CONTS

Computer-Based Mathematics Assessment of Engineering Students

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Abstract

HELM (Helping Engineers Learn Mathematics) is a major three-year curriculum development project undertaken by a consortium of five UK universities, sponsored by UK Government funding of £250,000 (about $550,000 Cdn). This paper describes how HELM uses computer technology to enhance the teaching and learning of mathematics to engineering undergraduates.

The HELM learning resources are briefly described. These consist of Workbooks, Computer-Aided Learning (CAL) courseware and Computer-Aided Assessments (CAA). The 50 Workbooks cover the mathematics in the first two years of UK engineering degrees. The CAL courseware, consisting of on-line interactive lessons to aid understanding, is web-delivered and complements many of the Workbooks. Essential to the success of the project is an extensive CAA regime; this takes two forms, either an integrated web-delivered version or an alternative stand-alone CD based version. The CAA regime facilitates the regular testing of large numbers of students. It incorporates both formative and summative aspects and thus powerfully encourages students to engage more in their own learning.

Finally, didactic issues raised by using CAA to drive mathematics learning are discussed by reference to trialling experiences at a number of UK universities, which have used the materials in a variety of ways.

1. Introduction

The importance of mathematics as a tool for the description and analysis of engineering systems and processes has long been acknowledged and the UK's Engineering Council rightly demands a high level of mathematical knowledge and skill in its accredited engineers. As a consequence, the design and delivery of an appropriate mathematical curriculum for engineering undergraduate students must be of central importance to engineering educators. That mathematics is a fundamental constituent of the education of an engineer is substantiated by the facts that it is often the only subject specified as a prerequisite, and it is a common thread in almost all UK engineering courses.

HELM (Helping Engineers Learn Mathematics) is a major three-year curriculum development project undertaken by a consortium of five UK universities, sponsored by UK Government funding of £250,000 (about $550,000 Cdn) for the period October 2002 – September 2005. The overall aim of the HELM project is to enhance the mathematical education of engineering undergraduates in England & Northern Ireland by the provision of flexible teaching and learning resources which may be integrated into existing engineering degree programmes by selection of individual stand-alone units or by adopting the whole scheme. The primary target group comprises departments and academics teaching mathematics to engineering undergraduates.

This paper describes how HELM uses computer technology to enhance the teaching and learning of mathematics to engineering undergraduates. First the learning resources are briefly described. These include Workbooks and web-delivered CAL segments which incorporate engineering exercises and case studies closely related to the mathematics presented. Second we describe the HELM CAA regime and its use to drive student learning of engineering mathematics. HELM provides both an integrated web-delivered CAA implementation and an alternative stand-alone CD based version. This CAA regime complements the other HELM resources, bringing out their full potential by allowing students to test their understanding and prepare for examinations.

HELM learning resources have been extensively trialled at Loughborough University over a number of years to teach mathematics to several thousand engineering students and trials are currently taking place at over twenty other universities and colleges in the UK. Finally we discuss the use of the HELM learning resources at other institutions and comment on some of the didactic issues raised.

2. HELM Learning Resources

The HELM learning resources consist of Workbooks, Computer-Aided Learning (CAL) courseware and Computer-Aided Assessments (CAA).

2.1 HELM Workbooks

The main student learning resource is 46 high quality printed Workbooks which cover the engineering mathematics required in the first two years of UK engineering degrees. They include syllabus requirements in calculus, algebra, Fourier analysis, Laplace and z-transform methods, ordinary and partial differential equations, complex analysis, numerical methods, probability, statistics and modelling. A complete list is shown in Appendix 1. Appendix 2 shows a sample page. They are written specifically for the typical engineering student and, in addition to the various mathematical topics and mathematical exercises, contain worked examples and related engineering exercises. The exercises include space for students to attempt the questions, and guide them through the problems in stages, where appropriate.

Workbooks incorporate engineering examples closely related to the mathematics. These encourage engagement in the learning process and are an important feature of many Workbooks. They help students learn to apply mathematics to solve engineering problems. Contexts specific to various branches of engineering such as mechanical, electrical and electronic, civil and chemical feature and typical examples include: Black Body Radiation, Bending Moments, Vibrations, Complex Impedance and Motion under Gravity. It is important to motivate the learning of mathematics for engineering students and we encourage this by the inclusion of two Workbooks devoted to engineering case studies, the first on 'Modelling Motion' and the second on 'Modelling Waves' and another Workbook on engineering applications.

