CONTS

Paraxial Approximation

Teaching Geometrical Optics with *DERIVE*

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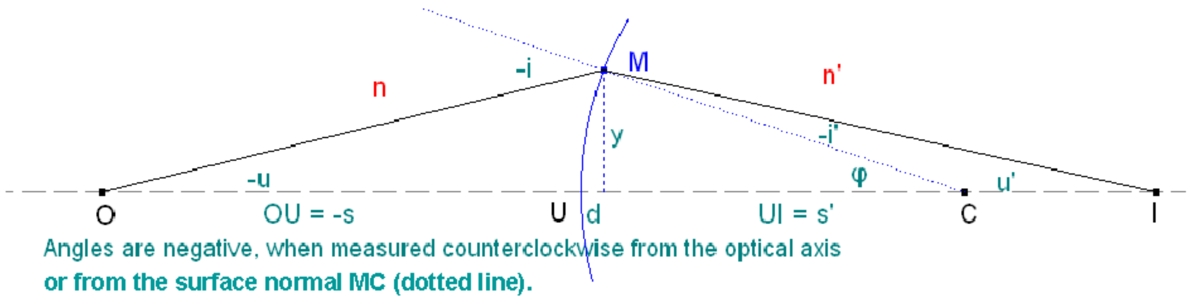
This lecture is based on a Leon Magiera´s paper and on a very extended exchange of emails between him and the presenter who produced the final presentation assisted by many comments of Leon. All graphics are Derive-2D-plots derived from Leon´s sketches.

Tracing a ray of light through an optical system consists on calculating the changes in ray slope angle caused by reflection or refraction and the changes in ray height on transition from one surface to the next.

The paraxial region of an optical system is a threadlike region about the optical axis which is so small that all the angles made by the rays (i.e., the slope angles and the angles of incidence and refraction) may be set equal to their sines and tangents. Under this approximation (first order optics) ray tracing formulae are very simple.

It will be demonstrated how to apply these simple formulae to calculate the important optical quantities (focal length, back focal length, aperture stop, entrance and exit pupil, size and image position ect.) and even for complicated multi-surfaces optical systems. Special cases of thick or thin lenses are considered and also multilens optical systems, in particular cemented and separated doublets.



Snellius: $n \sin(-i) = n' \sin(-i')$

Paraxial Approximation works under condition that all angles

i, i', u, u', φ are small and $d \ll r$.

$\Delta OMC \Rightarrow -i = \varphi - u$ and $\Delta IMC \Rightarrow -i' = \varphi - u'$

Snellius: $n(\varphi - u) = n'(\varphi - u')$ (under par. appr.)

Given are two refracting media separated by a spherical surface. From the point „*O*" an arbitrary ray *OM* is emitted (Fig 1). This ray is refracted at the point *M* and intersects optical axis at the point *I* (image point).

n- index of refraction on the left hand side of the surface,

n'- index of refraction on the right hand side of the surface,

r-radius of curvature of the surface,

s -object distance from the surface,

s' - image distance from the surface,

i - angle of incidence from the normal to the surface to the incoming ray,

i' - angle of refraction measured from the normal to the surface to the outgoing ray

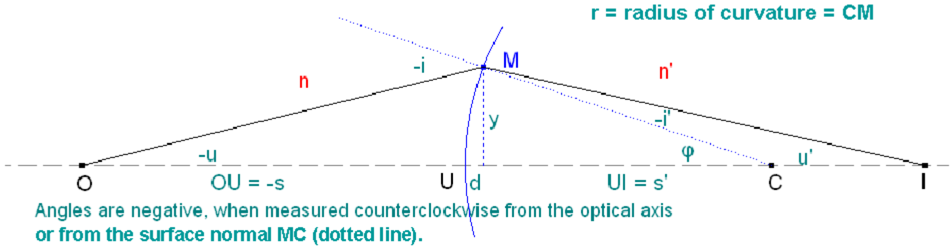
Sign Convention

1. Values on the left are unprimed, while values on the right are primed.

2. All distances are positive on the right of the surface from which they are measured.

3. Transverse distances are positive upward from the optical axis and negative downward, on the plane of the figure.

4. Angles are negative when measured counterclockwise.



$$n\,(\varphi - u) = n'\,(\varphi - u')$$

In Paraxial Approximation holds: $-u = \dfrac{y}{-s}$, $u' = \dfrac{y}{s'}$ and $\varphi = \dfrac{y}{r}$

$$n\left(\frac{1}{r} - \frac{1}{s}\right) = n'\left(\frac{1}{r} - \frac{1}{s'}\right)$$

Abbe - Invariant

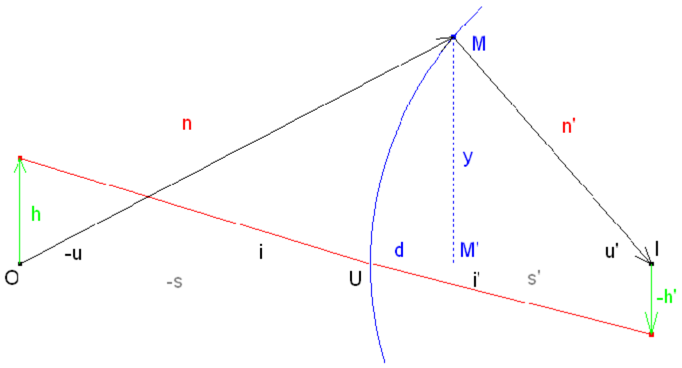
(cancellation of *y*!!)

