New Models in Assessment in Computer integrated Mathematical Instruction - First Results of the Austrian CA – Projects

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Since 1985 more & more teachers in Austria have been in a position to allow students to use electronic calculators in tests, if they had enough practice in class. Teachers of Mathematics at schools which are preparing students for university can choose and formulate their own test problems. Consequently their examples were gradually adapted to the new demands of computer algebra systems (DERIVE, MATHCAD, MATHEMATICA). But it is not easy for the teachers, because in big classes there are *often more students than computers available, not every student has a computer at home and not all lessons can be held in a computer lab.* Therefore three practicable models for tests with the help of computers have developed.

Model 1: ***different examples* for students with/without CAS**

model 2: ***time-sharing on the same computer*** *between two students*

model 3: ***50% team work alternating with 50% single work*** *on the same computer*

1.) **The model with *different examples* for students with/without CAS is a favourable interim model for classes,** where a strong minority is against using CAS in mathematics tests. It is a fact that after some weeks the handling of the computer and the finding of appropriate DERIVE-commands leads to a remarkable difference between those students who have the chance to practise at home and those who do not have this opportunity (Wurnig, 1992, 1996). In the tests the DERIVE-users would have had a great advantage over those students not using DERIVE if the latter had not been given a correct interim result with the help of which they could manage to continue their calculations in any case. At the end of the test the books and discs were collected and were printed out at home by the teacher. The time it took to print out the examples was easily made up by the advantage of the prints' good readability as well as the fact that - in case of a correct start and planning - the students' work was without mistakes.

2.) The second model ***time-sharing on the same computer*** *between two students is only sensible if 50% of the test and time can be worked on without a PC and 50% with a PC.* This model was partly used in the **Austrian CA I Project (DERIVE)**. The two students sharing the time of working on the computer, got different examples. Changing turned out to be no problem. The German report of the Austrian CAS I project says,
„After having finished his work on the PC, the student had to save the file and to quit DERIVE, thereby making the PC available for the second student." (Heugl, 1996)

3.) The third model, ***50% team work alternating with 50% single work on the same computer,*** is part of the concept MATHS & FUN with MATHEMATICA It is an educational experiment at the Business Academy I in Graz. Two out of three mathematics lessons per week take place in the computer lab where the students have to work in teams of two and this is the reason why they write their **tests in teamwork,** too. In **1998** this project was also carried out **in two sections** for the **final examination in maths** (A-Levels) for the first time. In the first two hours the students had to solve a problem in teamwork and the next two hours they had to use for their individual work on two different tasks given. If students of a team finished their teamwork before the end of the two hours, they could use all the remaining time for their individual work (www.mathsnfun.ac.at/mf/EnglischeVersion/index1.htm).

4.) But the real goal is one student per computer.

In the German report of the Austrian DERIVE-Project H. Heugl writes**:**
„It would be ideal if every student had a portable CAS-calculator, which could be linked up with the CAS in a computer lab, in his school bag.“

In the Austrian CAS II Project (TI92-Project) **1997/98** the students of the seventy research teachers wrote their tests in mathematics with the TI-92. At their final meeting in August 1998 the teachers collected their most important and sometimes unexpected results:
(Lechner/Wurnig, 1998)

- for solving problems it is very important **not always to insist on the use of the TI-92**.
- students find **new ways with the TI-92** → more work for the teacher
- the problems in tests have to be **more goal oriented** → **text longer** instead of shorter.
- **difficult decision**: What is to be the **minimum knowledge without** the TI-92?
- **difficult decision**: What **minimum knowledge of TI92-**commands is an absolute must?
- **modules** and **programmes** are a good chance **for good students**
  → **a new problem for bad students**.
- TI-92 has no floppy disc → **much documentation** in test book, therefore **fewer examples**.

5.) **The influence of the use of the tool CAS in the exam *situation***

Which is the more valid question? (Heugl, 2000)

Do the new ways of mathematics learning and teaching influence the exam situation?
or
Does the exam situation influence the new ways of learning and teaching?

