of the use of 'show that' questions. Further studies would need to be carried out to determine if there are changes in the proportions of CAS 'optional' and 'required' questions. Similarly, further studies are required to determine whether there are changes in the proportions of 'mechanical', 'interpretive' and 'constructive' skills assessed within CAS assumed examinations.

In conclusion, this small study indicates that the development of CAS assumed examinations has not brought about educationally significant changes in the assessment of mathematics at the end of high school.

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