So we find $n\,(\varphi - u) = n'\,(\varphi - u')$

The sketch shows that each ray emitted from O(bject point) will end in I(mage point). s' is independent of any y.

This leads to the Abbe Invariant. y is cancelled by calculation.

$-u$ and u' are tangent functions, φ is a sine function.



$$\text{Paraxial} \rightarrow \text{UM}' = \text{d} = \text{very small}: \quad \text{OU} \approx \text{OM}' \text{ and } \text{UI} \approx \text{M}'\text{I}$$

$$\text{Snellius:} \quad n \cdot i = n' \cdot i' \Leftrightarrow n\frac{h}{-s} = n'\frac{-h'}{s'}$$

$$\frac{h}{-s} = \tan i \approx i \text{ and } \frac{-h'}{s'} = \tan i' \approx i'$$

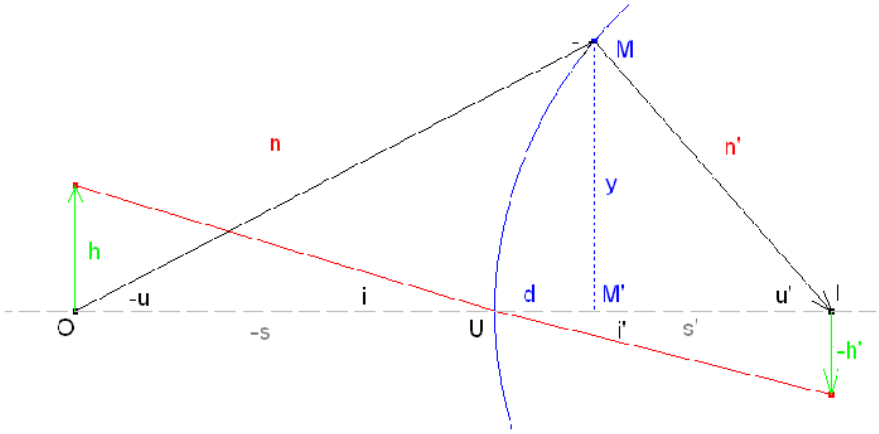
$$\text{and} \quad \frac{y}{OM'} \approx \frac{y}{-s} = \tan(-u) \approx -u \rightarrow y = s \cdot u; \rightarrow y = s' \cdot u'$$

What is the height (size) of the image of the vertical object with height h?

We eliminate s and s' doing some easy calculation which does not need now any trig function or trig identity we find the ....

Lagrange Invaríant ....

...... and maybe even interesting for practical application the Lateral Magnification given by an optical system.

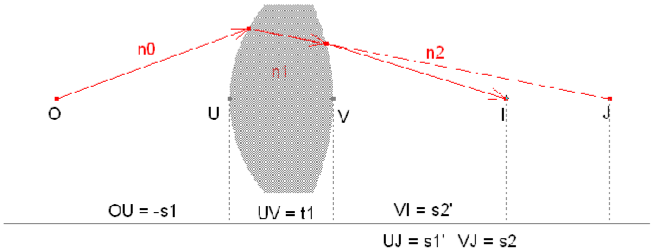


$$n \cdot h \cdot u = n' \cdot h' \cdot u'$$ **Lagrange Invariant**

$$\beta = \frac{h'}{h} = \frac{n \cdot u}{n' \cdot u'}$$ **Lateral Magnification**

Our first example is a single lens (two surfaces – spheres), three media, i.e. three indeces of refraction.

Single Lens



Our task is to find the image distance s2´:

$$[n0 :=,\ n1 :=,\ n2 :=,\ r1 :=,\ r2 :=,\ s1 :=,\ s1\_ :=,\ s2 :=,\ s2\_ :=,\ t1 :=]$$

$$eq1 := n0\cdot\left(\frac{1}{r1} - \frac{1}{s1}\right) = n1\cdot\left(\frac{1}{r1} - \frac{1}{s1\_}\right)$$

Abbe - Invariant

$$eq2 := n1\cdot\left(\frac{1}{r2} - \frac{1}{s2}\right) = n2\cdot\left(\frac{1}{r2} - \frac{1}{s2\_}\right)$$

$$s2 := s1\_ - t1$$

$y1/r1 = \varphi 1$, (sin); $y1/s1 = u1$ (tan) ..... the y´s are cancelled!!!