There will also be students' guide and a tutors' guide.

The Workbook writing team has been drawn from all five consortium universities. All Workbooks are critically read and then revised, if necessary.

2.2 HELM CAL

The CAL courseware was developed using Authorware[1], a visual authoring tool for creating web-delivered e-learning applications. This courseware consists of on-line interactive lessons to aid understanding and complement many of the Workbooks. These contain audio, interactivity and self-assessment features. At present 67 have been developed and more are being developed in response to feedback from triallists. They cover the mathematics theory in a form which is easy to understand and include worked examples and quizzes. Being web-delivered, these interactive lessons can be accessed by students at any time and anywhere if they have an internet connection.

The HELM resources can be used in various ways. Obviously lecturers can use them to support their teaching of a complete mathematics module, or part of it, to engineering students, and they are ideal for use with mixed-ability groups. However the Workbooks and associated interactive lessons can be used independently by students allowing them to work alone at their own pace. Consequently our learning materials are also ideal for self-learning.

A Tutor's Guide relating success stories and challenges and encapsulating good practice derived from trialling in a variety of institutions with their individual contexts and cultures is being written for use with the completed learning resources.

2.3 HELM CAA

Essential to the success of the project is an extensive CAA regime; this takes two forms, either an integrated web-delivered version or an alternative stand-alone CD based version. The CAA regime facilitates the regular testing of large numbers of students. It incorporates both formative and summative aspects and thus powerfully encourages students to engage more in their own learning of engineering mathematics. This CAA regime is essential to exploit the full potential of the other HELM learning resources.

Currently there are almost 5000 questions in a large number of question banks. These questions have been designed to match particular mathematical concepts in support of the topics covered by the HELM Workbooks and most have a page of feedback. It is anticipated that this number will rise to around 10000 on completion of the project. Originally questions were held in Question Mark[2] Perception (QMP) version 2.5 format, these are being reviewed and transferred to version 3.4, while new questions are being developed in version 3.4 directly. We present the body of each CAA question as a jpg image originated from a LaTex file; image quality is further enhanced using a graphics software application before being used in QMP. With this approach already in place to produce consistent high quality images we have maintained the same methodology for the development of new questions in QMP. The MathML approach was considered but in view of the additional expertise required to develop questions and the need for users to have MathML enabled browsers, its use was not deemed to be ideal at present.

The questions relevant to each mathematical concept have been structured into two sets, one for formative assessment, the other for summative assessment. Each set

[1] http://www.macromedia.com/software/authorware/
[2] http://www.questionmark.com/

contains at least 10 questions cloned from a single master question, thereby ensuring that the same level of difficulty is maintained, and justifying the random selection of questions from each set for test purposes. Several concepts covering an area of work may then be selected and randomly chosen questions covering the selected concepts may be presented within QMP to form a customised test.

CAA is an essential part of the project and this raises potential difficulties over transferability, as each institution would need to support CAA delivery on completion of the project to gain the full benefit. The adoption of QMP at Loughborough University has allowed us to deliver tests to large numbers of students over the web since October 2000. Other institutions planning to use the HELM CAA regime would need to put in place an appropriate system in order to properly administer student test taking and process the associated information through CAA.

Web-delivered CAA is a convenient method of delivery but an alternative implementation based on CDs has been developed. Currently all of the HELM tests (that is, all the questions and all the linked feedback) together with all the Workbooks easily fit onto one CD. Students, without an internet connection, provided with such a CD can then do the required work and complete the tests on the CD. This is easy to implement if only self-testing is required; formal testing is more challenging, requiring a network connection so that the students can submit their completed test results for processing.

