

# Lectures

(in alphabetical order of the presenters)

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## ***ATPCL.mth: Automated Theorem Provers for Propositional Classical Logic with DERIVE***

In this paper the file ATPCL.mth is presented. This file has been developed for using DERIVE as an automated theorem prover for Propositional Classical Logic by means of different algorithms such as “Quine”, “Semantic Tableaux” and “Short Normal Form + Resolution”.

The use of this utility file is specially useful for teachers and students in subjects on Computational Logic in Computer Science degree. In particular, the file can be used as a didactical tool for helping in the teaching and learning of automated logic deduction process.

In order to develop these algorithms, another utility file has been created to deal with tree structure using the new features of programming in DERIVE. In this paper this utility file (tree.mth) will be also described.

The use of these utility files will be presented by means of some examples about validity, satisfiability and deduction using both utility files. Finally, conclusions and future work will be shown.

[aguilera.pdf](#)

[aguilera.ppt](#)

[ATPCL.dfw](#)

[ATPCL.mth](#)

[tree.dfw](#)

[tree.mth](#)

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## ***Cognitive Tools for Exploring Linear and Exponential Growth***

Empowering teachers through the use of technology in mathematics exploration, open-ended problem solving, interpreting mathematics, developing conceptual understandings and communicating about mathematics is at the heart of BRIDGES (<http://education.wichita.edu/alagic/bridges/BRIDGES.htm>) professional development. Throughout the BRIDGES activities, concrete experiences have been provided to explore technology-based representations of data, graphs and functions. A particular focus of the professional development was two-dimensional: (a) deepening teachers’ understanding of linear and exponential growth via technology-based representations, and (b) providing effective context for students’ learning from the same technology-based representations, considering the fact that they do not have teachers’ standard representations in their toolbox. A variety of cognitive tools (spreadsheets, graphing calculators, hand-helds) provided for (a) visual and graphical multiple representations interconnected with appropriate simulations, (b) meaningful explorations of a variety

of cases in a smaller amount of time than if standard representations had been used and (c) nurturing learning environments that support priorities of teaching for understanding and teaching for transfer. This paper will report on lessons learned in attempts to explicate some bridges between classical and technology-based representations as well as on teachers' views on related indispensable and dispensable mathematical abilities and skills related to the concepts studied.

[alagic.pdf](#)

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### ***On the CAS and the coordination of semiotic representations***

The importance of the semiotic representations and their relations with the cognitive processes have been shown by many researches in Mathematics Education (Artigue, D'Amore, Duval, Mackie, Pavlopoulou, Tall). Deep learning, that is the conceptual acquisition of a concept, occurs when the pupil is able to pass from a representation in a given register to another one in another register or in the same register. In this paper we want to show how CAS, with direct and active involvement of the student, can improve learning in the above sense. This is because such environments are multiple representation systems, symbolic, graphical, numerical, parametric, logical, ... Students are often in front of diverse answers to the same questions (for example solving systems of linear equations in Derive can be done by SOLVE or SOLUTIONS or simply by PLOT) so they are stimulated to concentrate their attention to the meaning of the results obtained by the computer, to establish links among different ways of seeing same formal expression which acquire different meaning in diverse contexts. The ability to recognize such different representations and their common properties conduces to construct the "abstract" concept of a mathematical object or process. Such abstraction is fostered by CAS use.

[albano.pdf](#)

[albano.dfw](#)

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### ***Le calcul en images***

Le logiciel "Calcul en images" illustre de façon animée et interactive les concepts de: limite d'une fonction en un point, dérivée en un point, fonction dérivée, intégrale définie et fonction intégrale. Pour chaque animation les fonctions et les valeurs numériques sont affichées. Le logiciel permet ainsi d'explorer ces concepts sur la plupart des fonctions utilisées dans les cours de CEGEP. Le logiciel sera du domaine public sous peu.

Durant la communication nous présenterons le logiciel et nous ferons un bref compte rendu de notre expérimentation en classe. Quelques suggestions d'activités à faire avec les étudiants seront proposées. Nous présenterons aussi des simulations de situations concrètes accompagnées des graphiques des fonctions qui modélisent chaque situation. Toutes les animations sont écrites à l'aide du logiciel Maple. Une discussion avec les participants suivra l'exposé.

[antonius.pdf](#)

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### ***Technology Tools for Mathematics Classes***

Technology Tools for Teaching is a project designed to train teachers of public schools on the use of modern technologies in their classrooms. In this project, teachers make use of Microsoft Office programs and Internet to develop simple customized tools that fit their students' needs and abilities.

In this session, I will be demonstrating the use of three types of tools:

1. Electronic Worksheets developed with EXCEL. Samples of such sheets will be shown and discussed. These sheets are used for instructional purposes, both as teaching aids that help teachers in the course of explaining lessons or solving problems, and as independent learning activities.
2. Integrated Units developed with POWERPOINT, EXCEL AND WORD. Samples of such units will be shown and discussed. These units combine presentation features and hyperlinks to create integrated units that teachers can use in the instructional process and that students can use for independent learning.
3. Internet based activities that can be used to enhance independent learning and promote students computer skills. The Guided Tour is one such activity where teachers make use of educational Web sites to develop activities that allow students to explore concepts or themes related to their course of study. This is done through carefully guided steps that make sure that students know where to look for information, and at the same time, allow them to do some exploration on their own. A sample activity will be used for demonstration purposes.

[atallah.pdf](#)

[TIME\\_presentation.ppt](#)

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### ***Modeling some Dynamic Phenomena with Maple 6 in a CAS-Based Math Class***

We are interdisciplinary members of a research team supported by CONACYT #41596-Y, Mexico. We pursue some math teaching guidelines for modeling courses at a main Mexican University. At the Summer Academy, we will illustrate some classroom activities from a CAS-based curricula and teaching methods approach. The modeling problem is get the valve diameter of a pipeline entrance as a result of a decision making process from the residual water flow that is poured into one tank of a residual water treatment process. We designed a Maple6 program to generate a data table of the water flow historical records, the correspondent residence time and the valve diameter. The residence time is an intermediate variable and the valve diameter is the key decision. This is very important because is the control for water waste plant. We think that this is the type of problems that we should discuss in a CAS-based teaching of applied mathematics on transport phenomena.

[balderas.pdf](#)

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### ***Using the TI voyage 200 in Structural Analysis***

The speaker will propose a few examples of problem solving in structural analysis that illustrate the use of the TI voyage 200, a symbolic and graphic calculator. The examples deal with the classical methods of analysis for both statically determinate and indeterminate structures, within the scope of a first course on structural analysis at the undergraduate level. The use of the TI voyage 200 greatly reduces the mathematical difficulties in problem solving, and thus allows the students to spend more time in developing a good understanding of the behaviour of structures. The TI voyage 200 is a complex tool and the examples illustrate how to use the calculator in such a way that the students will want to take advantage of its computational power throughout their career as engineers. The examples are taken from the textbook used in structural analysis at the École de technologie supérieure (Samikian,1994) and are an attempt to update its contents by including the use of a symbolic calculator.

[bauer.pdf](#)

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### ***Exploration of Alternative Addition and Multiplication using the TI-89***

We define a new pair of operations on the real numbers by  $a \oplus b = a + b - 1$  and  $a \odot b = a + b - ab$ . These are not only binary operations on  $\mathbb{R}$ , but they give it a field structure that is isomorphic to the regular field operations. These new operations allow students to explore familiar ideas, specifically algebraic and analytic concepts, in an unfamiliar setting. This encourages them to look at the algebraic and analytic concepts beyond their intuitive understanding of the workings of the real numbers.

One of the strengths of the TI-89 is that we can define functions in two variables to match these operations. First, we use the TI-89 to explore the algebraic properties of the operations, such as associativity, commutative, and order of operations. The solve command can be used to find identities for the operations. (This leads to surprising results.) We also explore the inverses of elements under the two operations, including which elements are not invertible. This leads to the question of solving polynomial equations involving these operations. This, in turn, leads to an expanded understanding of what polynomials are.

We can now consider the formal derivative. One interesting question is how the formal derivative is related to the "limit" definition of the derivative from analysis. An exploration of the formal derivative leads to an appropriate limit definition of the derivative from analysis. Other explorations will be discussed.