We would like to find $s2$‘ (which appears in Derive notation as s2_)

We solve for s2' (= s2_):

#5: s1_ := RHS(SOLVE(eq1, s1_))

#6: s2_ := RHS(SOLVE(eq2, s2_))

nice expression!!

$$s2\_ = \frac{n2\cdot r2\cdot(n0\cdot t1\cdot(r1 - s1) + n1\cdot s1\cdot(t1 - r1))}{n1\cdot s1\cdot(n1\cdot(r1 - r2 - t1) + n2\cdot(t1 - r1)) - n0\cdot(r1 - s1)\cdot(n1\cdot(r2 + t1) - n2\cdot t1)}$$

For a lens in the air we use the data:

$$[n0 := 1,\ n2 := 1,\ n1 := n]$$

$$s2\_ = \frac{r2\cdot(t1\cdot(r1 - s1) - n\cdot s1\cdot(r1 - t1))}{n^2\cdot s1\cdot(r1 - r2 - t1) + n\cdot(s1\cdot(r2 + 2\cdot t1) - r1\cdot(r2 + s1 + t1)) + t1\cdot(r1 - s1)}$$

and in case of a thin lens and the object in infinity (negative because left) we get:

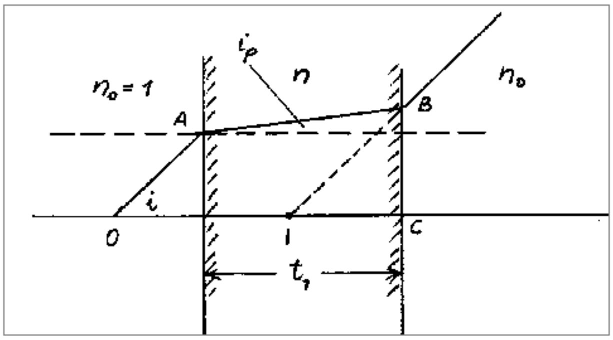
$$\lim_{s1\to-\infty}\ \lim_{t1\to 0}\ s2\_ = \frac{r1\cdot r2}{(n - 1)\cdot(r2 - r1)}$$

As you can see some manipulation is required, but our servant DERIVE has no problems and we obtain a „nice little“ expression for $s2$‘ = $s2$_

For a lens in air we can set $n0 = n2 = 1$, $n1$ = any arbitrary value n.

If the object has an infinite distance from the lens we need to find a limit.

Special Case: glass sheet
r1 = r2 = ∞



$$\lim_{r2\to\infty}\ \lim_{r1\to\infty} s2\_ = \frac{n\cdot s1 - t1}{n}$$

$$-s1 + t1 + \lim_{r2\to\infty}\ \lim_{r1\to\infty} s2\_ = t1 - \frac{t1}{n}$$

$$ip := \mathrm{ASIN}\left(\frac{\mathrm{SIN}(i)}{n}\right)$$

$$BC := y1 + t1\cdot \mathrm{TAN}(ip)$$

$$IC := \frac{BC}{\mathrm{TAN}(i)}$$

$$OI := \frac{y1}{\mathrm{TAN}(i)} + t1 - IC$$

$$n :\in \mathrm{Real}\ (0,\ \infty)$$

$$OI = t1 - \frac{t1\cdot \mathrm{COS}(i)}{\sqrt{(n^2 - \mathrm{SIN}(i)^2)}}$$

$$\mathrm{TAYLOR}(OI,\ i,\ 1) = t1 - \frac{t1}{n}$$

Take a special case:

The lens is a glass sheet:

Which is the distance OI??

Left bottom under paraxial approximation!

On the right we are doing it the classical way (trig functions) and finally applying Taylor Approximation.

Compare the results??

So why should we use cannon balls to shoot a mug??

Sweet memories on my (our?) school time physics classes.

Better: I „should“ remember, but Leon reminded me!

I remember from school:

$$\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{r1} - \frac{1}{r2}\right)$$

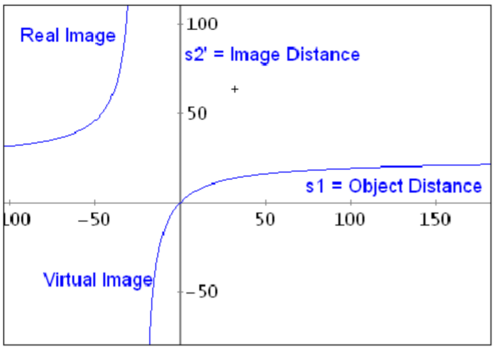
$$\text{SOLVE}\left(\frac{1}{f} = (n - 1)\cdot\left(\frac{1}{r1} - \frac{1}{r2}\right),\ f\right) = \left(f = \frac{r1\cdot r2}{(n - 1)\cdot(r2 - r1)}\right)$$

$$[r1 := 20,\ r2 := -30,\ n1 := 1.5]$$

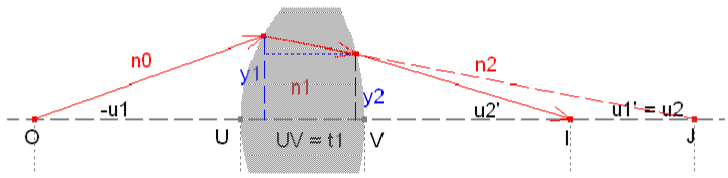
$$\text{SUBST}(s2\_,\ t1,\ 0) = \frac{24\cdot s1}{s1 + 24}$$

Object is left of surface: s1 < 0:

For |distance| <24 we don´t have a real image!