In the past the exam situation always had a great influence on the contents and on the didactic concept of mathematics education. So the emphasis sometimes placed on a specific maths topic can only be explained by the fact that it was easy to construct a suitable test.
Therefore it is understandable that one of the principles formulated by the American NCTM is called the „**Assessment Principle**":

> *Assessment should support the learning of important mathematics and furnish useful information to both teachers and students.*

Some tasks of the Assessment Principle:

- Assessment should be more than merely a test at the end of instruction to gauge learning.
- Teachers should be continually gathering information about their students.
- Assessment should focus on understanding as well as procedural skills.
- Assessment should be done in multiple ways, and teachers should look for a convergence of evidence from different sources.
- Teachers should ensure that all students are given the opportunity to demonstrate their mathematical learning.

Already in our former CAS projects we recognized that the way of assessment was not suitable to the new ways of learning which we observed in our CAS classes.

In traditional mathematics education written exams (5 or 6 one-hour-tests per year) dominate. As far as contents is concerned the emphasis is on calculation skills. This way of testing is suitable to the dominating style of teacher centred teaching, which causes a more reproductive way of learning. Often in two or three of the four examples of a test the same skills are tested - students have to be busy for one hour.

Some significant changes of the learning process in our CAS-classes which strengthen the necessity of changes in the exam situation:

- A more pupil-oriented learning process. More mathematical discussions among the students. The teacher is not the only source of knowledge, he supports the independent acquisition of knowledge by the students.
- Experimenting, the trial- and error method: We seldom find the „algorithmic obedience" where the teacher shows one way which all the students then accept and follow.
- Working in pairs or groups can be seen much more frequently.
- Besides the teacher there now exists a new, very competent expert - the tool CAS. That means pupils do not always need the teacher for examining the correctness of their ideas and results.
- Phases of „open learning" where the students are individually organizing the speed and the contents of their learning process.
- CAS is not only a calculation tool, students can also store knowledge by defining modules or using the text editor. Therefore it is not sensible to forbid the use of learning media, like books or exercise books during the tests.
- New emphasis on fundamental competence. A shift from calculation skills to other skills.
- Clearer emphasis on problem solving and more application oriented mathematics.
- More frequent cross curriculum teaching.

6.) The experiences in the CAS-projects require new models of assessment.

After a meeting of the project management, five topics were chosen for the CAS III Project. 2000 students in 70 experimental classes from grade 7 to 11 took part in this project. It was carried out in 1999/2000. One topic was the influence of CAS on the Examination Practice.
A team of teachers under the leadership of H. Heugl developed **some variants of a new model of assessment** (Heugl/Schirmer-Saneff, 2001). In accordance with the Ministry of Education, experimental studies to test the new models started in 1999/2000.

Model 1: short tests – problem solving tests

Shorter tests (15 to 30 minutes) to examine fundamental competence like calculation competence, visualization competence and abilities of using the available CAS. In some tests the use of any electronic tool, especially CAS, was forbidden, in others it was allowed.

Problem-solving-examinations (50 to 120 minutes) to check the competence of problem solving with more application-oriented examples, more open questions, with more emphasis on argumentation, reasoning and interpreting. During these examinations students are usually allowed to use their learning media like their maths school books or their exercise books.

For this model the idea of the two phases is significant (Heugl, 2000): First laying the foundation by focussing on certain fundamental mathematical competence like algebraic competence and then in a second phase using fundamental competence for problem solving.

Model 2: project work

A certain number of the **classic written exams are substituted by projects** which are partly done during the lessons but the larger part of the work has to be done at home. In some classes every single student has to work on his special theme, in others project work is given to a group of students. This practice can be especially observed in larger classes because the presentation of the project work of every student of the class would take up to much time.

The content of such project work concerns themes for which students have to apply formerly learned contents but there are also fields where the students are faced with new problems and contents on the basis of their project work. Students have to produce term papers which they hand out to the other students of the class after of the presentation of their results.

The assessment takes place in two ways:

- Observation of the learning process: independent activity and ideas of the student, necessary inputs of the teacher.
- Assessment of the results: quality of the term papers, quality of the presentation, competence during the discussion of the results.