3. HELM CAA Regime

Regular testing can play a very important part in the learning process. Carried out at the right level and at the right frequency it can aid the understanding of abstract concepts and be an encouragement to students to continue with a difficult topic. At the wrong level and at the wrong frequency it can be a disaster for the student, discouraging and stressful. Students must feel that the test is fair and that, given that they engage positively with the module, they would expect to perform well. There is little point asking a student a question if s/he is not in a good position to answer correctly. Our aim is to put students in a strong position to answer every question correctly. Of course, as educators, we must be sure we ask questions at an appropriate level; our tests contain no 'tricks' or 'twists', they are intended simply to measure (for our benefit and the students') what a student has learned about a given topic. In common with many UK universities and elsewhere, we use Computer Aided Assessments to ease the burden of frequent testing and to be able to effectively deal with an ever increasing number of students.

Having decided to ask a question or, in our case, a series of questions in a 'test', a number of relevant factors need addressing. We present these in no particular order of priority and ignore technical matters concerned with 'delivery'.

- **Frequency of tests**

  We feel it is better to test students regularly, often enough to get them used to being tested, in part to reduce stress levels, but not so often that the 'testing process' dominates the 'learning process'. We prefer to test a short time after a major topic has been covered in lectures whilst the subject is still fresh in their minds. In our case, teaching mathematics to undergraduate engineers at a UK university, which would include topics such as, say, vectors, matrices, calculus, etc., gives rise to a test 'naturally' occurring every 2 – 4 weeks. At Loughborough University in a 12-week semester we commonly test 5 times. Students prefer to be tested often. They can then better gauge how they are coping with the work. Regular testing gives further structure to a module, also welcomed by many students. Though time is not meant to be a significant factor in the testing process (we want our students to solve problems but not under a time constraint), we put a generous time limit on each test.

- **Value of tests**

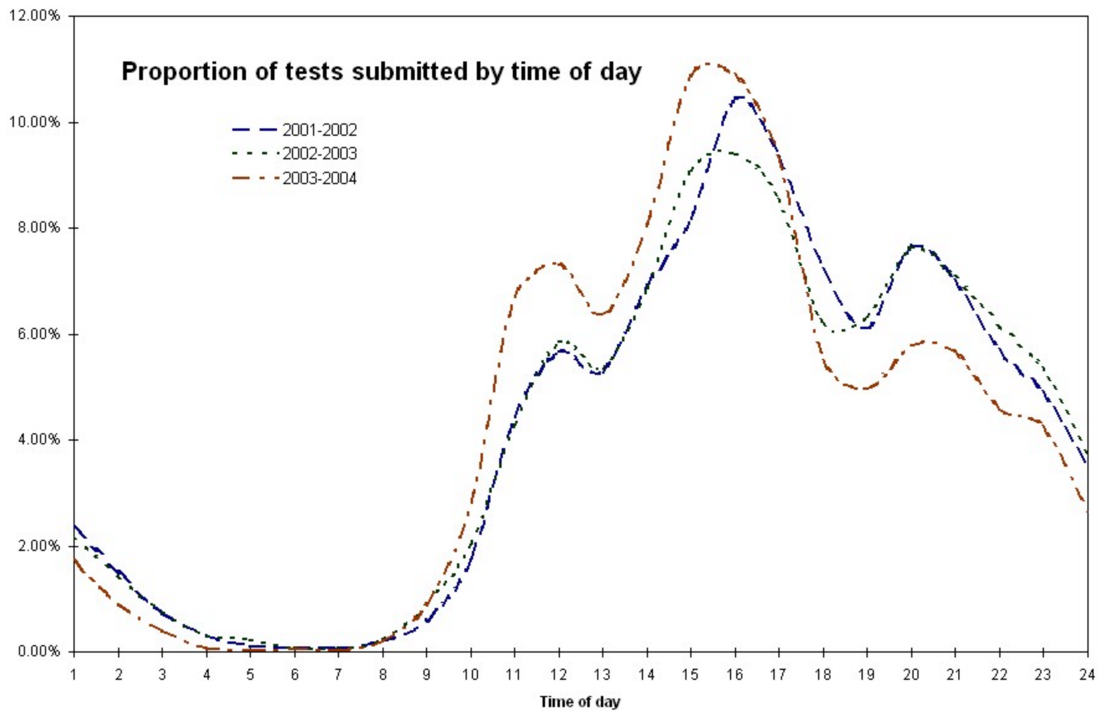
We use formative and summative tests. Formative tests can be taken as often as a student wishes and are available all day, every day for a week prior to the corresponding summative test; summative tests can be taken only once. As with all our tests they are web-delivered. Both summative and formative tests are similar in form, in level and in subject topic. For example, if question number *k* on the formative test required the calculation of a scalar product then so would question *k* in the summative test.