[beal.pdf](#)

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## ***Ten Years of International Derive Conferences***

Everything is going so fast. Mathematical computer programs are so powerful that we may think that it has always been the case. This lecture focuses on how technology has changed the way I teach mathematics, through specific examples.

A souvenir from Plymouth, U.K. in 1994: in version 3 of Derive for DOS, implicit 2D plots became available. Could you imagine investigating multiple variable calculus without implicit plots? Many ideas for using Derive as a teaching tool originated from this conference.

In Bonn, Germany, 1996, Bert Waits said, about the TI-92: “this machine will change the way we teach mathematics for the next fifteen years!”. This handheld technology allowed computer algebra to be used in the classroom by teachers and students as never before, changing forever the opinion of many mathematicians about graphic calculators.

From Gettysburg, USA, 1998, I will recall two independent souvenirs. First, TI-89 and TI-92 Plus Module with Flash technology, giving the possibility to upgrade to future software versions without buying a new calculator. Second: the collaboration between David Parker (Acrospin, Cyclone, DPGraph) and the authors of Derive announced upcoming interesting results.

In Liverpool, U.K. in 2000, Derive 5 and its new interface became a reality and spectacular 3D plots capabilities were added. New programming capabilities were introduced, “that have revolutionized the way that programs can be written and displayed in the Derive mathematical environment” as Terence Etchells said.

Finally, Vienna, Austria, 2002. I asked the following question: if we use both systems, Derive and the symbolic TI, can the two devices communicate? Some Derivers were concerned with another question: are you organizing the next conference? Well, Derive 6 now offers the link I was looking for (and even much more, for example, the ability to show the steps of many simplifications). And without the collaboration of my colleagues Gilles Picard and Kathleen Pineau, this TIME-2004 symposium would not have been possible.

[beaudin.pdf](#)

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## ***Calculatrices symboliques dans l'enseignement des mathématiques en génie à l'ÉTS : bilan et avenir***

Cela fera bientôt cinq ans, en septembre 2004, que les étudiants inscrits au baccalauréat à l'ÉTS utilisent une calculatrice symbolique de la compagnie Texas Instruments (TI-92 Plus ou TI-89 et maintenant la Voyage 200). En faisant un bilan de son utilisation, nous donnerons des exemples concrets d'apports positifs et négatifs d'une telle calculatrice et invoquerons certaines des raisons qui ont motivé son adoption. Il ne faut pas pour autant penser que cette utilisation est uniforme dans chaque classe, dans chaque matière enseignée : cela peut varier d'un enseignant à l'autre et il est tout à fait possible de continuer d'enseigner de façon enthousiaste sans nécessairement utiliser la technologie! Nous avons constaté que des appuis, des encouragements et des séances de formation auprès des professeurs s'avèrent insuffisants tant et aussi longtemps qu'un enseignant ne voit pas de lui-même un avantage à utiliser la technologie. Par conséquent, c'est nous les utilisateurs de la technologie qui devons répondre à la question suivante : en quoi la technologie vient-elle améliorer l'enseignement des mathématiques? Sans être absolument certains d'assister à une amélioration de l'enseignement des mathématiques, nous avons noté, sur le terrain, une évolution significative des contenus de cours, du matériel utilisé, l'apparition de nouvelles questions posées aux examens et dans les travaux aux étudiants. En d'autres mots, l'utilisation de la technologie est intimement reliée au désir de changer des choses : en abandonner certaines, en découvrir de nouvelles, en redécouvrir sous un angle différent. Réviser des trucs classiques dans un environnement stimulant et explorer de nouvelles avenues : voilà ce que nous avons fait et que nous voulons continuer à faire.

[beaudin\\_picard.pdf](#)

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## ***A Case for CAS***

A group of five Computer algebra System experts formed a "TI-Special Focus Group" to produce a book, which should convince reluctant teachers and school authorities as well to permit and support introducing handheld computer algebra in mathematics education. Colleagues from Scotland, Belgium, Switzerland, Denmark and Austria cooperated in gathering, selecting and commenting teaching units and assessment examples. They also tried to show the way how to prepare the students for technology supported exams. Valuable additional contributions from France and Australia accomplish the product which is embedded in a rich didactical discussion.

Guido Herweyers, Belgium and Josef Boehm, Austria will present the exciting history and the results of this challenging project.

[boehmcas.pdf](#)

Cabri-files are in folder \time\_contribs\boehm

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### ***DERIVE 6, Its Impact on Teaching Mathematics***

DERIVE 6 offers a bundle of new features which open a wide field of revolutionary possibilities in maths education.

It is not only the challenging concept of Interactivity between handheld and PC-Computer algebra, but also the meaningful use of slider bars in 2D- and 3D-graphical representations, the "Stepwise Simplification", loading external graphics on the Plot Window and others which provide a box of opportunities to enrich mathematics teaching and learning.

The lecture will demonstrate the new features by presenting selected examples from various chapters of secondary school mathematics.

[boehmd6.pdf](#)

All Derive-files are in folder \time\_contribs\boehm

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### ***L'utilisation de la calculatrice TI dans les cours en Génie électrique***

Depuis la réforme de 1999 à l'ETS qui visait à obliger tout nouvel étudiant à posséder une calculatrice symbolique, il s'est développé un intérêt marquant pour la programmation de ces ordinateurs pour les applications de génie. Dans cet exposé, je présenterai des programmes fait pour certains cours en Génie à l'ETS lors de mes études en génie.

[bois.pdf](#)

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### ***Logiciels de calcul des propriétés thermodynamiques pour l'eau et le réfrigérant HFC-134a pour les calculatrices TI-89***

En thermodynamique, les relations entre les variables d'état (pression, température, volume massique, etc.) sont typiquement très complexes. L'approche traditionnelle consiste alors à tabuler les valeurs de celles-ci. En conséquence, les étudiants en thermodynamique doivent se familiariser avec l'utilisation de ces tables et sont souvent contraints à de fastidieux calculs qui, quoique formateurs à certains égards, peuvent détourner l'attention de l'objet principal de la thermodynamique : les processus et dispositifs de transformation de l'énergie.



Des logiciels de calcul des propriétés thermodynamiques de l'eau et du réfrigérant HFC-134a ont été développés afin d'être utilisés sur les calculatrices TI-89, TI-92+ et Voyage 200. Ces logiciels qui s'appuient sur des formulations mathématiques reconnues sont maintenant utilisés dans le cours de thermodynamique à l'ÉTS. Nous présenterons ces logiciels et discuterons de certains points relatifs à la programmation en langage C sur ces calculatrices. Nous ferons aussi mention de certains des outils mathématiques que nous avons développés, notamment, une méthode simple de résolution de systèmes d'équations à deux variables appropriée aux situations que nous avons rencontrées.

[bordeleau.pdf](#)

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### ***The assessing of mathematics skills in a secondary school CAS environment***

Until recently much of the focus regarding the introduction of CAS into assessment at the end of high school has been on whether to, or not to, use CAS. However, now it is generally accepted that whilst there is still a long way to go, the danger presented by the introduction of CAS into examinations is not as significant as first thought. Of increasing interest, however, is whether the introduction of CAS will lead to a change in the skills assessed, and if so in what way.

This presentation will seek to address this issue by using data generated from the analysis of CAS allowed examinations in Denmark, Victoria (Australia) and the USA using a categorisation scheme which assess the types skills which are required to complete the question. The presentation will also indicate whether there has been a change in the testing of these skills as the examination question writers have become more familiar with development of CAS based questions.

[brown.pdf](#)

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### ***Developing Control Over the Use of a CAS : The Teacher's Perspective***

École de technologie supérieure (ÉTS) is a technical engineering school where the use of a handheld Computer Algebra System (CAS) has been integrated in the first year common core mathematics and science courses. Since the fall of 1999, the TI CAS (TI-89, TI-92 Plus or now the Voyage 200) has been mandatory for all first-year students entering calculus. Students are expected to use it in class, at home, and during exams.

A common culture for computer-assisted calculus has progressively emerged among teachers through the use of the same textbook for a given course, design of a common final exam for all groups,



exchange of “good problems”, and ad hoc discussions. However, some interesting individual variations exist in the teaching and in the design of learning tasks given to students. In particular, though the objective of helping students develop mathematical control over the tool seems to be shared by all teachers, the strategies deployed to achieve such an objective may vary substantially from one teacher to the next.

