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CAS influences the mathematical language

Finding the middle course between exact mathematical language and math-jargon induced by the use of technology

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Abstract:

When we use technology such as graphical calculators, computer algebra systems, or dynamic geometry systems for teaching and learning mathematics, we inevitably introduce the related technology jargon into the mathematics classroom.
How to find an acceptable middle course between the exact mathematical language and jargon of (math-)technology is one of the present-day problems of mathematics teaching. We discuss it in this presentation.

1. Mathematics as a language

Mathematics is a highly developed mental tool for model-representation and the investigation of phenomena and processes in nature.
„The book of nature is written in the language of mathematics" is a respective famous quote by Galileo Gallilei.
During the centuries and millennia different mathematical disciplines were developed – some which enable a very large spectrum of use (such as algebra, calculus,…) and some for very specialized problem areas (such as cybernetics, graph theory, probability, …). In mathematics in general as well as in each of these disciplines specific symbolic languages were developed ("mathematical language(s)") as a means of communication within mathematics (or its specific area) and about mathematics. In mathematics much attention (more than in any other scientific discipline) is directed towards an exact use of a symbolic language which is very precisely defined. In terms of establishing a common platform areas such as set theory and logics were developed.
The (symbolic) language aspect becomes one of the most important aspects of mathematics. By learning mathematics we inevitably learn also the "mathematical language" – and by teaching mathematics we inevitably teach also the mathematical language.

Already in very early stages of mathematics teaching we have to teach students to be precise in expression and we teach them to use appropriate symbolic descriptions:

a) First of all we teach the students to be precise regarding the **connection between mathematics and the spoken** (mother) **language** (e.g. to use appropriate words):

Here are a few examples:

- One should not say "the cube has 12 sides." (While a *side* can be part of a polygon, a solid has *faces and edges*)

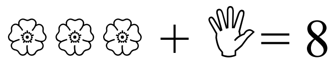
- A line segment is defined by it's *end points* and not by *vertices*

- The students learn to distinguish between "decomposing a number into summands" and "factorizing a number".

- The students should avoid to look for "… the zeros of an equation" or "the solutions of a function".

- Last but not the least, it is the matter of (mathematical) language (e.g. logic) to know what is the negation of the statement "a is divisible by 3, 5 and 7".

b.) The next step is to be precise **within mathematics** (in connection with the appropriate use of symbolic descriptions):

- One must not mix mathematical symbols and icons such as in: 

- There is an important difference between "$\forall x \in A, \exists y \in B, \ni: y=f(x)$ " and "$\forall x \in A, \exists! y \in B, \ni: y=f(x)$ " which the students should be aware of when defining a function $f: A \to B$

- When describing the notion of continuity, the symbols (mathematical language) have to be used very precisely:

$$\forall \varepsilon > 0, \delta \exists > 0 \ni: (| x - x_0 | < \delta \Rightarrow | f(x)-f(x_0) | < \varepsilon)$$

<u>2. Colloquial mathematics language</u>

One part of the mathematical language – the non-symbolic part – is closely related to the spoken language.

- Our tendency to speak very colloquial introduced some colloquial notions also in this part of the mathematical language – such as:
  - "f is the function given by $f(x)=3x+1$" - a mathematician knows that implicitly this includes also the domain A and the range of values B. This has to be made clear to the students!
  - "draw the function" really means "draw the graph of the function" – in math classes we have to stress this subtle issue again and again

- Slightly more dangerous are short cuts such as:
  - "Solve the equation $3x^2 + 27\,ax – 8 = 0$ ".
    Most of us (including our students) would "solve the equation with respect to x" without question, because it is a "common agreement" that "the variable is named x" – but is it always so?

- Our inconsistency in using mathematical language leads our students to invent and use even more dangerous short cuts such as:
  - "Solve the integral".

If not appropriately corrected and explained, such short cuts can lead to serious ambiguities and wrong conceptions regarding mathematical concepts.

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3. Math-jargon of technology

Long before we could do mathematics with the help of technology such as CAS, the most common users of mathematics – the technicians – made strong (over)simplifications of mathematics terminology. One of the most common such "simplifications" often is used also in school mathematics:

"Given is a sequence $u_1, u_2, u_3, \ldots$, the expression $\sum_{n=1}^{\infty} u_n = u_1 + u_2 + u_3 + \ldots + u_n + \ldots$ *, involving the terms of the sequence, is called infinite series or simply a series with terms $u_1, u_2, u_3, \ldots, u_n$."(Gullbery, 1997)*

(or: *"A series is the sum of a sequence $S_n = a_1 + a_2 + a_3 + \ldots + a_n + \ldots$" (Silverman, 1989)*).[1]

Today the "jargon of technology" threatens the mathematical language very seriously through the widespread use of CAS.

