CONTS

Experiences with the use of graphic calculators in Saxony

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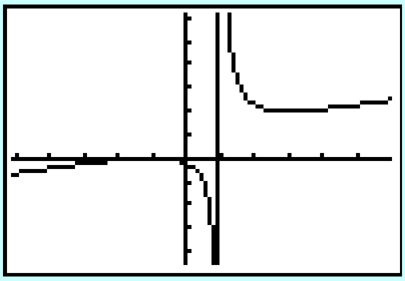
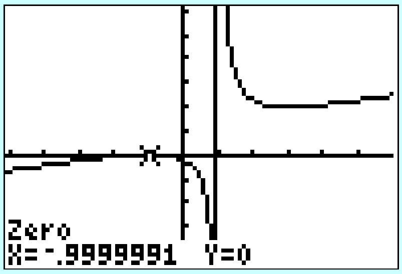
An example for an exercise in a German school-leaving examination in 1994

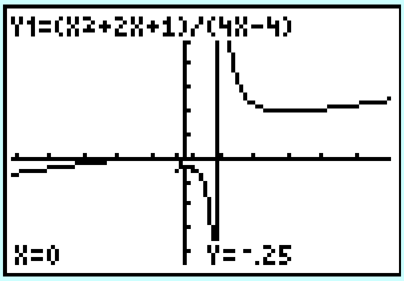
There is given a function with the equation $y = f(x) = \dfrac{x^2 + 2x + 1}{4x - 4}$.

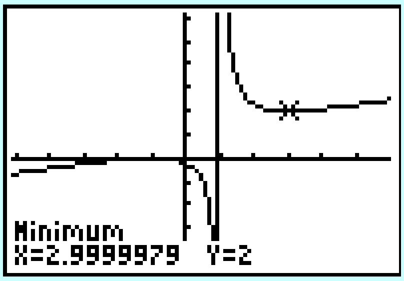
Discuss the characteristic attributes of the curve. Calculate the zeros, the coordinates of the intersection point with the y-axis and the coordinates of the local minimum-points or maximum points.

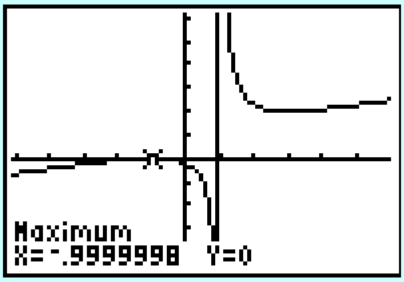
A solution with the grafic calculator:





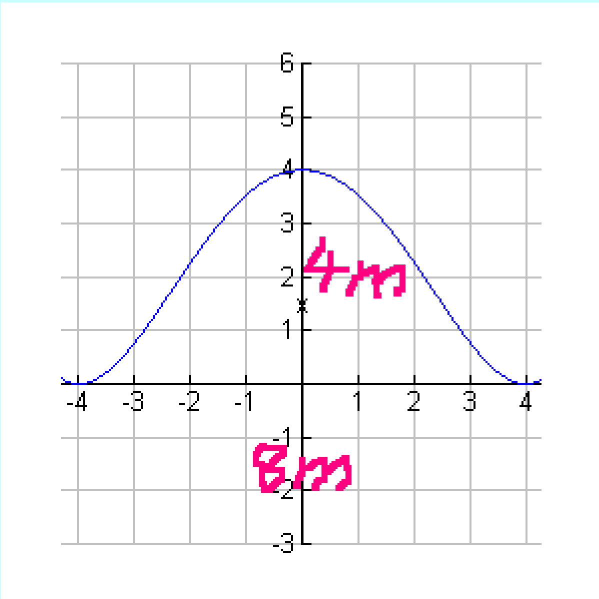






example for an exercise in a saxon school-leaving examination in 1999

The symmetrical gable of a house had to been reconstructed. The picture showed the gable in a coordinates-system. A symmetrical polynom-function describes the border of the gable. The x-axis is a tangent to the graph of f in the Points $P_1(-4;0)$ and $P_2(4;0)$. The altitude of the gable is 4,0m over the border of the roof.



a) Prove, that the degree of the polynom function is at least 4!
b) Determine a equation of the function.
c) The area of the gable should bee divide by an horizontal line into two parts with equal areas. (A painter will coloured the gable with two different colours.) Calculate, in witch altitude the border of the colours should bee.