In this presentation, we will characterize the common features and individual variations of CAS-integration in the first calculus course at ÉTS based on data collected from five teachers. This is the first of a two-part study which will later look into the effects of the different teaching strategies on the effective control students display in solving calculus problems.

[caron.pdf](#)

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### ***Technology in Mathematics Teacher Preparation Courses***

Common themes in requirements for prospective mathematics teachers include mathematical modelling, problem solving, effective use of technology, and communicating mathematics. In this presentation we will demonstrate how prospective teachers can use technology effectively to analyze the predator prey model. In our courses we assign student projects to provide the opportunity for students to communicate mathematics. We require that students work in teams to make a formal presentation to the class with the technology of their choice. Some students use a computer with presentation software and/or access to the Internet. Others use calculators, posters, and transparencies on overhead projectors. For example, the students can use the technology of the TI-89 or Voyage 200 computer algebra system to explore the differential equations of the predator prey model and interpret solutions from three different points of view: graphical, numerical, and analytical. Slope fields and graphs of solutions or direction fields and solution curves in the phase plane contribute to better understanding of long term behavior of the model. Tables of approximate solutions using Euler or Runge-Kutta methods also provide information. Graphs and tables of exact or approximate solutions can be compared on a split screen. Exact symbolic solutions to many 1st- and 2nd-order ordinary differential equations can be computed easily. Matrices, eigenvalues and eigenvectors are also easily handled to determine the exact solutions to systems of ordinary differential equations.

[connors.pdf](#)

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## ***Assessment Issues in the introduction of a CAS pilot in the International Schools***

An international pilot using computer algebra systems enabled (CASE) calculators in the International Baccalaureate Organization's Diploma Programme Mathematics higher level course.

The history of the use of GDC calculators in the UK is not impressive. International schools, predominately Eurocentric in pedagogy, have made some limited use of GDC calculators but have not moved to CASE based teaching or assessment. The IB introduced the GDC in the mid nineties and the new mathematics curriculum, starting 2004, fully supports GDCs in course guides and with teacher support material.

As part of the commitment to evolving and introducing appropriate technology, the IB has committed to a pilot program that is slated for start September 2004. The pilot will follow the mainstream mathematics higher level course, but allow CASE technology in all assessment. The pilot course will explore the feasibility of supporting and assessing courses that include CASE technology on all examinations in an international setting.

This presentation will explore the issues in creating / adjusting an assessment model in the international setting using a pilot program within the setting of the Diploma Programme at the IB. The IB grading system is criterion-referenced. This means that each student's performance is measured against well-defined levels of achievement. Top grades reflect knowledge and skills relative to set standards applied equally to all schools. The question is "What impact does CASE have on such an assessment system?" "What lessons have already been learned from exposing the program to CASE thinking in the development of the pilot?"

[corbeil.pdf](#)

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**Panel: Roger Brown** (edxrb@bath.ac.uk)

## ***Panel Discussion: Boon or Bust: What are the implications of calculator applications?***

Many systems and boards are meeting the challenge of incorporating CAS enabled devices in mathematics programs. But what about other subject areas? Is it the CAS feature that will ultimately decide the matter? Perhaps not. The recent addition of the TI84 has brought to the forefront a new issue to the whole calculator game: applications. The new 84s apparently will come preloaded with a number of APPS that are currently available for the 83. These APPS may challenge / break system / examination board rules for subjects other than mathematics. For example, periodic includes a wealth

of information for chemistry and certainly a policy needs to be formulated on this issue. This could be an assessment boards night mare.

- 1) Is it reasonable to expect teachers to deal with advanced memory management issues? Does this mean that an exam board must ban the 84? But, is the 83 so different?
- 2) Is a technology free portion of an exam a reasonable practice?
- 3) Is an fully active exam or "Open Book" approach viable for other subjects?

More at issue, what is the possible issue of APPS in future examinations, including mathematics. This may be more than an "other subject" issue. Do we want students to be able to have access to a worked solution?

Some practices and problems of an international exam board will be discussed and participation from other boards is encouraged.

[corbeilbrown.pdf](#)

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### ***Promenades, bombardements aléatoires et distribution d'échantillonnage avec Cabri 2 Plus***

#### ***Random Walks, Random Shots and Distribution of Samples with Cabri 2 Plus***

On montrera les différentes applications possibles du générateur aléatoire de Cabri 2 Plus pour illustrer des problèmes statistiques :

- 1) Simulations de promenades aléatoires : exemples sur une droite, dans le plan et dans l'espace.
- 2) Bombardements aléatoires : exemple d'estimations d'aires dans le plans et de volumes dans l'espace.
- 3) Courbes de fonctions aléatoires pour illustrer des distributions d'échantillonnages et modéliser leur fluctuation de manière dynamique.

[dahanrand.pdf](#)

**Paper is in English.** All Cabri- and Excel files are in folder \time\_contribs\dahan\random\_files.

Trial version of Cabri 2 Plus can be downloaded from the Cabrilog-website

<http://www.cabri.com/web/nsite/html/logiciels.html>

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## ***Visualiser les solutions d'équations différentielles et de systèmes différentiels d'ordre 1 avec Cabri 2 Plus***

### ***Visualisation of Solutions of Differential Equations and Systems with Cabri 2 Plus***

Une introduction traitera d'abord de la génération des courbes des primitives d'une fonction donnée formellement avec le logiciel de géométrie dynamique Cabri 2 Plus. On montrera comment le fichier réalisé pourra être utilisé comme un imagiciel pour ce problème dans la mesure où la formule de la fonction pourra être modifiée en direct sur la page de géométrie dynamique.

La partie principale montrera comment l'idée de base (méthode d'Euler adaptée) pourra permettre de générer les courbes solutions de toute équation différentielles  $y' = f(x;y)$  (à condition que  $f$  soit éditables par les opérations formelles usuelles). On montrera aussi bien des champs de solutions que des balayages du plan pour générer les solutions avec conditions initiales modifiables à l'envie.

Dans cette même partie, on utilisera une technique analogue pour les visualisations des solutions de système du type  $x' = f(x;y)$  et  $y' = g(x;y)$ .

Les explorations porteront tant sur les équations classiques que sur des exemples originaux qu'on créera à la demande.

On essaiera de conclure sur une possibilité nouvelle d'aborder l'enseignement des équations différentielles mais aussi sur une nouvelle manière plus expérimentale d'aborder ces problèmes aussi bien par les enseignants que les étudiants

[dahandes.pdf](#)

**Paper is in English.** All Cabri files are in folder \time\_contribs\dahan\des\_files.

Trial version of Cabri 2 Plus can be downloaded from the Cabrilog-website

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## ***Three-fold activities for discovering conceptual connections within the cognitive neighborhood of a mathematical topic***

New technologies provide an efficient tool for the discovery of a broader mathematical landscape than what is afforded through purely traditional ways of learning. Compound activities, including numerous techniques, should be developed by educators, in order to enhance the epistemic value of the learning process and enlarge the student's knowledge of the internal connections within the cognitive neighborhood of learned topics.

We propose a canvas for multi-task activities, involving hand-work, CAS assisted computations and related websurfing.

The talk will be illustrated by the following example:

A definite integral is given, depending on an integer parameter. This integral can be studied in two ways:

1. Paper-and-pencil work leads to an induction formula, and further, to a closed form for the parametric integral, involving factorials.
2. CAS assisted work does not lead immediately to such a closed form, but shows the first terms of integer sequences.
3. A suitable web-search provides an interpretation of the obtained sequence in terms of Catalan numbers.

[dana.pdf](#)

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### *A Course of ODE with a CAS*

In this lecture we present a proposal for a basic course of Ordinary Differential Equations (ODE). In that proposal, using a Computer Algebra System (CAS), we try to show, in a general and integrated way, exact and approximate methods to solve ODE. The main goals of the course will be the following:

- To know basic terminology.
- To use a few exact methods to solve specific ODE, mainly linear ODE.
- To obtain information: direction field, qualitative analysis, isoclines, asymptotic behaviour, etc., only using the ODE.
- To be able to implement some numerical methods and to interpret the obtained results.

The systematic utilization of all possibilities of a CAS (the chosen CAS used Maple, Mathematica, etc. is not important) allows a deep change in the contents and in the methodology regarding traditional teaching where different courses were devoted for exact methods, including in such courses a wide number of methods to solve ODE, and numerical methods.