Use of CAS as a tool for doing and teaching mathematics has a strong influence on the mathematical language being used.
Even the users who use CAS only for doing mathematics accept the language of their mathematics tool as their math-jargon.
But the use of CAS in the school – in particular in mathematics classes – creates the mathematical language we teach. If we use CAS for teaching mathematics, we must take into account that inevitably also the language of the tool becomes part of the *"mathematical language"*.

Therefore the teacher should pay enough attention to the language used by their mathematics tool.

Originally CAS were developed for doing mathematics (to be used by trained mathematicians who know mathematics and the mathematical language well). Therefore the programmers use a very simplified mathematics vocabulary as the basis for commands or dialogues (- with supposed sophisticated users the use of a simplified vocabulary appears acceptable).
The vocabulary used by a CAS seems suggested by the desire to obtain a certain functionality such as a wider applicability of an operator compared to what is normally allowed in mathematics. Therefore:

– Sometimes the CAS language is very precise, such as demanding the user to specify all arguments required by an operator. – *Example(Fig.1)*:

[1] This really should be formulated as follows: » Let $u_1, u_2, u_3, \ldots$ be a given sequence. Form a new sequence $S_1, S_2, S_3, \ldots$ where
$S_1 = u_1$, $S_2 = u_1 + u_2$, $S_3 = u_1 + u_2 + u_3$, ..., $S_n = u_1 + u_2 + \ldots + u_n$, ... where Sn, called the nth partial sum, is the sum of the first n terms of the sequence (un). The sequence $S_1, S_2, S_3, \ldots$ is symbolized by

$u_1 + u_2 + u_3 + \ldots = \sum_{n=1}^{\infty} u_n$ which is called an infinite series.« (Murray A Spiegel (1963))

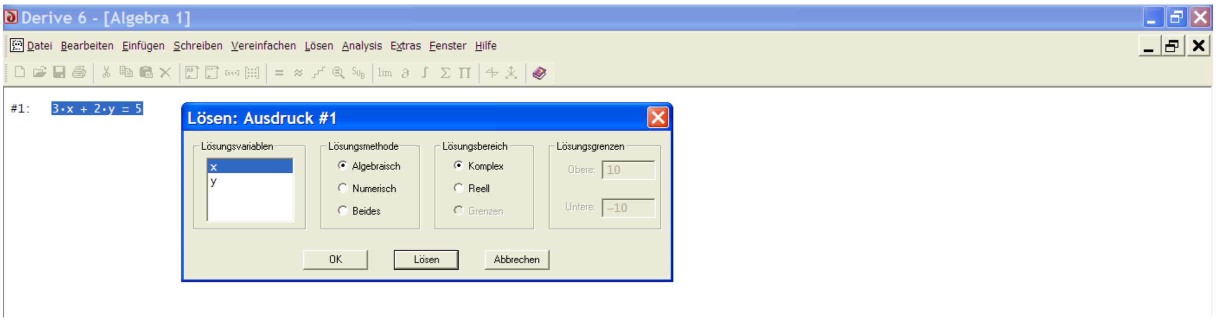


Fig.1

- Sometimes the CAS language is imprecise, such as using inadequate mathematical terminology. (For example using the phrase "Solve Expression".) – *Example (Fig1).*
- Sometimes the CAS language is sloppy, such as allowing illegal applications. (For example solving an expression which is not an equation.) - *Example(Fig2)*.

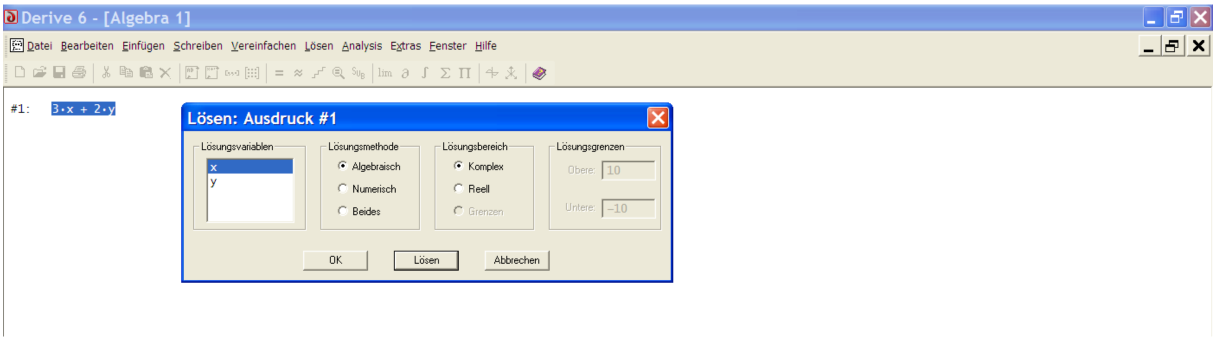


Fig.2

To some extent this is the price one has to pay for ease of use. The stricter the tool's language, the more difficult is its use. The more convenient you want to make the user interface, the looser the language will become.