Only the left part of the hyperbola is of relevance, because only for object distances with absolute values > 24 we will obtain real images!



slope angles u can be used instead of distances s →

$$n(\varphi - u) \approx n\left(\frac{y}{r} - \frac{y}{s}\right) \approx n\left(\frac{y}{r} - \tan u\right) \approx n\left(\frac{y}{r} - u\right)$$

$$n_0\left(\frac{y_1}{r_1} - u_1\right) = n_1\left(\frac{y_1}{r_1} - u_1'\right)$$

$$n_1\left(\frac{y_2}{r_2} - u_2\right) = n_2\left(\frac{y_2}{r_2} - u_2'\right)$$

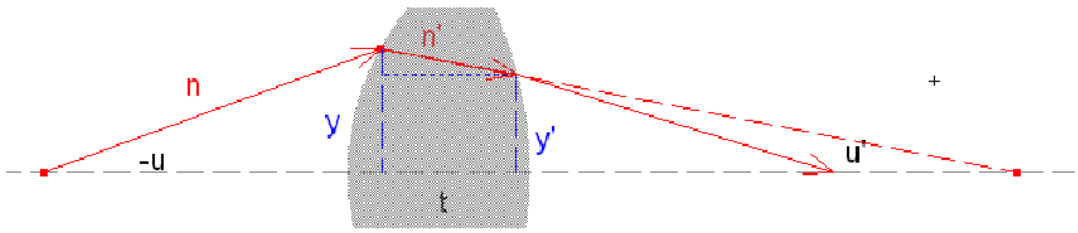
$$y_2 = y_1 - u_1' t_1 \quad \text{with} \quad u_2 = u_1'$$

Applying **Paraxial Approximation** we cans use the slope angles *u* and *u*‘ instead of distances *s* and *s*‘.

We again benefit from the Abbe-Invariant!! .... as we will do in the future!!

Paraxial ray tracing is based on two relations describing:

- the height of the ray passing the next surface (transition to the next surface)



$$y' = y - u' \cdot t$$

- the slope angle after refraction

The formula for y' can easily be derived from the sketch. Take into account that u' replaces tan(u')!!!

The slope angle after refraction:

#1: $\mathrm{SOLVE}\left(n \cdot \left(\frac{y}{r} - u\right) = n\_ \cdot \left(\frac{y}{r} - u\_\right),\ u\_\right)$

#2: $u\_ = \frac{n \cdot r \cdot u - y \cdot (n - n\_)}{n\_ \cdot r}$

$\mu = n/n' \rightarrow n = \mu * n'$

#3: $u\_ = \frac{(n\_ \cdot \mu) \cdot r \cdot u - y \cdot (n\_ \cdot \mu - n\_)}{n\_ \cdot r}$

#4: $u\_ = \frac{y \cdot (1 - \mu)}{r} + \mu \cdot u$

Abbe Invariant solved for u' (slope angle after refraction) angle between the refracted ray and the optical axis.

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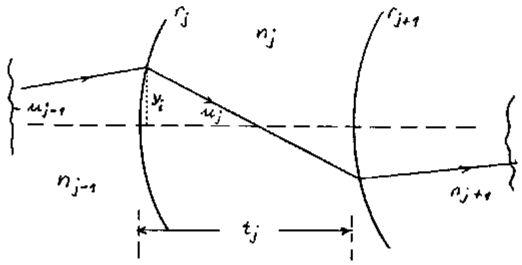
Second example: A Multi Surfaces Optical System

Multi – Surfaces Optical System

$$u_j = u_{j-1} \cdot \mu_j + \frac{y_j}{r_j}(1-\mu_j)$$

$$y_j = y_{j-1} - u_{j-1} \cdot t_{j-1}$$



The ray tracing formulae can be rewritten in general form for rays passing from one media to the other.

y_j is the height of the paraxial ray at the j-th surface and t_{j-1} is the distance between surface #j-1 and surface #j.

We obtain a system of mutual recursive equations. This seems to be an excellent occasion for working with Derive

$$u_j = u_{j-1} \cdot \mu_j + \frac{y_j}{r_j}(1-\mu_j)$$

$$y_j = y_{j-1} - u_{j-1} \cdot t_{j-1}$$

and its recursive Derive realisation:

```
u(j) :=
   If j = 0
      u0
      u(j - 1)·μ↓j + y(j)/R↓j·(1 - μ↓j)

y(j) :=
   If j = 1
      y1
      y(j - 1) - u(j - 1)·T↓(j - 1)
```

$$\mu := \text{VECTOR}\left(\text{IF}\left(j = 1, \frac{n0}{N_1}, \frac{N_{j-1}}{N_j}\right), j, 1, \text{DIM}(R)\right)$$

We find all quantities from above (the *u*s, the *y*s, and the μs and try our Derive realisation by repeating solving the problem from above.