A test covering, say, 10 concepts, with a question randomly chosen from perhaps 10 questions available in each library set, thus provides a determined student with the possible opportunity to attempt up to 100 different questions, and to receive question specific feedback prior to formal assessment. Since students know that the summative test questions are of a similar nature they gain great confidence from this opportunity to practice prior to summative testing.

In our experience most students access the formative test many times so they are better prepared to tackle the corresponding summative test. In order to get (street-wise) students to engage in the assessment process there has to be a sufficiently attractive 'carrot'. At Loughborough University the summative CAA tests are classified as coursework. Overall the coursework element for a mathematics module comprises between 30-40% of the whole, so that an individual test is worth between 6-8%, with the remaining 60-70% usually being for a written assessment upon completion of the module. Our experience is that this level of CAA is enough to keep activity levels very high (>95% of students take every summative test). However, it is not so high as to have an adverse effect on how serious the students regard the traditional end-of-semester written examination. Giving too much credit for the coursework element would (as we have observed) allow students to pass the module without having to pass the written assessment.

- **Availability of tests**

Our tests, being web-delivered, are available from any suitable PC (of course, the tests are password protected). Their free availability is a major attraction to students and is often mentioned in student feedback. Formative tests can be taken when a student is ready (and in the right mood). Our records show that many tests are taken out of 'normal' working hours as seen in Figure 1 which shows the percentage usage by time of day.



- **Figure 1: Usage by time of day**

(Source: Bryan Dawson, Professional Development, Loughborough University.)

- **Robustness**

There is a widely held view that computer-delivered tests are less robust than traditional written-based tests. At Loughborough the demand for CAA testing has increased significantly over the last 3 years as Table 1 shows, and now thousands of tests are delivered each year. They can be scheduled at the start of term and then delivered automatically with little or no further significant human interaction. Our evidence is that 99.9% of tests are delivered successfully. There are mechanisms in place for dealing with the remaining 0.1%.

	2001-2	**2002-3**	**2003-4 (Up to May 2004)**
Tests submitted	**25145**	**51798**	**58286**
Users (as %)	**1431 (12.3%)**	**2631 (20.7%)**	**2601 (19.5%)**
Number of tests	**185**	**295**	**337**
Number of modules	**24**	**56**	**87**

Table 1: Usage statistics

(Source: Bryan Dawson, Professional Development, Loughborough University.)

- **Integrity of tests**

  A major concern on the part of academics involved with testing is the problem of cheating. At Loughborough University all CAA tests are unsupervised (but not the end of semester written assessment). Students can take the summative tests at any time within a designated (2-day) time period. However, they are only allowed to access this test once. In our tests a question is randomly chosen from a set of clone questions (distinct questions though similar in type and level of difficulty). Thus, two students sat next to each other are very unlikely to see exactly the same questions on the same test. Our experience is that cheating is not a significant problem. For some groups we have run equivalent **supervised** paper-based tests instead of their usual CAA test. Results from each type of test show good correlation. To a large extent it is easier for the student to do the work than to cheat.

- **Student preparation**

  Our tests are advertised well in advance. Each summative test is preceded by formative tests which can be accessed an unlimited number of times over a seven day period. The summative test can be taken anytime during the following two day period. If students engage in the process (and they do) they practice the test a number of times. Most are then well prepared when they access the summative test which they are only allowed to do once.

  Testing in this way, allowing students adequate, focussed preparation and allowing free access to trialling versions of the test goes some way to removing the high stress levels most students experience in tests. They are less stressed, better prepared and so generally perform well.

- **SENDA[3] issues**

  For students facing difficulty in reading information from a screen paper-based versions (identical except for size) can be prepared. Care is taken with fonts and font sizes to minimise screen fuzziness. Dyslexic students can be allocated (individually) more time to complete tests.