In this course we include new contents which is not possible to handle with a traditional way of teaching. In order to enhance our thesis, we will propose, in the lecture, several examples concerning numerical integration, Picard method, some deduction of algorithms, etc.

[delavilla.pdf](#)

Accompanying files in folder \time\_contribs\DeLaVilla:

Maple worksheets:      [ode1.mws](#)      [ode2.mws](#)      [ode3.mws](#)      [ode6.mws](#)  
Mathematica notebooks:      [ode4.nb](#)      [ode5.nb](#)

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## ***Adding an Interactive Component to Computer Algebra in Differential Equations***

The need to add an interactive/dynamic component to computer algebra systems such as Maple, Derive or Mathematica is just beginning to be addressed. For example, Maple now has an Interactive ODE Analyzer, which provides a minimal graphical user interface (GUI) for working with differential equations. I will demonstrate some (platform independent) interactive programs that I have developed for investigating first and second order differential equations. With these programs one can dynamically change parameters, initial conditions, window settings, step size, etc., and see the results immediately in multiple views (phase or time plots). The results can then be imported into Maple for further refinement there, and for report writing. Algebraic results from Maple can also be exported back for further interactive graphical manipulation. Thus, for example, the exact solution to an ODE that has one or more parameters can be graphed and manipulated, and compared to numerical approximations. In addition I will demonstrate how you can create your own customized GUI using tools that I have developed. I will conclude the talk with some projects that I have used in differential equations courses that demonstrate how students benefit from the synergy created by combining dynamic software and computer algebra.

[decker.pdf](#)

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## ***Représentations des nombres réels et représentations graphiques des fonctions***

### ***Real Numbers Representations and Charts***

Une difficulté souvent rencontrée par les enseignants et qui restent inexplicable pour la plupart tient dans la façon dont les calculatrices traitent la représentation graphique des fonctions puissances. Pourquoi par exemple alors que mathématiquement on définit " $x^\pi$ " uniquement pour des réels positifs, les calculatrices tracent-elles un arc pour  $x$  négatif ? Pourquoi cette remarque vraie pour " $x^{\sqrt{2}}$ " mais pas pour " $x^{\sqrt{5}}$ " ? Pourquoi dans le cas de " $x^x$ ", la fenêtre graphique intervient-elle sur le résultat du tracé ? C'est dans la représentation des nombres sur la calculatrice que l'on trouvera des pistes de réponses ? Comment cette représentation intervient-elle dans un logiciel "à précision variable" comme Derive ? L'ensemble de ces questions et des quelques réponses que l'on peut leur apporter permettent aux enseignants de lever des doutes sur les outils qu'ils utilisent, mais également apportent une interrogation mathématique pertinente qu'il faut partager avec les élèves.

[eggerreal.pdf](#)

pdf-file of paper version

[eggerpresengl.pdf](#)

pdf-file of English PowerPoint presentation

[eggerpresfr.pdf](#)

pdf-file of French PowerPoint presentation

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## ***Divers aspects du calcul approché d'intégrales***

### ***Approximation of Integrals***

Les diverses méthodes d'approximation d'intégrales sont en France dans les programmes des premières années d'enseignement supérieur. Elles sont souvent présentées d'une façon très théorique, en particulier en ce qui concerne leur précision. L'utilisation du logiciel Derive, en particulier avec la version 6.0, permet une démarche plus dynamique dans l'examen de la convergence. Elle permet de surcroît une approche expérimentale de la précision de chaque méthode. La séquence pédagogique se décompose selon diverses modalités : dans un premier temps les étudiants sont "spectateurs" : il s'agit pour l'enseignant de montrer graphiquement chacune des méthodes d'approximation, en utilisant les possibilités dynamiques de Derive 6 (approximation d'une fonction par un polynôme avec la formule de Taylor, méthodes des rectangles, des trapèzes, du point-milieu, de Simpson). Les étudiants sont ensuite acteurs : au travers d'une activité en salle informatique, ils vont pouvoir examiner sur des exemples la précision de chaque méthode et ainsi préparer les résultats théoriques qui leur seront démontrés en cours.

[eggerintengl.pdf](#)

(English Version)

[intengl.dfw](#)

Derive 6

[eggerintfr.pdf](#)

(French Version)

[intfr.dfw](#)

Derive 6

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## ***Derive, Neural Networks and the discovery of rules in data***

This presentation describes how Derive was used to extract rules from trained Neural Networks that successfully classify data. A brief overview of Neural Networks and the role of rule extraction will be followed by a description of the datasets that the rule extraction method was applied to; including data that classifies breast cancer tumours into benign or malignant. In particular the presentation will deal with the tools and programming strategies that needed to be developed in Derive so that the rule extraction algorithm and validation of the rules could be efficiently implemented. In addition, the presentation will show how the 2 and 3 D plotting facilities of Derive can be used to visualise the rules extracted from data via a neural network.

[etchells.pdf](#)



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### ***Mathematics and the Web: Lessons Learned***

Our presentation will describe how we integrated the use of the Internet into traditional mathematics courses at Manhattan College. Our goal was to create a pedagogically sound experience for the students. We have spent years integrating Maple into the mathematics curriculum at Manhattan College. Our experience proved to be very valuable when we started to integrate the web into our courses. We tried not to make the same mistakes twice. The lessons that we learned when we introduced Maple into the classroom were easily extrapolated to help us avoid similar pitfalls when the web was introduced. In our presentation, we will give specific examples of how the use of technology has enriched our mathematics courses. Perhaps more importantly we will provide specific examples of what did not work at all. Our presentation will include examples using the computer algebra system Maple and the course management system Blackboard.

[farley.pdf](#)

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### ***Modeling with Sketchpad in the Teaching of Mathematics***

Mathematical modelling as the process of finding and validating a mathematical model that explains a phenomenon you want to study with the main purpose of understand it, and apply this understanding in a more general way, is an excellent vehicle for the teaching of mathematics from Upper Middle School (pupils between 15 and 17 years old) to College since it gives meaning to the mathematical content we are teaching, covering the gap between the mathematical knowledge acquired by our students in non school environments and the formal knowledge we want to teach in school. Mathematical modelling also awakes the student interest on branches of human knowledge other than mathematics. Now, the use of Dynamical Geometry software like The Geometer's Sketchpad (V4.0) puts an additional attraction in the students perspective, an attraction that you can not achieve with others computerized media.

The main objective of this lecture is to reflect on the pros and cons of this approach through the development of a modelling activity, and to take into account the potentialities of DG software in the teaching of mathematics while commenting some results the author has gotten with his own students.

[flores.pdf](#)

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## ***Random\_Distributions.Mth: Random Samples from Distributions with Derive***

In this paper the file RANDOM\_DISTRIBUTIONS.mth is presented. This file has been developed for generating random values from main continuous and discrete distributions.

The macros contained in the file can be grouped within the following blocks:

- Random values from uniform distribution: the macro RANDOM\_UNIFORM returns an uniform random deviate between 0 and 1. This macro is the base of the other generations. Different algorithms have been used to develop this macro in order to improve the DERIVE RANDOM function.
- Random values from discrete distributions: some generic algorithms for any discrete distribution have been implemented as well as specific algorithms for some discrete distributions (Uniform, Poisson, Binomial, Geometric, Negative Binomial, ...).
- Random values from continuous distributions: specific algorithms for main continuous distributions have been implemented (Uniform, Exponential, Normal, Lognormal, Cauchy, Chi-square, Student's t, F, Gamma, Beta, ...).

The use of this utility file is useful for simulating any process which follows a specific distribution. Moreover, this file can improve the teaching and learning process in Computational Statistic subjects.

[galan.pdf](#)

[galanpres.ppt](#)

[presentation.dfw](#)

[random\\_distributions.dfw](#)

[random\\_distributions.mth](#)

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## ***Animated lessons with TI-Interactive***

TI-Interactive is an ideal software to teach and learn mathematics.

Some of its characteristics as the automatic updating of documents and the existence of sliders allowing to change the values of parameters, makes TI-Interactive an excellent tool to illustrate the different situations that eventually occur in a given problem. Moreover, as TI-Interactive is simple to use even for a secondary school student, worksheets can be passed on to the students, completed by them then passed on to the teacher again, all this, in the same environment. These worksheets may contain various links to other documents (partial solutions, hints, additional informations, ...) that the

students are free to consult according to their own needs. This gives them the opportunity to work in a relatively autonomous way.