$$[r1 := 20,\ r2 := -30,\ n1 := 1.5]$$

$$\mathrm{SUBST}(s2\_,\ t1,\ 0) = \frac{24 \cdot s1}{s1 + 24}$$

$$[n0 := 1,\ N := [1.5,\ 1],\ R := [20,\ -30],\ T := t1]$$

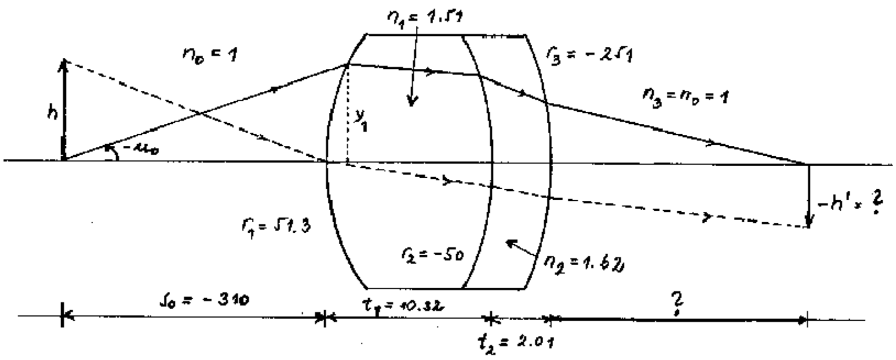
$$\left[y1 :=,\ u0 := \frac{y1}{s1}\right]$$

$$\lim_{t1 \to 0} \frac{y(\mathrm{DIM}(R))}{u(\mathrm{DIM}(R))} = \frac{24 \cdot s1}{s1 + 24}$$

*s*2' = image distance = hyperbola from above.

Next example is a bit more difficult:

Double Cemented Lens



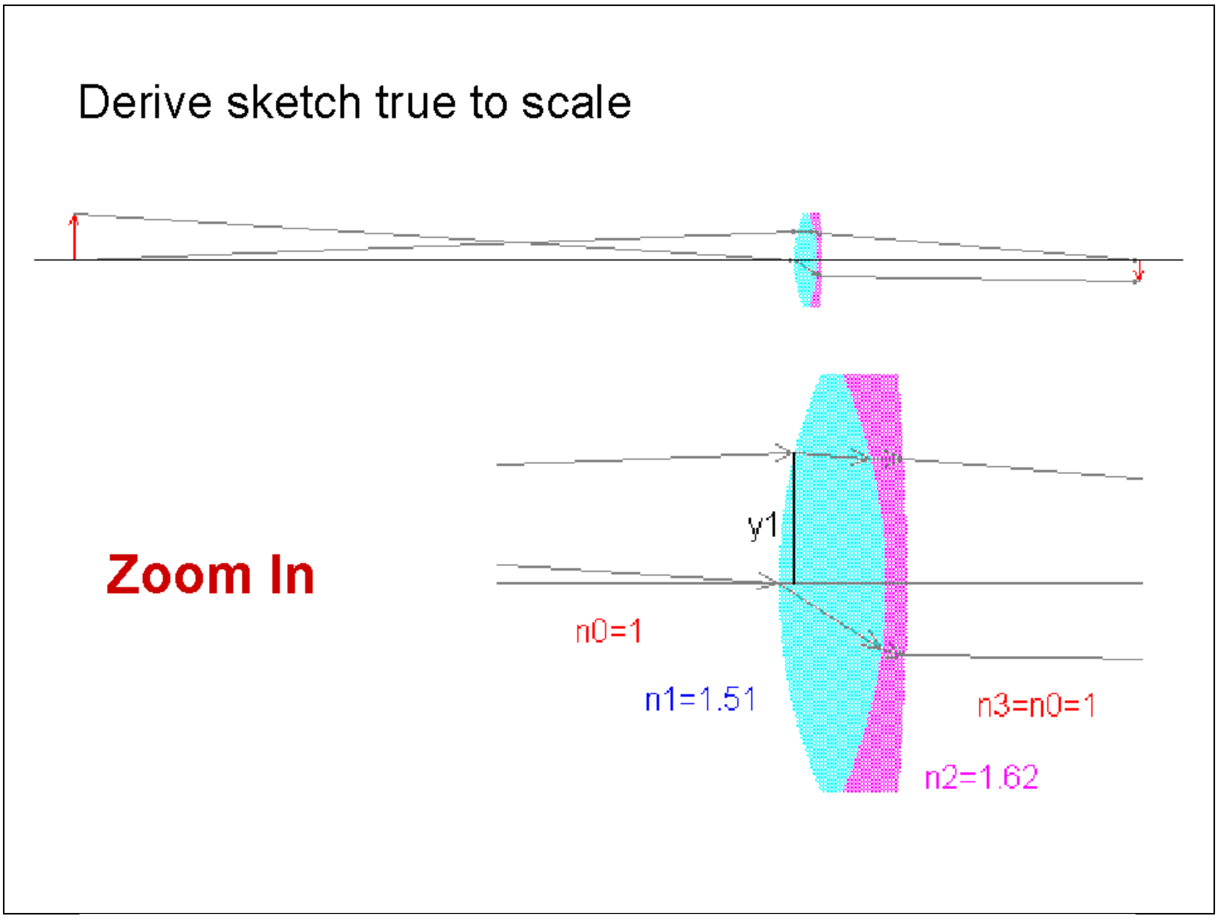
h = 20mm; h' is negative!!

Sketch of the problem

The optical system consists of three surfaces (radii, distances and refr. indeces are given), the object is located 310 mm to the left of surface #1 and is 20 mm high. It is surrounded by air ($n0 = 1$).

Questions: Image distance *s*3'?
size –*h*'?

To demonstrate the problem more detailed I produced shaded derive graphs.