- **Presentation**

  All our questions (presently approaching 5,000 in number) are prepared using LaTex specifically useful for typesetting mathematics.

  Appendix 3 shows a sample question. As the reader will see, our questions are high quality, entirely equivalent to that which might be expected in a written version. All our questions are designed to fit onto a standard template. On the left-hand side the question numbers are listed (with scroll bar if required). By hitting the appropriate button students can access any question they wish. On the lower left is a clock indicating the time remaining in the test. Inevitably, some questions (usually multiple choice) are too long to fit onto a single screen. In these cases a scroll bar is used. We endeavour to minimise the use of a scrollbar in question presentation. Students can return to the questions and change their

[3] **SENDA** : Special Educational Needs and Disability Act 2001, UK

answers any time before submitting the test. When a student is satisfied s/he has completed the test the 'submit' button is hit. The test score and, if available, any other feedback are then displayed to the student.

Question feedback is an option which may be enabled in both formative and summative type questions. We always use it with formative questions as a motivational aid which drives student learning.

Feedback is of three kinds:
specific: a full-worked solution to the given question
exemplar: a sample worked solution to a similar question
generic: an algebraic-type solution to a complete class of similar problems.

Care must be exercised with generic feedback as some (average to weak) students can find such feedback more difficult to 'untangle' than the original question. As far as possible we attempt to give specific solutions whenever we can (though this implies a not inconsiderable increase in the preparation). Students often prefer to see the solution to the problem presented.

On summative tests we choose to give no feedback (except the test score).

- **Fairness**

For the kind of questions we developed initially (most questions were 'numeric') the student was marked either right or wrong. In our initial approach no credit was given for 'working'. Missing out a minus sign, incorrectly rounding or simply mistyping the answer would incur a zero mark for the student even though they might have well understood how the problem is solved. Students viewed this as being unfair. Actually, though this comes high on the students' gripe list, in our experience, it appears not to occur very often in practice. We have some sympathy with this view. We are countering this 'negative' aspect of our initial approach by taking the following actions:

- ✓ Numeric answers (except whole number answers) are now acceptable in a range of values. If the 'correct' answer is given the student will obtain full marks for that question. If the student's answer is 'incorrect' but within a small tolerance of the 'correct' answer they are given partial or full credit.
- ✓ Multiple choice questions. We are including more of these questions which, to some extent, minimise student input errors.
- ✓ Multiple numeric input. For these questions two or more numeric answers are expected. All the marks allocated for the question are shared out giving credit for those students getting some of the working correct.
- ✓ Staged questions. Here a single question is divided into two or more parts. A student is asked to work through a problem in stages. Marks are allocated for correct answers to each stage; each stage question might be numeric, multiple choice etc. Unknown quantities or expected answers from stage (n) are presented to the student at stage (n+1) so that they can proceed to answer other parts of the problem without hindrance. Using this approach, more complicated problems can be tackled. To a large extent this kind of question tackles the 'credit for working' problem.

4. Trialling Experiences

An important aspect of the HELM Project is the evaluation of all the learning resources produced. This is essential to ensure that they are accurate and appropriate for the needs of the academics and students who will be using them. The learning resources have been extensively trialled at Loughborough University over a number of years to teach mathematics to several thousand engineering students and trials are currently taking place at over 20 universities and colleges in the UK. This evaluation is being conducted through an extensive set of activities which consults academics and students who have been engaged with the resources. These academics, and some others, are also contributing to checking the resources for accuracy by acting as Critical Readers and reviewing the Workbooks. The CAA questions have mainly being checked in-house.

Triallists who have used some of the materials with at least one group of students during the academic year 2003/2004 have all been contacted by telephone, have completed a written questionnaire and have been visited by a member of the HELM Project Team. This three-pronged approach has ensured that a thorough and reliable set of feedback has been received; this will enable improvements to be made to the materials before the start of the next academic year.

The feedback has in general been very positive. Most of the triallists have used the Workbooks as supplementary notes for their students rather than as the course text, although a small number have taken the Workbooks as their core notes. Some of the triallists have produced hard copies of the Workbooks for all of their students but others have only made them available on request or as an electronic resource.