In our presentation, we will illustrate the various characteristics of TI-Interactive indicated above on the following two subjects that have been experimented in secondary school classes (16 to 18 years old students).

The tale of the horse and the cheetah : taking as starting point the equations relative to rectilinear movements with constant velocity or with constant acceleration, the problem consists in simulating a situation in which a cheetah is pursuing a horse and in determining algebraically and graphically the issue of the pursuit.

Can you imitate the flight of a bird ?: in this work, the students are asked to simulate the flight of a bird using appropriate parametrical functions.

### ***Leçons animées avec TI-Interactive***

TI-Interactive est un logiciel idéal pour l'enseignement et l'apprentissage des mathématiques.

Certaines de ses caractéristiques comme la mise à jour automatique de documents et l'existence de glissières ou ascenseurs pour faire varier des paramètres, font de ce logiciel un excellent outil permettant d'illustrer de manière dynamique des résolutions de problèmes ou certains chapitres du cours de mathématique. De plus, comme TI-Interactive est relativement facile d'emploi pour les élèves du secondaire, il est possible de proposer des documents de travail qui seront complétés par les élèves à l'école ou à domicile, remis au professeur et corrigés par lui, tout cela dans un seul et même environnement. Ces documents de travail peuvent contenir des liens vers d'autres documents (informations utiles, solutions partielles, problèmes résolus analogues au problème posé, ...) que les élèves consultent selon leurs besoins, ce qui les amène à travailler de manière autonome.

Dans notre exposé nous illustrerons les différentes caractéristiques du logiciel citées plus haut sur deux sujets qui ont été expérimentés dans des classes du secondaire (élèves de 16 à 18 ans) : LA FABLE DU CHEVAL ET DU GUEPARD : partant des équations du mouvement rectiligne uniforme ou rectiligne uniformément accéléré, le problème consiste en une simulation de la poursuite d'un cheval par un guépard et en la détermination algébrique et graphique de l'issue de cette poursuite. SAVEZ-VOUS IMITER LE VOL D'UN OISEAU ? : dans ce travail, il est demandé aux élèves de simuler le vol d'un oiseau en trouvant les fonctions paramétrées appropriées.

[gossez.pdf](#)

[TI-I-comments.pdf](#)

The TI-InterActive-files are in folders

\time\_contribs\gossez\bird and \time\_contribs\gossez\horse and cheetah

**Trial version of TI-InterActive 1.2 is in folder \time\_soft\interactive**

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Students (Electrical Engineering, ÉTS), Carignan  
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### ***Using TI symbolic calculator, Derive and DPGraph to ease comprehension***

We are both students in electrical engineering at ETS since 2 years. We had the opportunity to use various tools such as Texas Instruments symbolic calculator, Derive CAS and DPGraph in order to improve the comprehension of some mathematical concepts. During this presentation, we will show you our interest in using these tools from a student's point of view.

We will show the usefulness of these tools to solve concrete problems. Starting with the TI-92+, we will do a simple simulation for better understanding, then a graphical resolution of the problem. Moreover, this handheld calculator enables us to use mathematical software features without having to use a computer. Then we will present an example in which we use Derive as well as DPGraph. With Derive, we will show the clear organization and development of the problem, while DPGraph offers a good rendering of three-dimensional solids.

[gouinguilb.pdf](#)

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### ***AP Calculus using -Calculus Made Easy- and Learning Cryptography interactively***

2 presentations in 25 minute lecture:

1) Teaching AP Calculus using the TI-89 software "Calculus Made Easy" : some ideas, suggestions from past experience.

[http://www.islands.vi/~shawn/ti\\_89.htm](http://www.islands.vi/~shawn/ti_89.htm)

2) Introduction of the "Interactive Cryptography Tutorial" that allows learners to interactively encrypt, decrypt and break ciphers. By doing so students learn conventional and modern public key systems.

<http://www.antilles.k12.vi.us/math/cryptotut/home.htm>

[hahnfeld.pdf](#)

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## ***Computer-Based Mathematics Assessment of Engineering Students***

HELM (Helping Engineers Learn Mathematics) is a major three-year curriculum development project undertaken by a consortium of five UK universities, sponsored by UK Government funding of £250,000 (about \$550,000 Cdn). This paper describes how HELM uses computer technology to enhance the teaching and learning of mathematics to engineering undergraduates.

The HELM learning resources are briefly described. These consist of Workbooks, Computer-Aided Learning (CAL) courseware and Computer-Aided Assessments (CAA). The 50 Workbooks cover the mathematics in the first two years of UK engineering degrees. The CAL courseware, consisting of on-line interactive lessons to aid understanding, is web-delivered and complements many of the Workbooks. Essential to the success of the project is an extensive CAA regime; this takes two forms, either an integrated web-delivered version or an alternative stand-alone CD based version. The CAA regime facilitates the regular testing of large numbers of students. It incorporates both formative and summative aspects and thus powerfully encourages students to engage more in their own learning.

Finally, didactic issues raised by using CAA to drive mathematics learning are discussed by reference to trialling experiences at a number of UK universities, which have used the materials in a variety of ways.

[harrison.pdf](#)

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## ***Experiences with the obligatory use of graphic calculators like TI-83 in Saxonia***

Saxonia is a little country in Germany and play an important role in Germany in the question of integrating technology into mathematics education.

It is obligatory to use graphing calculators like TI-83 in all schools from 8class on since 1996. The lecture will show clearly some experiences in connection with the use of graphic calculators, for example consequences for the curriculum, the teaching methods, the culture of mathematical exercises and the valuation criterias in the central examination.

The lesson also will make curious about the next place of ACDCA summer-academy and Derive-Congress in 2006: Dresden, capital of Saxonia.

[heinrich.pdf](#)

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## ***Réflexions sur les potentialités des logiciels et des calculatrices symboliques pour l'enseignement***

L'utilisation croissante de calculatrices symboliques et de logiciels impose une réflexion sur leur influence dans l'enseignement et dans l'apprentissage des mathématiques. À partir de quel point de vue devons-nous analyser les potentialités à priori des calculatrices symboliques et des logiciels destinés à l'enseignement des mathématiques ? À partir de celui de la construction des concepts ? De la résolution de problèmes ? De ce qu'enseignent et évaluent les enseignant(e)s ? Ou de ce qu'apprennent les étudiant(e)s ?

Dans le présent document nous voulons faire une réflexion sur le rôle des représentations dans le développement des mathématiques, dans l'apprentissage des mathématiques et dans l'enseignement des mathématiques. Pour quoi les mathématiques sont-elles devenues une science anti-illustrative ? Est-ce qu'on a besoin de l'utilisation des représentations visuelles pour comprendre les mathématiques ? L'utilisation de différentes représentations des objets mathématiques est-elle importante pour l'enseignement ? Nous allons essayer de discuter de cette problématique en faisant un lien avec les outils technologiques. Mais nous pouvons avancer que la visualisation mathématique a été présente à l'origine même des mathématiques. L'utilisation de figures et de l'intuition ont joué un rôle prépondérant. Nous retrouvons ceci, par exemple, dans la mathématique grecque.

[hitt.pdf](http://hitt.pdf)

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## ***Rule dialogue in problem solving environment T-Algebra.***

T-Algebra is an environment for solving algebraic problems enabling step-by-step problem solution in four fields of mathematics: calculation of the values of numerical expressions; operations with fractions; solution of linear equations, inequalities and linear equation systems; simplification and factorisation of polynomials. Work in the T-Algebra environment proceeds by the steps described in the algorithms taught at school.

Each solution step in T-Algebra consists of three stages: selection of the rule; determination of the object(s) of the rule; entering of the result of the application of the rule. The last stage may occur in three different modes: direct input of the result; structured input of the result – entering in the fixed template; construction of the result from the components. The scheme above requires the students to notify to the program how they want to modify the expression, which provides the possibility to check.

T-Algebra enables the checking of both the knowledge of solution algorithms and the skills of applying different rules. The system diagnoses the student's errors and offers help at each stage of the step. Checks are performed on the correspondence of the selected rule to the algorithm; the suitability

of the marked elements to the selected rule; the understanding of how a new expression is built; and the finding of the components of the new expression.

The report introduces in more detail the rules obtained and examines some of the characteristic rules stage-by-stage. Examples are given of problem solution in different modes to demonstrate the program's capabilities to identify the mistakes made and provide advice to correct them.