We define the data and ......

```
#5:   R := [51.3, -50, -251]
#6:   T := [10.32, 2.01]
#7:   n0 := 1
#8:   N := [1.51, 1.62, 1]
```

$$\#9: \quad \left[s0 := -310,\ u0 := \frac{y1}{s0},\ h := 20 \right]$$

receive the location of the image.

(distance from the last surface to image plane)

$$\boxed{\frac{y(DIM(R))}{u(DIM(R))} = 136.275}$$

and we again use the previously generated tool.

See that it works properly – as expected!!

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There are three ways to calculate the size of the image h´:

(1) using the Lagrange Invariant

(2) using paraxial ray tracing from the top of the object

(3) including the image plane as last surface

(1) Using Lagrange Invariant

$$n \cdot h \cdot u = n' \cdot h' \cdot u'$$

$$n_0 \cdot h \cdot u_0 = N_{DIM(R)} \cdot h' \cdot u_{DIM(R)}$$

$$\mathrm{SOLVE}(n0 \cdot h \cdot u0 = N_{\mathrm{DIM}(R)} \cdot h\_ \cdot u(\mathrm{DIM}(R)),\ h\_)$$

$$h\_ = -9.271 \vee y1 = 0$$

$$\mathrm{SOLVE}(n0 \cdot h \cdot u0 = N_1 \cdot u(1) \cdot h1,\ h1) = (h1 = -9.607 \vee y1 = 0)$$

$$h1 := -9.607$$

$$\mathrm{SOLVE}(N_1 \cdot u(1) \cdot h1 = N_2 \cdot u(2) \cdot h2,\ h2) = (h2 = -13.975 \vee y1 = 0)$$

$$h2 := -13.975$$

$$\mathrm{SOLVE}(N_2 \cdot u(2) \cdot h2 = N_3 \cdot u(3) \cdot h3,\ h3) = (h3 = -9.271 \vee y1 = 0)$$

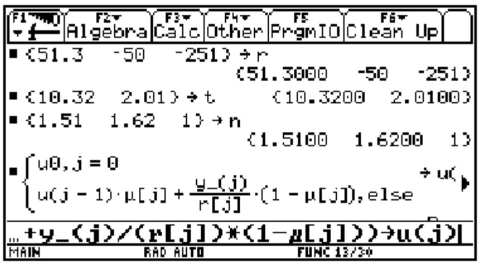
The product presenting the first medium remains the same in the last medium $n * h * u$!!

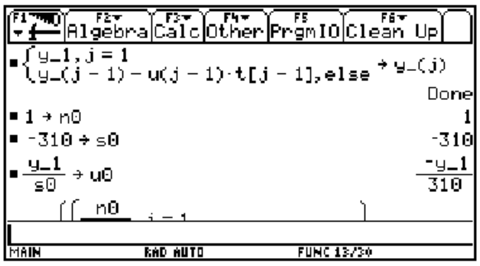
The final result is 9.271 mm upwards from the optical axis.

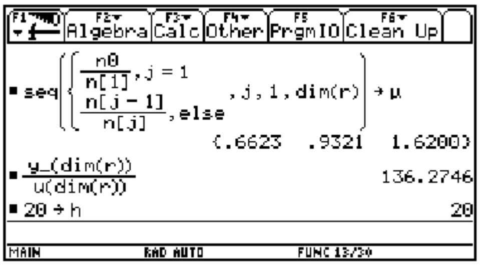
I find it nice to proceed stepwise from one surface to the other and follow the change of the image size, making the recursion much more understandable.

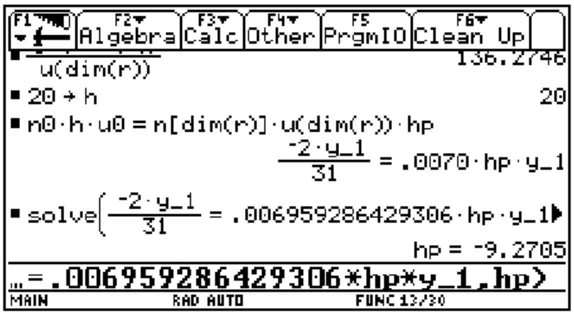
Let´s make a side step to CAS-Calculators (because it is also a TI-CAS-Conference!!)

Let´s try on the Handheld (CAS-TI)

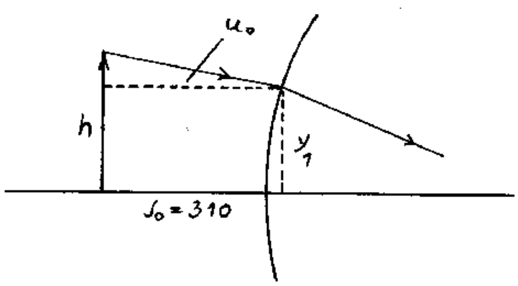








(2) Using paraxial ray tracing from the top of the object



$$u0 := \frac{h - y1}{-s0}$$

$$y(DIM(R)) - 136.274 \cdot u(DIM(R))$$

$$0.00000396 \cdot y1 - 9.27$$

No trig functions here ....

As $y1$ is small it can be neglected, hence the size is again -9.27 mm.