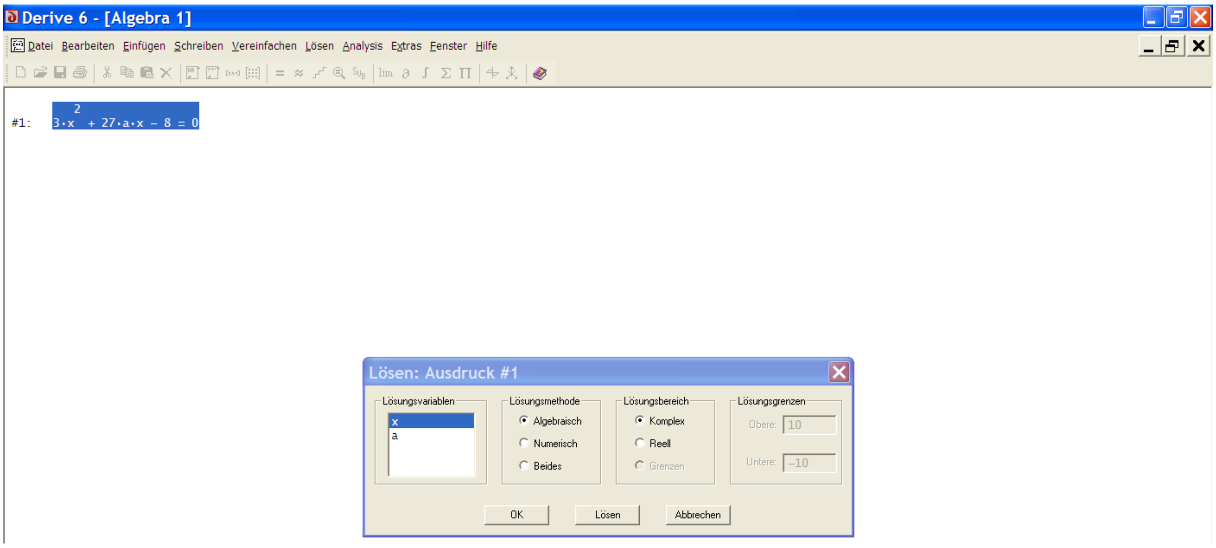
But the terminology used by CAS becomes relevant (in fact **very** relevant) if we use CAS for school mathematics **teaching**.
The goal of school mathematics teaching still remains to introduce (and teach) the exact mathematical language as it is used in the frame of the mathematical science.
Therefore we should be aware of all weaknesses (to be able to avoid them) and strengths (to be able to make use of them) of the CAS-(math-) jargon.

4. Examples

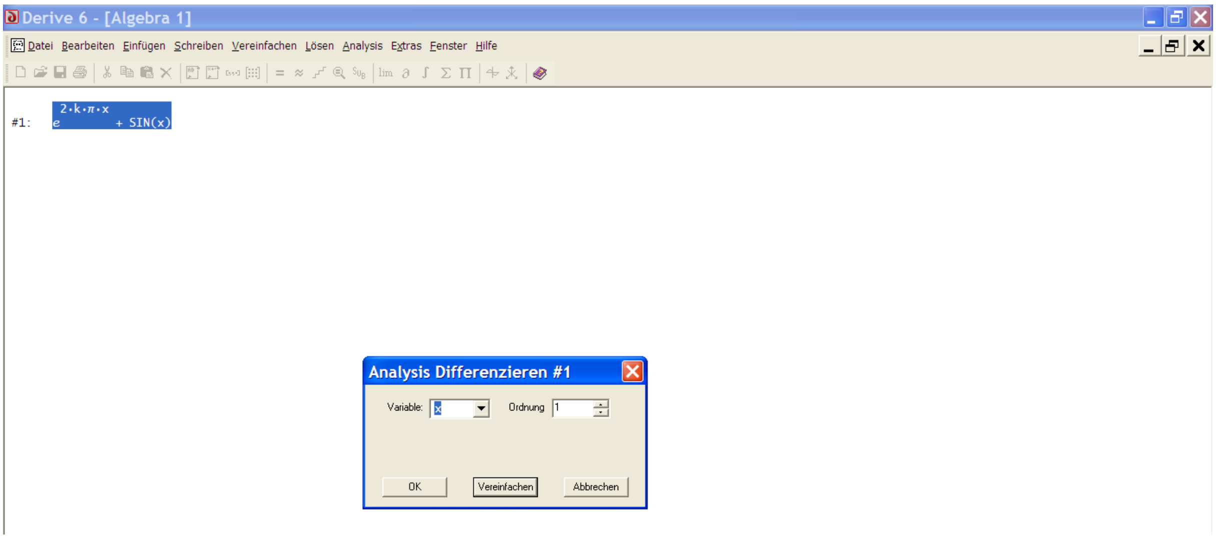
4.1. Use of CAS can improve a student’s poor image of mathematical concept:

a.) PrtScr - Voyage 2000 (“this is no equation”)

b.) Solve the equation $3x^2 + 27\,ax - 8 = 0$ – with Derive



c.) $\int (e^{2kx} + \sin(x))dx$



These are examples, made with Derive 6, to show how CAS can force the user to be precise. Using Derive (or any other CAS), one must tell the system very precisely, regarding to which unknown it should solve the equation; the system permanently reminds the user of the importance of the function variable (students using a CAS will hardly forget the dx such as in $\int x^6 = \frac{x^7}{7} + c$).

This is an important contribution to the (school) development of mathematical concepts.

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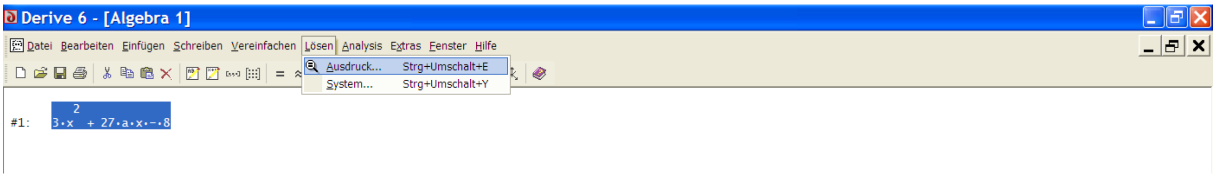
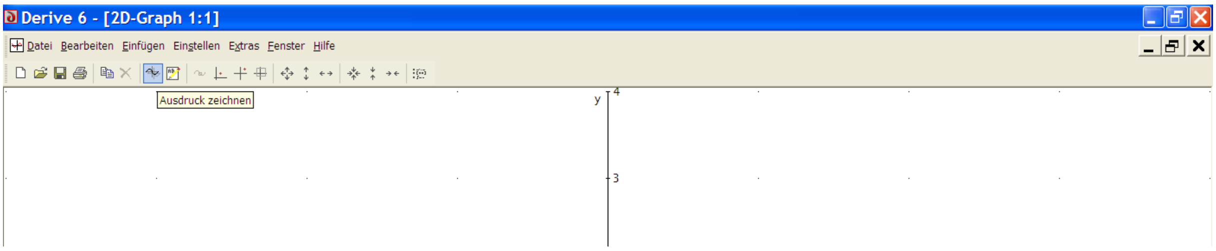
4.2. Use of CAS can lead to inappropriate use of mathematical terminology:

a.) “Solve expression”: $3x^2 + 27\,ax - 8$



b.) “Plot expression”: sin(x)



If the teacher does not draw the students’ attention to the dangerous part os the tool’s jargon (such as in examples above) and if the teacher does not make the necessary corrections and explanations, then the students are likely to adopt this and make it part of their mathematical language.
In that case the tool’s jargon will badly influence the students’ concept image of certain mathematical concept. This can result even in misconceptions.

But the teacher can turn this from bad to good: through the permanent awareness of imprecise formulations (stimulated by the teacher), the students will learn to pay attention on various (important) aspects (characteristics, qualities) of the respective mathematical concept.

5. Conclusion

Use of CAS in school mathematics inevitably influences the mathematical language, which is taught as part of school mathematics.

The math-jargon of technology influences mathematical language learned by the students, as well as the students’ concept image of the particular mathematical concepts.

The duty of the teacher is to pay attention to this issue and to take precaution to make pedagogical **use of CAS** also with respect to the mathematical language.

"We should strive to makes things as simple as possible - but not simpler than that."
(Albert Einstein)
(The teacher becomes even more important than in traditional teaching, when the students use CAS in math classes, also in the case of mathematical language.)

References

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