The fundamental idea of the **model project work** is to remove the existing separation between written and oral exams and the separation between product-oriented tests and process-oriented oral exams.

Model 3: cross curriculum tests

One of the main tasks of the school of the future is a greater emphasis on the **training of networked thinking**. One type of exam within the school leaving exam ("Matura" in Austria) is an oral exam connecting two subjects. Only a few students choose this type of exam, because they cannot experience curriculum phases of learning and because cross curriculum testing has not existed at school so far. In comparison with traditional classes we watch a **growing importance of cross curriculum phases** in our CAS-classes and consequently it was reasonable to consider this fact in the exam situation of our research classes.

Using the possibilities of the TI-92, especially CBR and CBL, a connection of mathematics and science is obvious (Laughbaum, 2000). The questions of such cross curriculum tests are based on both subjects and their assessment is included in the final grades of the two subjects.

Model 4: written group tests

The use of the tool CAS causes a **growing frequency of cooperative learning phases.** Using the new expert, ´CAS´, students more often work in pairs or groups, share their tasks, discuss mathematical problems and explore mathematical themes together. This cooperative way of learning needs a suitable method of assessing students abilities and competence.

As long as every student gets his individual grade, it is absolutely necessary not only to assess the group-competence. **The individual competence must be the central competence for reaching a valid grade**. One rule in our model 4 is that a passing grade can only be obtained by a student if neither the individual competence nor the group competence is negative.

When using the project method as a didactic concept for group work, the learning of the students must obviously be checked in a process-oriented way. When having to assess a written group test, however, the individual partner distribution is not obvious and consequently difficult to transfer into an individual mark.

7.) Evaluation by the Centre of School Development

(**Z**entrum für **S**chul-**E**ntwicklung, Grogger/Svecnik, 2001)

In the school year 1999/2000 the 305 students and the 13 teachers from the 17 project classes at the 8 grammar schools and 3 business academies had to fill in a questionnaire handed out by the ZSE, a department of the ministry of education. In addition also the German and English teachers and all the maths teachers not involved in the CA-project were questioned (altogether 142 teachers).

Results at the student level:

When asked about the 4 models (short tests - problem solving tests, project work, cross curriculum tests, written group tests), it turned out **that the cooperative forms of assessment met with general agreement among the students.** The interdisciplinary forms, but also the replacement of a written tests by written paper + a presentation of the paper as well as the short written tests which force the students to be up to date were declined.

With the exception of the cross curriculum tests, **students at the business academies** find the alternative models more attractive than the traditional system.

With the exception of the cooperative forms, **students at the grammar schools** prefer the regular form of tests.

The different acceptance of the tried and tested forms of assessment is expressed by the fact that given completely free choice a little less than **75% of the students at the business academies would decide for the new models again whilst among the students at grammar schools almost 50%** would do so.

The attitude towards the cooperative forms of work is definitely positive among all the students, but agreement is more pronounced among the students at the business academies**.**

Students at business academies like to work together with other students and wish to have more partner and group work in their maths lessons. They think that group work leads more quickly to better results with problems that require much time. They do not like to work on complex problems on their own.

Students at grammar schools are of the opinion that when doing group work only the best students work and the others just look on.

Students at business academies do not at all agree with this.

Results at the teacher level:

From the point of view of the **teachers who were not involved** in the project, the need for adapting the assessment system does not really exist.

As to the appropriateness of the traditional written tests in a **more student oriented teaching** there clearly exist **signs of two groups** of more or less equal size with different opinions according to the view of ZSE. **But in general the teachers are of the opinion** that the traditional way of assessment works quite well**.**

Among the teachers who try out one of the four presented models or a combination of them, **the model of the short written tests** which force the students to be up to date **is the clear favourite**. Positive effects are ascribed to this model in regard to facilitating teaching the different groups of students in the classroom.

The written group test to which the students express a clearly positive attitude, **is generally met with refusal by the teachers**.