Triallists have found that the Workbooks have been well produced, have few, if any, errors in them and provide an excellent resource for their students. There is a fair consensus that the Workbooks should contain more exercises, including more stretching examples at the end of each section, plus a revision set at the end of each Workbook. The issue of contextual examples is more important to some than others, with some triallists being content to provide the context for their own particular applications whilst others feel that this should occur more frequently in the Workbooks.

There is some debate about the length of the Workbooks and the depth of material covered. There have been contradictory comments about the approaches taken to introduce students to the mathematical topics, some preferring the theory element to be glossed over whilst others would prefer a more expansive approach than is taken in the Workbooks. This is a significant challenge for the HELM Project which is not aiming to provide a set of textbooks but an interactive resource that benefits engineering students in their learning of mathematics. To include all of the elements that triallists have mentioned would expand the size, sometimes already lengthy, of the Workbooks, making them too cumbersome and expensive to reproduce for some potential users.

There has been less interest in the CAL elements to date, although the triallists are beginning to make them available to their students as an additional resource.

In spite of the high level of interest expressed in CAA, take up has been quite low in the first year of trialling mainly due to lack of support to set up an online system at the triallists' home institutions. However, there is significant interest in adopting the system in the next academic year either as an extra set of questions for the students to tackle in preparation for formal assessments or as part of the overall assessment process. There have been a few concerns about the style of questions and the lack of marks available for intermediate stages in more complex questions. This is being addressed by introducing staged questions into some topic areas.

Feedback from students has been gathered by conducting focus groups at the triallists' institutions. Students tend to be very positive about the Workbooks and the testing regime, where it is used. They find the Workbooks to be very useful to aid their learning, but are particularly pleased with them when it comes to revision for their examinations. They feel more confident because they have a complete set of notes to hand and do not have to rely on possibly partial notes they have taken themselves during lectures and which may have errors in them. Students who have taken a gap in their learning or are part-time or distant learning students are particularly happy with the HELM Workbooks as they are able to review the material at their own pace. Students with dyslexia have also commented favourably that they can focus on learning the mathematics from a complete and accurate set of notes, something that they struggle to achieve when having to construct their own notes. They have made some detailed comments about the numbering system employed which they found confusing. They also felt that answers should be placed at the rear of each section rather than underneath the exercises to prevent the temptation to look at them before attempting the questions themselves.

The CAA regime is particularly welcomed by the students, as it provides clear interim targets and immediate feedback on how they are progressing. However, as we discussed earlier, students were unhappy about not getting any marks for a "nearly right" answer. We are overcoming this by awarding partial or full marks to answers within a certain specified range of the correct answer.

5. Conclusions

The HELM learning resources, high quality Workbooks, Computer-Aided Learning courseware and Computer-Aided Assessments have the potential to be of major benefit to the learning and teaching of mathematics to undergraduate engineers. The Workbooks facilitate more student engagement during lectures. The web-delivered CAL courseware, which is closely aligned to the Workbooks, is useful as a tool for self-learning and further aids understanding.

At Loughborough we have over seven years experience of delivering CAA to literally thousands of undergraduate engineers and we can certainly claim to be at the forefront of CAA testing of mathematics in the UK. The CAA regime facilitates the regular testing of large numbers of students. It incorporates both formative and summative aspects and thus powerfully encourages students to engage more in their own learning.

Feedback exercises at Loughborough University and elsewhere indicate that students appreciate our learning regime; quality Workbooks, Computer Aided Learning segments and our Computer Aided Assessment regime including a generous practice period, extensive on-line feedback and a flexible summative testing approach.

Moreover, the learning resources can be used as an alternative to lectures and tutorials for students who prefer independent learning or who have to work in distance learning mode.

Appendix 1: HELM Workbooks - as at 1st July 2004

The **HELM** project is now in its second year and more and more resources are becoming available. Keep up to date with progress by visiting our website at: **http://helm.lboro.ac.uk/**.