(3) Including the image plane as last surface

$$R := [51.3,\ -50,\ -251,\ \infty]$$

$$T := [10.32,\ 2.01,\ 136.274]$$

$$N := [1.51,\ 1.62,\ 1,\ 1]$$

$$y(DIM(R)) = 0.00000396 \cdot y1 - 9.27$$

$$-\frac{9.27}{h} = -0.464$$

Lateral Magnification

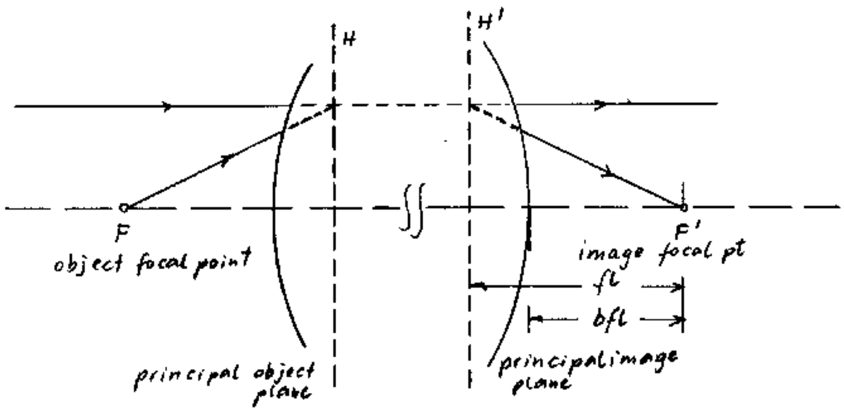
And no trig functions here again!!
And again the same result.
For me this also is a kind of the principle of „window shuttling“, i.e. changing the representation forms to get a better insight for concepts.
Having calculated the image size we obtain the lateral magnification as a fraction.

I browsed the Internet to inform myself on optics and Paraxial Approximation and I alway came across these expressions.

An optical system can be replaced by two principal planes H and H‘ (principal object plane & principal image plane or 1st and 2nd principal plane).

The way of the light ray within the optical system is of no interest for us (Black Box)!

Principal Planes,
Image Focal Length (fl) and
Back Focal Length (bfl)



Modelling the situation in Derive is easy:

```
R := [51.3, -50, -251]
T := [10.32, 2.01]
N := [1.51, 1.62, 1]
[u0 := 0, n0 := 1]
```

$$\left[\mathrm{bfl} := \frac{y(\mathrm{DIM}(R))}{u(\mathrm{DIM}(R))},\ \mathrm{fl} := \frac{y(1)}{u(\mathrm{DIM}(R))}\right]$$

```
[fl, bfl] = [98.337, 90.693]
```

Now let´s change into object space!

We describe the optical system from before (3 surfaces) and try to find fl (image fl) and bfl (back focal length).

We try to find distance s0 from the first surface to the object focal point F and

Then we use this distance for calculating the object focal length.

We are still using the same tools!

$$\left[s0 :=,\ u0 := \frac{y1}{s0}\right]$$

SOLVE(u(DIM(R)) = 0, s0)

$$s0 = -97.8496 \vee y1 = 0$$

$$s0 := -97.8496$$

$$\frac{y(DIM(R))}{u0} = -98.3371$$

2nd Method: Ray tracing in opposite direction (from image to object space). In order to do this correctly we have to change order and signs of the optical parameters. We trace the parallel ray (u0 = 0).

$$R := [251,\ 50,\ -51.3]$$

$$T := [2.01,\ 10.32]$$

$$N := [1.62,\ 1.51,\ 1]$$

$$u0 := 0$$

$$\left[\frac{y(DIM(R))}{u(DIM(R))},\ \frac{y(1)}{u(DIM(R))}\right] = [97.8496,\ 98.3371]$$

We recognize reverted signs.

What about fl and bfl for a thick lens?

$R := [r1,\ r2]$

$T := [t]$

$N := [n,\ 1]$

$[n0 := 1,\ u0 := 0,\ y1 :=]$

Repeat from above:

$$\left[bfl := \frac{y(DIM(R))}{u(DIM(R))},\ fl := \frac{y(1)}{u(DIM(R))}\right]$$

$$\left[\frac{r2\cdot(n\cdot(r1 - t) + t)}{(1 - n)\cdot(n\cdot(r1 - r2 - t) + t)},\ \frac{n\cdot r1\cdot r2}{(1 - n)\cdot(n\cdot(r1 - r2 - t) + t)}\right]$$

The Optical Power is

$$\frac{1}{fl} = \frac{(1 - n)\cdot(n\cdot(r1 - r2 - t) + t)}{n\cdot r1\cdot r2}$$

In Internet I found $-n/f$ = *"Power of the Surface"*

Separated Doublet

We investigate a system composed of two lenses with focal lengths fa and fb, separated by distance tx.

What is the focal length of this system?

Again we don´t need a new tool.
Paraxial Approximation does the job without any problems.

First of all let´s define the parameters of the system.

We evaluate focal length of a system composed of two lenses of focal legths *fa* and *fb* placed at distance *tx*.

Can you imagine how the system does look like?