[issakova.pdf](#)

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### ***Using CAS to Develop Precalculus Concepts in a US Curriculum***

Computer Algebra Systems can be used as a pedagogical tool to help high school students learn mathematical concepts. With a CAS, equivalence transformations help to solve  $n$ th root and exponential equations as well as to derive inverse functions. Making conjectures from patterns produced on a CAS help in investigating the binomial theorem, exploring the division and remainder theorems, and factoring sums and differences of powers. CAS contributes to the use of multiple representations (tables, graphs, and the symbol manipulator) to provide various ways to consider the definition of even and odd functions. Four years of anecdotal evidence in a US high school suggests that appropriate use of a CAS enhances students' understanding and performance.

[jakucyn.pdf](#)

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### ***New Linear Algebra features in Derive 6***

Derive has supported matrices and vectors for some time, but in Derive 6 several new features have been added. Matrix factoring is one important example. I shall describe the new factoring options and show how they can be used.

The factoring offered includes Turing (LU) factoring and Gram-Schmidt (QR) factoring. Turing factoring is a generalization of LU factoring that is especially developed for computer algebra systems. With a system such as Derive, teachers and students are keen to try problems that are more interesting than just the row-reduction of matrices with numerical entries. Unfortunately, when a symbolic entry is included in a matrix, the traditional methods of row reduction can break down and give misleading results. This is where Turing factoring comes in. The structure of the factors will be described and the way they can be used.

[jeffrey.pdf](#)

[gramschmidt.dfw](#)

[LS.dfw](#)

[pivot.dfw](#)

[rref.dfw](#)

[turing.dfw](#)

[visible.dfw](#)



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### ***Testing the Accuracy of DERIVE's "RK" Routine***

A computer laboratory component has become part of many beginning courses on ordinary differential equations. Here students are asked to numerically solve initial value problems, but little attention is given to the accuracy of such numerical solutions. Indeed, many text books only pay lip service to this important part of numerical analyses, possibly because the authors believe that the topic is too advanced for beginners. In this article we offer a simple DERIVE procedure that provides a measure of the accuracy of the Runge-Kutta order 4 DERIVE routine "RK". The procedure has a nice spin off too: it often provides a superior numerical result from the algorithm.

[joubert.pdf](#)

[Accuracy\\_RK\\_routine](#)

Scientific Workplace file

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### ***Problems Corrected by Computer and Available on the Net***

The project consists in the elaboration of a series of problems corrected by computer and available on the Net. For the moment, examples come from linear algebra.

A learner (not necessarily a student) will be able to solve problems with approaches that aim a clear solution. It consists in defining in details the different steps by subdividing a problem in smaller and simpler ones in order to guide gradually the learner towards the solution. The problems are instantly corrected by computer.

Because the answers are written in Maple language, the learner is first introduced to Maple (it could be another language) and gains the basic knowledge of Maple through exercises.

The software for an automatic correction requires a 'random' generation of data and a dynamical data base allowing the comparison with the answers of the learner and computer.

[joyal.pdf](#)

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### ***Preparing for the “Full Monty”***

In the UK, and probably elsewhere, we are experiencing an increased diversity in the mathematical capabilities of students entering our universities. For example we encounter students who:

- have a minimal mathematical background
- are adult returners and have not studied Mathematics for some time
- have managed to scrape through previous Mathematics courses and lack confidence
- are forced (usually reluctantly) to take Mathematics to support a degree in Engineering or Physical Science
- are reasonably competent in Mathematics but have some gaps
- need to brush up on topics met previously

In addition to these students, we have:

- undergraduates who need extra support whilst taking taught courses in Mathematics – an increasingly large population
- students who miss classes through illness etc.

Some Institutions attempt to address this issue by providing Foundation or Bridging Courses, Mathematics Clinics or other form of remedial help. This type of provision needs additional resources and therefore cannot always be implemented.

At Anglia Polytechnic University (APU), the author has begun a project to address the mathematical needs of the students described above. The aim of the project is to offer students an on-line learning menu for topics that have been identified as “troublesome” such as algebraic simplification or rearranging formulae. Currently the menu consists of:

- Diagnostic quiz – to establish the extent of knowledge
- Notes - such as those available for lectures and can be printed out or copied
- Power Point Presentation – including audio features
- Video based tutorial – this could be incorporated into Power Point
- Assessment and feedback

In other words, the student is offered the “full Monty” of learning experiences for each topic and, beneath this, for each contributing sub-topic. The student can thus choose the learning option to suit their needs. This is all provided within a WebCT environment and can be developed to provide discussion forums and other WebCT facilities. The author will endeavour to demonstrate some of this work which is still in the preparation stage, hence the title.

[kempski.pdf](#)

[kempski.ppt](#)

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### ***Finding the middle path between exact mathematical language and math-jargon of technology***

When we use technology such as graphical calculators, computer algebra systems, or dynamic geometry systems for teaching and learning mathematics, we inevitably introduce this technology jargon in the mathematics classroom. How to find an acceptable middle course between the exact mathematical language and jargon of (math-)technology is one of the present-day problems of mathematics teaching. We will discuss it in this presentation.

[kokol.pdf](#)

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### ***Derive 6 - A Computer Program for Teaching and Learning Mathematics***

Derive is a very mature computer algebra system for PCs, its roots go back to the late Seventies. It is the first computer algebra system which was widely used as a tool for teaching and learning mathematics. Many ministries and school authorities throughout Europe have adopted Derive as the primary computer tool for mathematics education.

Derive 6 is the newest version with a lot of features which support teachers and students in a mathematics class. We give an overview of Derive 6 with emphasis on these more pedagogical product features. We demonstrate these features with appropriate classroom examples from lower and upper secondary school mathematics and put them into a general didactical framework.

[kutzler.pdf](#)

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### ***Preservice Mathematics Teachers' Beliefs about Teaching with Technology***

This lecture will report on the findings of a study that investigated preservice secondary mathematics teachers' (PSTs) beliefs about teaching mathematics with technology (primarily computers and graphing calculators). First, PSTs' experiences with technology will be described in terms of their ownership of learning mathematics with technology. Experience, knowledge, and confidence were the primary factors that constituted ownership. Second, the primary dimensions of the PSTs' core beliefs with respect to technology, referred to as their beliefs about the nature of technology in the classroom,

will be discussed. These core beliefs consisted of beliefs about the availability of technology, the purposeful use of technology, and the importance of teacher knowledge of technology. Third, the lecture will focus on the roles the PSTs envisioned technology playing in their classrooms. Motivational roles of technology were nonmathematical in nature and were closely tied to the PSTs' beliefs that effective teachers motivated their students to learn and used a variety of teaching methods. Procedural roles involved using technology to execute calculations or procedures that could also be (and often were) done by hand. Conceptual roles facilitated the visualization and exploration of mathematics. The more PSTs wanted to focus on conceptual understanding and wanted students to take responsibility for that understanding, the more they were concerned about their own ability to facilitate such learning and the need for technology availability. The more PSTs focused on procedural understanding in mathematics and on teacher-centered lessons, the more they were concerned about students misusing technology and failing to learn the procedures. The lecture will conclude with a discussion of the implications of these findings for mathematics teacher education.

[leatham.pdf](#)

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### ***Are They Being Served? Using a CAS for Teaching a First Year Mathematics for Biologists Course***

Many times when we design a mathematics for biologists course for our client departments, Mathematics Departments select the mathematical topics and then search for applications to illustrate their usefulness. This stands the process on its head. We need to understand the content of the client discipline and the mathematical problems that it presents and then design our course.

This is especially true of Biology, whose use of mathematics and computing has undergone a great metamorphosis in recent years. Biologists are interested in growth, migration, evolution, and change. These are terms appropo of the derivative. They are interested in frequency, probabilities, commonality, and, best fit. These are tools from Probabilities & Statistics. More recently they are interested in sequencing DNA and Protein strings, and pattern matching. These are tools from finite mathematics as well as Computer Science.

In this presentation I will present some Derive tools for addressing problems from Biology. I will also discuss the mathematics and mathematical facts that the Biology students need to know in order to use their CAS in an intelligent way when solving problems and to interpret their results. I will look at the Hardy-Weinberg Equations from genetics and evolution. I will examine the role that the CAS can play in performing an analysis of the results. I will also look at a strategy for DNA sequence matching and determining a best match for a sequence.