The teachers interviewed saw the advantage of project work and of the cross curriculum tests in an increase of the students´ motivation and pleasure in learning, **but in their opinion they meet the demands of the better students rather than those of the less able ones.**

Reactions of the research teachers on to the ZSE-Evaluation:

At the first presentation of the results by the ZSE in autumn 2000 **the research teachers were surprised about the different opinions of students and teachers**. But what really astonished them were **the different reactions of the students at grammar schools and those at business academies.**

Böhm, a research teacher at a business academy, mentions two reasons to explain the different attitude towards cooperative teaching forms and assessment forms: "*One reason might be that our students (business academy) are more used to collaborate work. Another one could be that our students were one year older than the others.*" (Böhm, 2003)

Böhm gave a written group test with three problems. He formed his class of the business academy with 26 students in 8 groups (2 x 4 and 6 x 3). Then he observed the students in organizing their group work. He described his observation and was really fascinated:
"*First they read and discussed the problems. Then they tried to distribute the work according to their abilities and preferences. This took much more time than I had expected, but I saw that this was a very necessary and positive aspect. So I decided immediately to extend the working time from one to two hours. No one single student did only copy the others´ work or was not really busy on his part. Each group had to deliver one paper. All members of a group received the same marks. Finally only one group had a negative mark which was accepted by the students without any claiming about grading to hard."*

8.) The model of assessment I chose.

I chose to use **a combination of model 1 and model 2** in form 11 at a grammar school (1999/2000):

The fundamental idea of my model variant is to use the pre-set time for written tests in a school year - 350 minutes in form 11 - in different ways (Wurnig, 2001):

For short tests *- up to a maximum of 25 minutes -* ***to check reproductive skills*** *or reproductive knowledge with or without CAS.*

For one longer test per semester*,100 minutes, -* ***to check problem solving skills****. There should be sufficient time to experiment and to use materials which have been worked out at school or at home.*

For working out a short chapter of mathematics (a little project work) *which has not been dealt with at school. Each student was to prepare his short chapter in written form at home and present it to his classmates at school.*

The **shorter tests and longer problem-oriented tests** had a very different impact on my students of the 11th form. Many students did not work hard enough for the short tests in which the basic skills were examined by means of easy short problems and so the test results were bad. The problem-oriented tests, which were written later on, were taken more seriously, because they wanted to get good marks in their reports and consequently the achievements were much better.

The preparation of **a short chapter of mathematics at home and the ensuing presentation at school** proved to be the most difficult part on behalf of the students. **Most of the students had never before prepared a disposition for a theme at school**. It took me two lessons and many discussions to make even good mathematicians understand how to prepare an acceptable written and oral presentation.

To illustrate the **model short tests – problem-oriented tests** I will give an example of each.

A short test

In the **2nd short test** the students had to use the new formulas to solve standard problems and at the same time they had to use the TI-92 in an efficient way if the wanted to get a good mark in the time given. The four problems set were prepared in the mathematics lessons. The aim of the 25-minute-test was to check the minimum knowledge in analytic circle geometry in a reproductive way. In problem 1 the TI could only be used to check the result. I/O means input/output of the TI-92.

1) Circle: $x^2 + y^2 - 8x + 10y = 0$ Calculate M = (m, n) and r

I: *$x^2-8x+16+y^2+10y+25=16+25$*
I: *factor($x^2-8x+16$,x)* O: *$(x-4)^2$*
I: *factor($y^2+10y+25$,y)* O: *$(y+5)^2$*

2) What is the distance from line g: 3x - 4y = 12 to k: M=[-5, -3], r = 2

I: *dotp([-5,-3]-[4,0],unitv([-3,4])* O: *3*

3) Calculate one of the two intersection points of the line g with the circle k!

g: X = [7, -4] + t . [3, -2] k: $(x-2)^2 + (y+5)^2 = 65$ X = [x, y]

I: *$(x-2)^2 + (y+5)^2 = 65$ | x=7+3t and y=-4-2t* O: *$13t^2+26t+26=65$*
I: *solve($13t^2 + 26t + 26 = 65$, t)* O: *t = 1 or t = -3*
I: *[x, y] = [7, -4] + t . [3, -2] | t = 1* O: *[x = 10 y = -6]*