Now a fictitious example for an exercise in a saxon school-leaving examination in 2005

Describe the form of the gables with mathematical methods!

(Dresden-morning-post: 30.1.2004):

Schon 5 000 Ermittlungsverfahren in diesem Ja

DVB gnadenlos! Jeder Schwarzfahrer wird angezeigt

Von Jens Jungmann

The streetcar company in Dresden is merciless!

institute 5000 court proceedings against “Black drivers”

Passenger without ticket: “Black drivers”

Every passenger should bee checked average at least once in three month.

facts:

Everyday use 480 000 passengers the streetcar. With a honest passenger the company earn 0,40€, with a black driver 40€ fine (punishment).
The 456 000 honest passengers pays on every day 182 400€, the 24000 “Black drivers” had to pay 960 000€, if a streetcar-company guards all of they would find out.

But:
If all 480 000 passengers would pay honestly, the streetcar-company would earned only 480 000 • 0,40€ = 192 000€.

The Problem is:
The company had to check the tickets exact so often, that the company get sufficient money and the “Black drivers” would bee preserve.

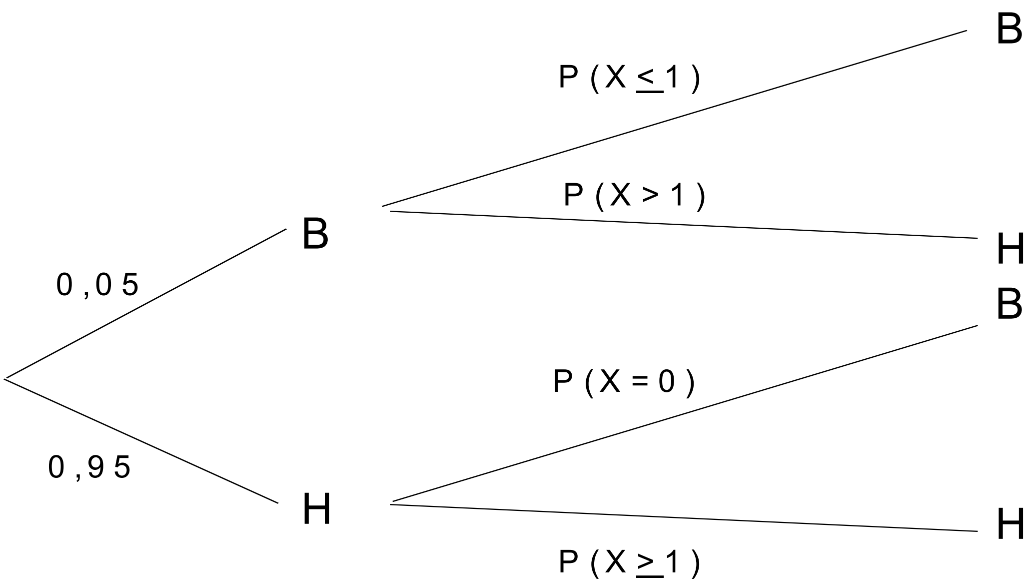
Which number of inspectors is necessary for the biggest profit of the company?

We look at a period of time of three mounth, nearly 180 drives for a student.

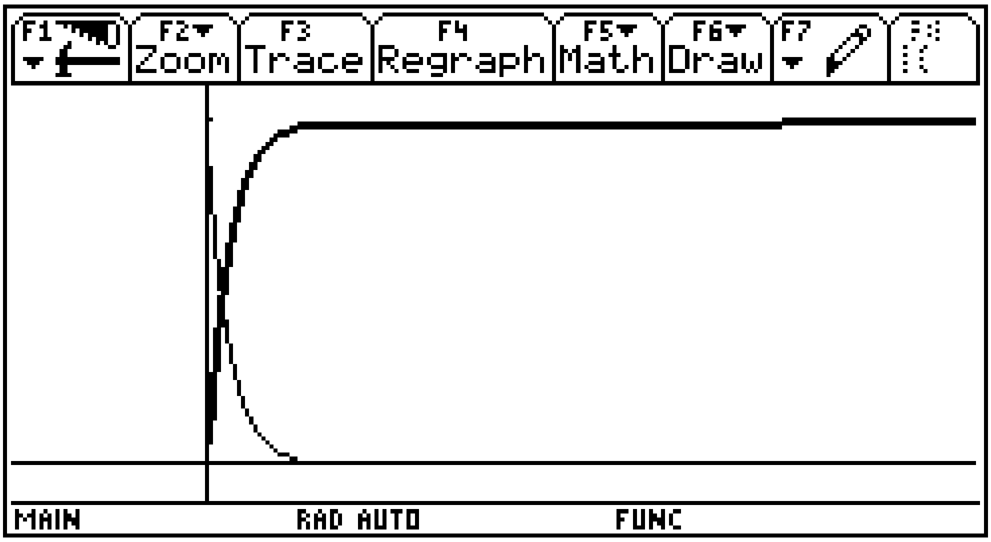
Assumptions:

- At the beginning there are 5% “Black drivers”
- If a „Black driver“ would bee catcht at least two times in the period, he will change to a honest passenger
- If a honest passenger would bee never checkt in the period, he will change to a black driver in the next time.

X is the number of checks in the period for any passanger. X is Distribution of X is binomial with n=180 and unknown probability p. p is the probability, that a passenger would bee checkt.

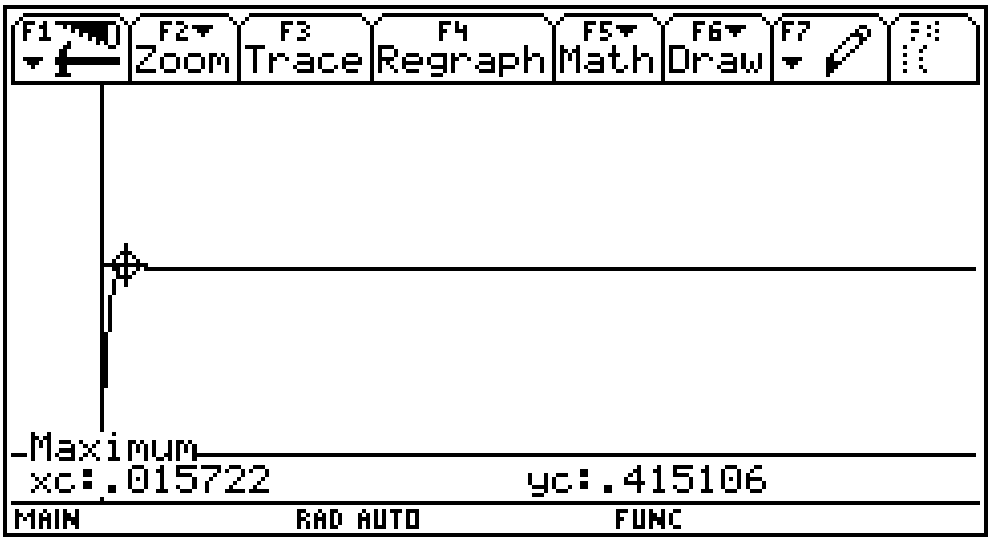


$y1 = 0.05 \cdot \text{binomcdf}(180, x, 1) + 0.95 \cdot \text{binomcdf}(180, x, 0)$ und
$y2 = 0.05 \cdot (1 - \text{binomcdf}(180, x, 1)) + 0.95 \cdot (1 - \text{binomcdf}(180, x, 0))$.



random variable Z: profit of the streetcar-company in the period for any passenger in dependence of the probability p

z_i	0,40€	40,00€	0,00€
$P(Z = z_i)$	$y2(p)$	$p \cdot y1(p)$	$1 - (p \cdot y1(p)) - y2(p)$



For p = 1,6% earn the company the highest profit.

Compare the model and the reality:

	model	reality
number of inspectors	13	30
number of checks (600 per inspector per day)	7800	18000
check-probability p (480 000 passengers)	0,01625	0,0375
average earn of the company per passenger (Z)	0,415€	0,40€
daily earn of the company (480 000 Fahrgästen	199 200€	192000€

result: If the streetcar-company would dismiss 17 inspectors, they would earn daily 7200€ more.