[leinbach.pdf](#)

[HardyWeinberg.dfw](#)

[leinbpres.pdf](#)

[bioinf.dfw](#)

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<http://science.kennesaw.edu/~jlewin/2203/data/law->

## ***Using a Computer Screen as a Whiteboard while Recording the Lecture as a Sound Movie***

The purpose of this presentation is to demonstrate the process of using a laptop computer screen as a whiteboard, while, at the same time, recording all screen activity, and everything that is being said, as a sound movie. The result is a clear, complete, and reliable set of lecture notes that the students can pick up from the instructor's web site, and also a complete movie record of the actual proceedings of the classroom.

When viewing a movie, students have the option to pause it at any time while they collect their thoughts; or they can pull the cursor back and forth to view the specific parts of the movie that are most important to them. The movie recording of each lecture, and other course materials, are made available on a CD which is duplicated in a CD copier, and is ready to be picked up by the students very soon after the lecture ends.

Lecture material is written with Scientific Workplace or Scientific Notebook which combine the advantage of a dynamically produced clear and complete set of lecture notes with instant access to a computer algebra system. The movie is created, either with the recording utility in Camtasia Studio by TechSmith, or by ScreenCorder 4 by Matchware. Then the CD is mastered by the MenuMaker utility in Camtasia Studio and, within minutes, a complete, friendly self-contained CD has been mastered and can be carried to the instructor's CD copy machine.

The presentation will demonstrate how these software products can be used to produce the results that have been described here, and will discuss the advantages of giving students the option of duplicating the actual lecture room experience rather than merely reading a set of lecture notes.

The URL below gives an example of a sound movie on an elementary mathematical topic. The download is about 44 megabytes. Press alt + enter to show or hide the cursor while the movie is playing.

[lewin.pdf](#)

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## ***Teaching geometrical Optics with Derive / Paraxial Approximation***

Tracing a ray of light through an optical system consists on calculating the changes in ray slope angle caused by reflection or refraction, and the changes in ray height on transition from one surface to the next.

The paraxial region of an optical system is a threadlike region about the optical axis which is so small that all the angles made by the rays (i.e., the slope angles and the angles of incidence and refraction) may be set equal to their sines and tangents. Under this approximation (first order optics) ray tracing formulae are very simple.

It will be demonstrated how to apply these simple formulae to calculate the important optical quantities (focal length, back focal length, aperture stop, entrance and exit pupil, size and image position ect.) and even for complicated multi-surfaces optical systems. Special cases of thick and thin lenses are considered and also multilens optical systems, in particular cemented and separated doublets.

[magiera.pdf](#)

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### ***Weighted Mean Approximation for Integration***

The fundamental theorem of Calculus states that the definite integral over an interval is the difference of the anti-derivative at the two endpoints of the integration interval. However, we need to know the anti-derivative, but in too many interesting cases (for example in computing the length of curves) the anti-derivative does not exist, or is difficult to find.

In this paper we present a didactical sequence, which introduces a geometric approach for integration using the notion of Weighted Mean Approximation for computing the definite integral of any function (this approximation is exact for polynomials up to the third degree). This WMA depends only on three values of the integrand (no anti-derivative is needed). Refining the approximation opens the road to Simpson's rule for computing numerically definite integrals.

[mann.pdf](#)

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### ***The great revelation***

Student enrolled in the Nature Science Program at the collegial level in Quebec are initiated to calculus and higher-level mathematics through two courses. These two courses, named "Differential Calculus and Integral Calculus" are 75 hours each and cover a large spectrum of subjects. Some students will also go through a third course named "Advanced Calculus" for a total of 225 hours. These three courses being introductory, a large spectrum of interesting questions is never addressed and powerful mathematical tools are left for future discoveries.

Ten years ago, I created an advanced physics courses directed mainly to the students who intend to pursue studies in engineering at the university level. The main objective of this course is to adequately prepare students for their first year in a degree in applied sciences. Combining subjects from physics and mathematics, this course makes a large use of Maple for producing graphs and accelerating certain mathematical developments.

For certain problems, the software offers solutions making use of complex numbers or esoteric functions. In others cases, solutions can only be found through the use of numerical analysis. Now, all of these mathematical tools are not part of the regular curriculum for these pre-university students. Instead of avoiding them, I have chosen to use these situations as an occasion for smoothly introducing what's to come in their training as a future scientist.

[marcheterre.pdf](#)

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### ***Learning College Calculus in a CAS Environment: Theory and Practice***

This study is a qualitative case study focusing on the question “What are the processes of learning in a Computer Algebra System (CAS) environment for college students learning calculus?” The study is designed to research the impact on student learning of new and revolutionary software available for mathematics education. There is a gap in the research on CAS in the area of investigations into the process of student learning and their development of concepts while using CAS. The importance of investigating these processes lies in understanding the impact of CAS on learning so that an appropriate balance can be found in the integration of technology into teaching ensuring that the merits of traditional methods are not lost and that the merits and demerits of CAS use are fully understood. The primary data for the study consists of audio and video of tape of students in a college calculus course learning calculus using CAS software. These data are supplemented by interviews with the students and instructors as well as analysis of the students’ homework and tests. The study provides a detailed description of the process of learning mathematics with the use of CAS as well as the emergence of conceptual development arising from collaboration among the students in the collective interaction with the software. The major findings of the research are that Adapted Rotman Model of Mathematical Reasoning is an accurate model for understanding the place of technology in the learning of mathematics; and that the Pirie-Kieren Model of Mathematical Understanding can be fruitfully applied to learning in a CAS environment. Among the particular gains afforded by the technology are increased experimentation by students in mathematical activity and increased focus on concepts rather than technical competence. Among the obstacles presented by the technology are difficulties in understanding computer representations of mathematical objects.

[meagher.pdf](#)



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## ***Using Derive to Explore the Mathematics of the JPEG Image Compression Algorithm***

Many photographic images displayed on Web pages and saved on CDs have been compressed via an algorithm developed by the Joint Photographic Experts Group (JPEG). The algorithm employs several mathematical techniques, but at its heart is the application of the Discrete Cosine Transform (DCT) -- a variant of the classic Fourier Transform -- to 8x8 blocks of pixels. While standard graphics programs can be used to compare the image degradation corresponding to different, vaguely defined levels of compression, Derive allows one to choose precise values for numerous algorithm parameters and to examine in detail what takes place at each step in the process. Insight into frequency-space representations of data can be gained by generating "test images" (8x8 matrices) with various properties to see how the crucial DCT expresses regularity and diversity. Preceding the DCT step are an obligatory transformation from red-green-blue color-coordinates to luminance-chrominance coordinates and an optional downsampling (loss of chrominance information). Following the DCT step is a quantization step, which allows the customization of the way information (especially "high frequency" data) is discarded. Even a "lossless" step may contribute round-off error; this can be investigated by running the step and its inverse with different numbers of digits of precision. Derive makes it possible to automate investigations by performing one or more steps with multiple choices for images or parameters via a single simplification command.

[michael.pdf](#)

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## ***Mathematical Reasoning and its formalization within a Dynamic World***

Recent curriculum reforms have pointed out the relevance of using technology in Mathematics Education. The NCTM (2000) identifies the use of technology as one of the key organizer principles of the curriculum. Working within a dynamic environment (Cabri, for instance) has been important to enhance students' capacity to explore geometric problems and to provide them with adequate tools for conjecturing and proving mathematical propositions. Computational representations allow the exploration of mathematical objects with new strategies, different from the strategies in a world of paper, as mathematical objects to be identified with the (executable) representation provided by the dynamic software. In consequence, mathematical thinking is enriched. Within the framework sketched here, we will present empirical work done with students (18-20 years olds) related to activities of exploring, conjecturing, and proving. Our results are part of an ongoing longitudinal study that has already led to several master and Ph.D theses and workshops in Mexico. To substantiate our assertions, we will present a genuine formal proof within Cabri World, of the Napoleon Theorem and

explore other related results. We understand we are studying the “ecology” of mathematical propositions and the ways to find the natural context of generalization. Working within a computational environment forces us to adopt a different strategy: we have to resort to the nature of the mediating tools we have at our disposal, for instance the internal mathematical universe residing in the computer/calculator.