Which parameters do we need to know?

$$R := [r1, r2, r3, r4]$$

$$T := [ta, tx, tb]$$

$$N := [na, 1, nb, 1]$$

$$n0 := 1$$

First lens (thick lens)

$$\#68: \quad \frac{1}{fa} = \frac{(1 - na)\cdot(na\cdot(r1 - r2 - ta) + ta)}{na\cdot r1\cdot r2}$$

$$\#69: \quad \mathrm{SOLVE}\left(\frac{1}{fa} = \frac{(1 - na)\cdot(na\cdot(r1 - r2 - ta) + ta)}{na\cdot r1\cdot r2}, r2\right)$$

$$\#70: \quad r2 = \frac{fa\cdot(na - 1)\cdot(na\cdot(r1 - ta) + ta)}{na\cdot(fa\cdot(na - 1) - r1)}$$

$$\#71: \quad r2 := \frac{fa\cdot(na - 1)\cdot(na\cdot(r1 - ta) + ta)}{na\cdot(fa\cdot(na - 1) - r1)}$$

Second lens (thick lens)

$$\#72: \quad \frac{1}{fb} = \frac{(1 - nb)\cdot(nb\cdot(r3 - r4 - tb) + tb)}{nb\cdot r3\cdot r4}$$

$$\#73: \quad \mathrm{SOLVE}\left(\frac{1}{fb} = \frac{(1 - nb)\cdot(nb\cdot(r3 - r4 - tb) + tb)}{nb\cdot r3\cdot r4}, r4\right)$$

$$\#74: \quad r4 = \frac{fb\cdot(nb - 1)\cdot(nb\cdot(r3 - tb) + tb)}{nb\cdot(fb\cdot(nb - 1) - r3)}$$

$$\#75: \quad r4 := \frac{fb\cdot(nb - 1)\cdot(nb\cdot(r3 - tb) + tb)}{nb\cdot(fb\cdot(nb - 1) - r3)}$$

CAS is a useful tool for performing elementary calculations eliminating parameters *r*2 and *r*4)

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Resulting in an extended and bulky expression:

Combine both to the full system using the set of data from above and calculate fl:

$$fl =$$

$$\frac{fa \cdot fb \cdot na \cdot r1 \cdot (nb \cdot (r3 - tb) + tb)}{fa \cdot (na \cdot (r1 - ta) + ta) \cdot (nb \cdot (r3 - tb) + tb) + na \cdot r1 \cdot (fb \cdot nb \cdot r3 + nb \cdot tx \cdot (tb - r3) -} \sim$$

$$\overline{tb \cdot (r3 + tx))}$$

Nice little expression?? Would you like to do it by hands??

Let´s take thin lenses!

$$\lim_{tb \to 0} \lim_{ta \to 0} fl$$

$$\frac{fa \cdot fb}{fa + fb - tx}$$

For thin lenses *ta* and *tb* tend to zero ....

Conclusion for an astronomic telescope (composed of two thin lenses (objective lense and ocular) with fl = ∞:

$$\frac{fa \cdot fb}{fa + fb - tx}$$

$$tx = fa + fb$$

Distance between ocular and objective is the sum of the focal lengths (Kepler Telescope).

..... and for an astronomic telescope focal length equals infinity, hence denominator equals zero .....

In a one of my son´s textbooks on Physics I found that Johannes Kepler used Paraxial Approximation doing his calculations .....

..... but he was not in the lucky situation to have *DERIVE* as a servant.

I will close with a short summary of this lecture provided by its father Leon Magiera.

The fundamental procedure in geometrical optics calculations is the ray tracing. Ray tracing of a ray through an optical system consists of calculation of the slope angle of the refracted or reflected ray (on the base of the Snellius law) and the change in ray height on transition to the next surface.

In many computational problems it is sufficient to restrict the consideration to the paraxial region i.e. the region around the optical axis of the optical systems in which the angles made by rays (i.e. the slope angles and angles of incidence and refraction) may be set equal their sinuses and tangents. This approximation (first order optics) leads to the Abbe invariant, the ray transition formula and also to the Lagrange invariant. The relations obtained appear to be very simple.

The fundamental quantities of the optical system are: focal length, back focal length, the size and image location and hence the magnification.

In the presented examples we have demonstrated how to calculate these quantities by using paraxial formulae. In particular analytical formulae for these quantities for thick and thin lens have been derived. It was demonstrated that paraxial formula can also be used for evaluation of the image position given by parallel plate (single lens for which curvatures radii are equal to infinity becomes parallel plate). The derivations for thin lens and parallel plate has been done via applying *lim* function.

All these computations can be obtained "step by step" or automatically in one step or what is very convenient by applying recursive procedure for paraxial formulae.

The optical system is specified by its constructional parameters (curvature radii, distances between surfaces and refractive indices). The *Derive* program accepts parameters data of an optical system entered even in the form of complicated analytical expressions. This fact have been demonstrated on the example of 'separated doublet' by using "*lens maker equation*". Its solution allows to express the curvature radius in the form of expression containing focal length of a single lens and its remaining parameters.

The image size evaluation have been demonstrated not only by applying the method of tracing of the ray from the top of the object to the image plane but also by the method based on the axial ray tracing and applying the Lagrange invariant.

I enjoyed the far distance collaboration with my friend from Poland and I hope that I could transmit at least partially our wonderful "working atmosphere"

Thanks for your attention

and if you have any questions, then please contact

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