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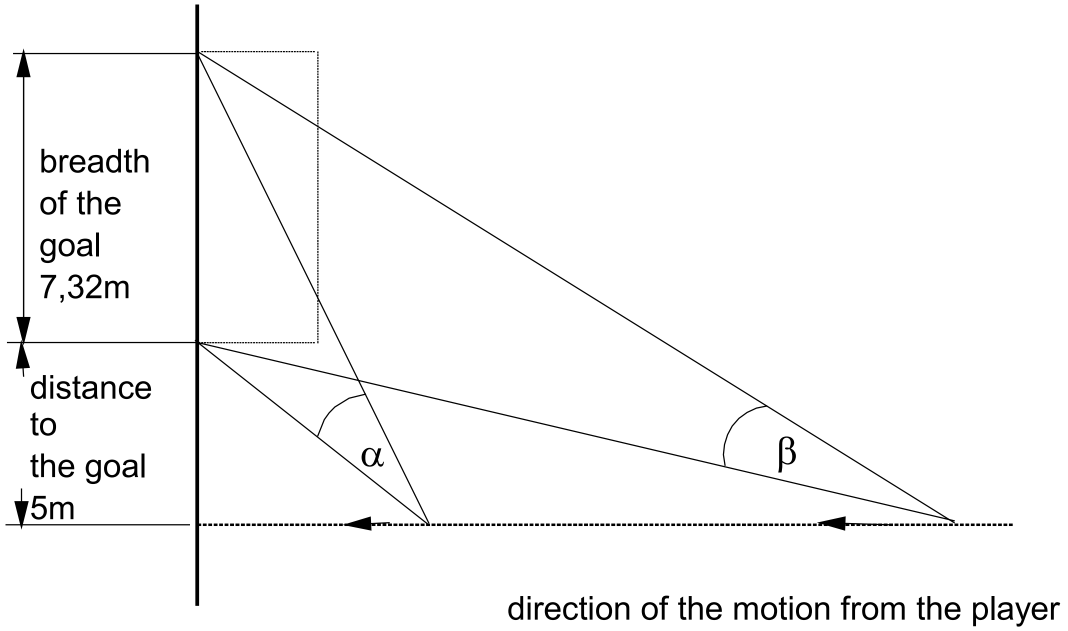
Why soccer teams don´t use the TI-83?

In october 2001 was a match between germany and finland. the result was 0:0 and the press was disappointes and enraged.
That was not particulary funny.
Especially the german player Oliver Bierhoff was in the centre of the criticism, because he did´nt hit the goal from a distance of 8 meters.
A student asked “Why 8 meters? Is it more terrible to miss the goal from a distance of 6 meters or 10 meters?



The forward goes along a imaginary line, parallel border of the field in the direction of the opposing goal.
We estimate the distance from this line to the goal about 5m.
The breadth of the goal is 7,32m.

Look for the function: “distance of the forward from the demarcation of the field in line with the goal → angle of the shoot into goal”.

distance → angle

For example: α and β in the figure.

Exist a maximum for this angle for the shoot into the goal?

What is the optimal distance for shooting?