4) Find the equation of the circle tangent in point B of the circle k:

k: $([x, y] - [7, -2])^2 = 20$ B = [3, -4]

I: *dotp([x,y]-[7,-2], [3,-4]-[7,-2])=20* O: *-4x -2y + 24 = 20*

A problem oriented test

After dealing with the conic sections, calculus and the binomial and poisson distribution in the mathematics lessons the last test in the 2nd semester was **a problem-oriented test of 100 minutes**. The students had to solve the following **three problems**:

Problem 1:

A segment of a hyperbola is enclosed by the line x = 7 and the hyperbola, which is given with the equation $25x^2 - 4y^2 = 100$. Inscribe into this segment a rectangle of maximum area.

Sketch a diagram oriented by the graphic window of the TI-92!

a) **Solve this problem with the help of calculus in the algebra window!**

b) **Solve this problem in the graphic window! Documentation!**

In Austrian traditional teaching students often have to solve such a problem, but it is not usual for them to have to do so in a graphic way, too.

Problem 2:

Supposing a point object is moving on the curve of the function

$$f: x \to (1/4) . (x^3 + 2x^2 - 3x), \quad [-2;2] \to R$$

and is striking a wall **(x=2)**. Find out the size of the angle of impact?

Sketch a diagram oriented by the graphic window of the TI-92!

The following problem could be solved with the new concept of calculus with the possibilities of the TI-92. It is amazing how many different ways of solving this problem the students have found.

Problem 3:

The death rate of newborns, this means the probability of babies to die during their first year of life, is 1,8% in a country. What is the probability that of 1000 newborns chosen at random more than 950 and less than 980 will live to see their first birthday ? **Solve this problem a) with the binomial, b) with the poisson distribution !**

This problem normally is solved with the binomial distribution (p = 0.982). But you can also solve it with the poisson distribution, if you calculate the probability that of 1000 newborn babies less than 21 will not live to see their first birthday. After doing this you have to find the contrary probability.

9.) Acceptance of the new assessment models as seen by the students in 2002.

At the end of the CAS IV project (2001/2002) the students of all research classes of form 11 had to answer 27 questions concerning the acceptance of the new assessment models. The result is very encouraging (Fürst, 2003).

Nearly 100% of the students recommend the new assessment models and would like them to be part of the normal school assessment.

Short tests are seen as a positive innovation by both boys and girls. (89%).

Problem solving tests are seen more critically. 74% find that they have a high level.

74% think they should get a positive mark in problem solving tests if they are able to solve exercises similar to those done at school.

Concerning the use of auxiliary materials the results are as follows:

Formulary	should be admitted	100%
TI – 92		98%
Exercise/ homework books		75%
Mathematics book		57%

Project work is seen as a very useful innovation (92%), but the students want to decide by themselves in which of their subjects they do the project work (79%).

As to the last question asked, the students had to consider whether and in what way they had personally profited from the new experimental way of teaching with the help of CAS.

Almost all the students were of the opinion they had profited..

Some answers from among my students translated from German into English:

- Girl, best student of the class, Maths not among favourite subjects, but very good mark

"*It's fun working with the TI-92. I was able to reduce my original dislike of the subject. My attitude to Maths changed. It lost the character of a subject for freaks for me when solving application-oriented problems of mathematics."*

- Girl, Maths not among favourite subjects, but a good mark.

"*I got to know modern media and learned to do calculations in a faster way. I like the use of the TI-92 and I appreciate the use of the supporting materials when writing tests.*"

- Girl, Maths rather a favourite subject, average mark.

"It is easier to understand the learning content because of the many short tests. They help to create a readiness to cooperate."

- Boy, Maths among favourite subjects, mark below average.

"*I have learned how to take meaningful notes so as to be able to use them when needed. I have learned to work with the TI-92 and this has helped me to expand my knowledge.*"

- Boy, Maths rather a favourite subject, good mark.

"*My learning has become more independent and more problem oriented. I no longer copy mechanically, I now solve my problems individually.*"

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