No.	Title
1	Basic Algebra
2	Basic Functions
3	Equations, Inequalities and Partial Fractions
4	Trigonometry
5	Functions and Modelling
6	Exponential and Logarithmic Functions
7	Matrices
8	Matrix Solution of Equations
9	Vectors
10	Complex Numbers
11	Differentiation
12	Applications of Differentiation
13	Integration
14	Applications of Integration 1
15	Applications of Integration 2
16	Sequences and Series
17	Conics and Polar Coordinates
18	Functions of Several Variables
19	Differential Equations
20	The Laplace Transform
21	The Z Transform
22	Eigenvalues and Eigenvectors
23	Fourier Series
24	The Fourier Transform
25	Partial Differential Equations
26	Functions of a Complex Variable
27	Multiple Integration
28	Vector Calculus 1
29	Vector Calculus 2
30	Introduction to Numerical Methods
31	Numerical Methods of Approximation
32	Numerical Solution of Initial Value Problems
33	Numerical Solution of Boundary Value Problems
34	Numerical Optimisation
35	Sets and Probability
36	Descriptive Statistics
37	Discrete Probability Distributions
38	Continuous Probability Distributions
39	The Normal Distribution
40	Sampling Distributions and Estimation
41	Hypothesis Testing
42	Goodness of Fit Tests and Contingency Tables
43	Correlation and Regression
44	ANOVA
45	Nonparametric Methods
46	Quality Control and Reliability
47	Case Studies 1 - Modelling Motion
48	Case Studies 2 - Modelling Waves
49	Case Studies - Engineering Applicat
50	Tutor's Guide
0	Student's Guide

4. Examples of Fourier Series

We shall obtain the Fourier Series of the "half-rectified" square wave shown.

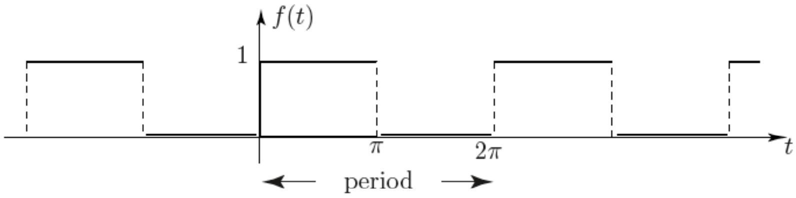


Figure 4

We have

$$\begin{aligned} f(t) &= \begin{cases} 1 & 0 < t < \pi \\ 0 & \pi < t < 2\pi \end{cases} \\ f(t+2\pi) &= f(t) \end{aligned}$$

The calculation of the Fourier coefficients is merely straightforward integration using the results already obtained:

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos nt\,\mathrm{d}t$$

in general. Hence, for our square wave

$$a_n = \frac{1}{\pi}\int_0^{\pi}(1)\cos nt\,\mathrm{d}t = \frac{1}{\pi}\left[\frac{\sin nt}{n}\right]_0^{\pi} = 0 \qquad \text{provided } n \neq 0$$

But $a_0 = \frac{1}{\pi}\int_0^{\pi}(1)\,\mathrm{d}t = 1$ so the constant term is $\frac{a_0}{2} = \frac{1}{2}$.

(The square wave takes on values 1 and 0 over equal length intervals of t so $\frac{1}{2}$ is clearly the mean value.)

Similarly

$$b_n = \frac{1}{\pi}\int_0^{\pi}(1)\sin nt\,\mathrm{d}t = \frac{1}{\pi}\left[-\frac{\cos nt}{n}\right]_0^{\pi}$$

Some care is needed now!

$$b_n = \frac{1}{n\pi}(1 - \cos n\pi)$$

But $\cos n\pi = +1 \qquad n = 2, 4, 6, \ldots,$

$$\therefore \qquad b_n = 0 \qquad n = 2, 4, 6, \ldots$$

However, $\cos n\pi = -1 \qquad n = 1, 3, 5, \ldots$

$$\therefore \qquad b_n = \frac{1}{n\pi}(1 - (-1)) = \frac{2}{n\pi} \qquad n = 1, 3, 5, \ldots$$

i.e. $b_1 = \frac{2}{\pi},\ b_3 = \frac{2}{3\pi},\ b_5 = \frac{2}{5\pi}, \ldots$

Appendix 3: HELM CAA – sample question