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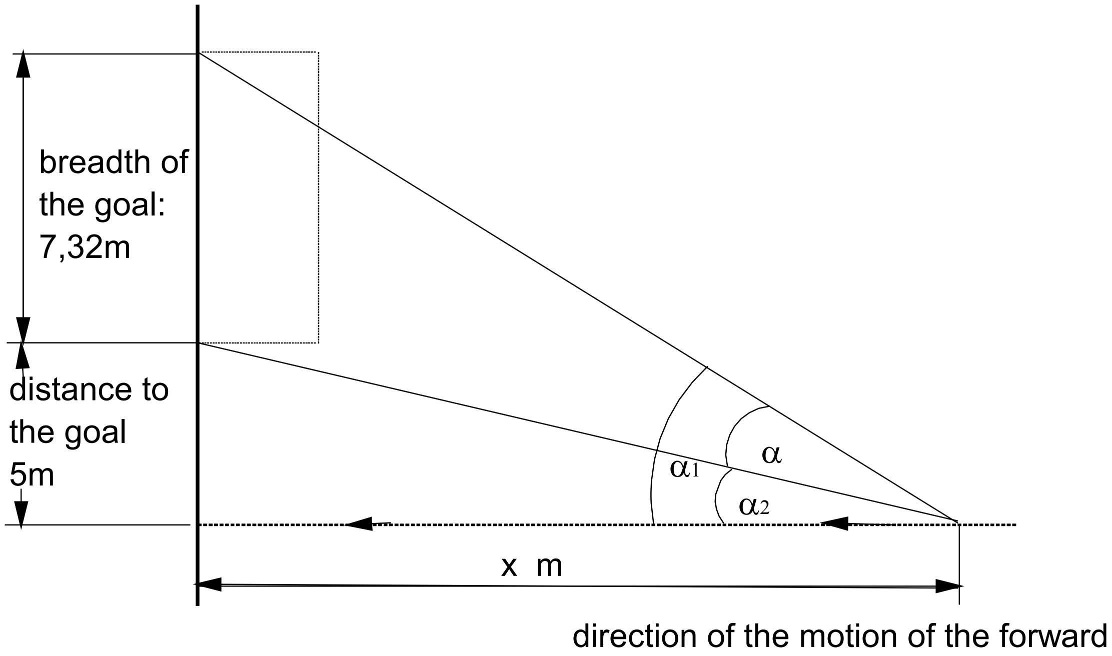
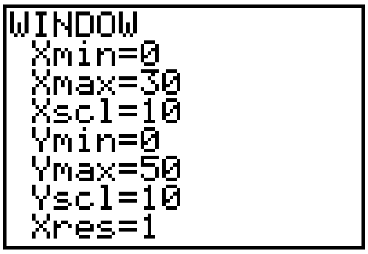


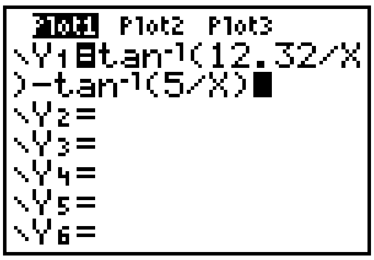
$$\alpha = \alpha_1 - \alpha_2 = \tan^{-1}\left(\frac{12{,}32\text{m}}{x\text{m}}\right) - \tan^{-1}\left(\frac{5\text{m}}{x\text{m}}\right).$$

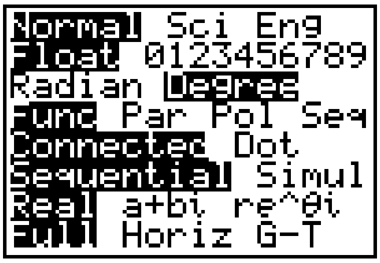
TI-83 Plus:

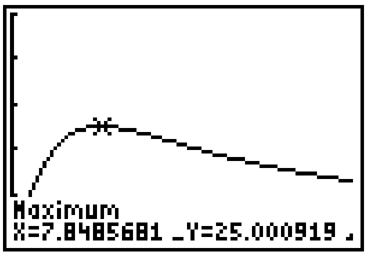
Distance for shooting: x-axis   angle: y-axis

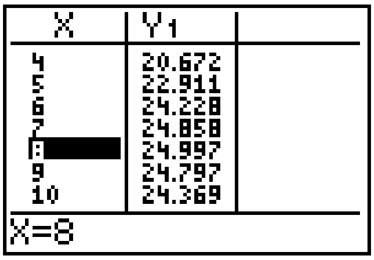
Adjustment of the TI-83





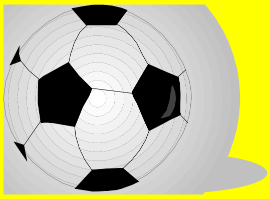


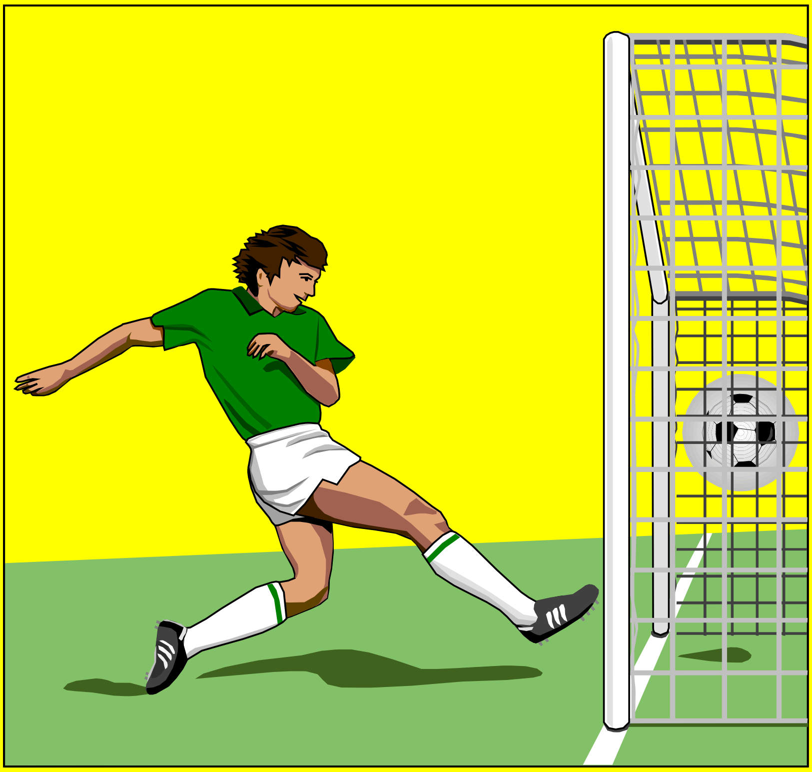




result:

The optimal shooting distance is 8 metres, the Dresdner morning post was on the right track!!
The optimal angle is about 25°.

What a surprise! 



The sum of the square number of three one after another following natural numbers is 590. Calculate this natural numbers.

teacher´s expected solution:

$$n^2 + (n+1)^2 + (n+2)^2 = 590$$

$$n^2 + n^2 + 2n + 1 + n^2 + 4n + 4 = 590$$

$$3n^2 + 6n + 5 = 590 \mid -590$$

$$3n^2 + 6n - 585 = 0 \mid \div 3$$

$$n^2 + 2n - 195 = 0$$

formula

$$x_{1;2} = -\frac{p}{2} \pm \sqrt{\frac{p^2}{4} - q}$$

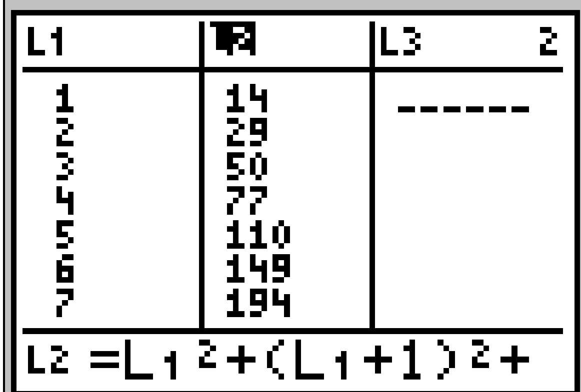
$$= -1 \pm \sqrt{1 - (-195)}$$

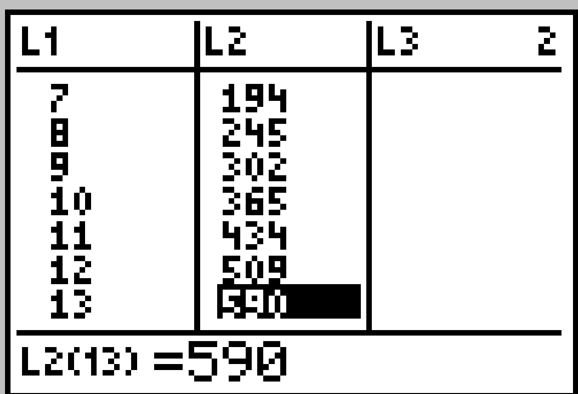
$$= -1 \pm \sqrt{196}$$

$$x_1 = 13 \text{ (trifft zu)}$$

$$x_2 = -15 \text{ (entfällt)}$$

student´s solution





Reactions of students

Sascha A. (14):
We have concluded to found a programmer-club – and you are our boss.

Claudia Ö. (16):
I thing, mathematics is just as cool like music.

Nicole G. (14):
My mother had said, I should like to thank you for introduce the calculators in math – lessons. And I should give you a kiss, but I don´t dare.