[moreno.pdf](#)

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### ***Representations and Graphic Calculator In Mathematical Teaching. A Study with Calculus Tutors***

A group of Mathematical Tutor's productions, who participated in a Formation Program, based on the use of Representation Systems and Graphic Calculator in the teaching and apprenticeship process of Differential Calculus, are analyzed. The mentioned Program was implemented through a course- workshop. The participants in the study were Mathematical Tutors from the basic cycle of the Economical and Social Sciences Faculty of the Carabobo University, La Morita Nucleus in Maracay, Venezuela. The participants productions, withdrawal from their note books and reflections emitted by the tutors, at the end of every working session, are analyzed from a qualitative perspective. These productions analysis, concluded that the tutors showed up changes in the Didactic Knowledge, expressed in their reflections and productions during the activities carried on. It likewise, manage the participants to apply the representation systems, systematically, in the resolution of problems with the graphic calculator assistance, and also identified options for the Differential Calculus Teaching.

Key Words: Graphic Calculators, Representation Systems, Mathematical Tutors, Differential Calculus, Didactic Knowledge.

[ortiz.pdf](#)

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### ***A mathematics and science domain e-learning platform IWT based***

The purpose of this paper is to introduce an innovative e-learning platform customized for mathematics and science domain. Our experience in e-learning field suggests us that in order to offer an high quality support for mathematics and science domain it is not enough offer Web based contents, but it is necessary porting of tools and methods of particular domain in new didactic context.

Starting point to realize a domain platform is IWT (Intelligent Web Teacher). IWT is an e-learning platform fully adaptable and extensible be in contents be in services.

The platform for mathematics and sciences supplies to the final user a flexible tool for teaching and learning in the field of the mathematics and sciences. To realize such feature we equipped IWT of useful tools for presentation and treatment of data of mathematician-scientific type. First of all we integrated and customized inside the platform power engine for numerical and symbolic calculation: WebMathematica and Simulink/Matlab, the software XpressMP for linear optimization and some mathematical software library: LEDA and NAG. Moreover all the communication channels of IWT between teacher and student (messages, chat, forum etc.) are extended in order to allow editing of formulas with MathML standards or Tex and the visualization of mathematical expressions in symbolic way.

[pagano.pdf](#)

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### ***Classical and computer methods in elementary geometry***

On the base of a personal experience in teacher training in mathematics at the Pedagogical Faculty, University of South Bohemia, Czech Republic a comparison of classical and computer methods in elementary geometry is given. Whereas classical methods show e.g. the beauty of geometry, enable better insight into the geometric situation and better understanding the problem, computer methods (or solving a problem with the help of a computer) make possible to carry out complex computations, dynamic presentation of geometric situations, automatic proving theorems of elementary geometry, automatic derivation and discovery of geometric formulas. On a few examples from geometry of a triangle and quadrangle advantages and disadvantages of the both methods are presented. It is shown that both classical and computer attitude is necessary in teaching geometry to solve geometric problems. The presentation will be accompanied by using dynamic software Cabri and CAS CoCoA.

[pech.pdf](#)

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### ***Pedagogical Uses for Symbolic Algebra in a Numerical Analysis Course***

It is very easy to think that a numerical analysis course will use a computer or calculator for only numerical computations. Certainly that is a major emphasis of the course. However there are many opportunities to make use of symbolic algebra in the derivation and analysis of numerical algorithms. I will use the Voyage 200 to demonstrate some of these topics. I tend to prefer the Voyage 200 for my in-class demonstrations because of how easy it is to carry this handheld device and the viewscreen into my classroom (which has no other easily available technology). Similar things can be done on any

CAS, and my students often go to a computer lab to complete assignments that involve symbolic mathematics.

Here is one example. I had a high school student doing some independent reading on polynomial interpolation. On a TI-89, he programmed the standard algorithms for computing the divided difference table and then for evaluating the polynomial in the Newton form. Normally the input for the evaluation routine is the floating-point number desired for  $x$  in the evaluation of  $p(x)$ . However when he ran the routine with a symbolic  $x$ , he got the symbolic form of the polynomial. There are many other symbolic tasks related to polynomial interpolation that can be explored at the same time you are presenting the numerical algorithms.

[pence.pdf](#)

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### ***The application of CAS in teaching calculus***

The presentation gives information on the experience with the usage of CAS, specifically in MAPLE, in teaching of mathematical analysis in teacher training in the Czech Republic. New software products and information technologies bring quite new problems, but also new challenges especially in didactics and teachers' training. The aim of the computer-aided instruction of mathematics is to develop the feeling of the students for estimation of the solution, their ability to resolve a non-standard task, to develop their functional thinking. It is necessary to focus preparation of the students on the fact that they should not search for the solution to the problems according to learnt algorithms, into which they have incorporated standardized solutions, but that they should orientate themselves in the problem, model, express their opinions and defend them.

[petraskova.pdf](#)

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### ***5 years of teaching mathematics to students with mandatory symbolic calculators : the good, the bad and the ugly!***

Since the fall of 1999, every students in our classrooms have the same symbolic calculators (TI-89, TI-92 Plus or Voyage 200). We have witnessed just about every possible negative reaction to having this technology available for all students. From ridiculising it, to seeking its weaknesses, to asking students not to use it, up to using it for everything even to solve a simple equation like  $x + 2 = 5$ . After almost 5 years, the dust has settled down.

First, on the good side, we will show examples, in various topics, of what students can now do in classrooms and how these calculators have changed the way we teach part of the math curriculum.

On the bad side, and this is true in general, the more we use technology, the more we tend to depend on it. Students now rely more on these calculators to do basic mathematics. They are less skillful in manual calculations. We will show examples of calculations which are best left to the calculators, examples our students would have some problems resolving manually; this is caused mainly by a lack of practice. We had to make room for the intelligent use of technology and cutting back on some long manual calculations was an easy way to do it.

Finally, the ugly side of this on-going experience. Good students tend to be even better but bad students tend to be even worse. At the low end of the scale, some students cannot do mathematics without a symbolic calculator and usually don't understand much of what the technology is doing for them.

We have seen, in the last few years, an evolution in course topics, in exam questions linked to the use of symbolic calculators. The major part of the curriculum remains the same but with an intelligent use of the technology, we can now change the way we view and explore, with the students, some aspects of mathematics.

[picard.pdf](#)

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### ***Creating Visualizations using Maple***

Many of the ideas that we teach in mathematics can be moored to a changing image. This is particularly true in calculus: rectangles under a curve become thinner and more region-filling as their number increases; a point travels along a curve, its velocity and acceleration vectors changing as it moves; the slope of the tangent line to a surface varies as a direction vector swings around a point on the surface; a horizontal plane descends through a surface, cutting it in varying level curves. These are images that we have in our mind's eye, and that we try to convey to our students. When we are successful, we give them something concrete to associate with a concept.

Computer animation is a very effective means to accomplish this, and Maple has that capability. I will show several ways to make animations using Maple, assuming that the audience has at least casual familiarity with a computer algebra system. In particular, I will show how to use the ANIMATE, ANIMATE3D, and SEQ commands as well as FOR loops to accomplish this, the choice of method depending on the complexity of the frames. These will be coupled with the DISPLAY command and its various options. Every example will be an animation that can be used directly as a classroom demonstration to help students visualize a concept.

John Putz: Everything I talked about at the conference is in my book, "Maple Animation", and is, therefore, copyright-protected by the publisher. For this reason, I did not write up my presentation as a paper.

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### ***Assessment using Technology: A Case Study in Computer Aided Drafting.***

We report on a case study project about assessing with technology in a small ( $n=12$ ) middle level mathematics class. The goal was to assess middle level students understanding of proportional reasoning, in a problem based approach, using MLCAD (CAD program for creating Lego™ structures.) The concept is first explored on a concrete level through the comparison of different scale building materials. To demonstrate their understanding of proportional reasoning, students are guided to: (1) create a design on a computer drafting program and give dimensions of the finished product in the three scales of building materials, (2) build a structure based on the design given to them by a peer. Project assessment consists of two parts, the student's computer aided design and predicted measurement correctness and their ability to accurately build a structure from a given design, or find possible flaw in that design. During this process the students are developing additional information technology abilities and skills, such as visualization, connections between two-dimensional representations and three-dimensional structures, computer-based modeling, and problem solving. This presentation reports on the challenges and implications encountered during the implementation of the project. Examples of student designs and structures will also be demonstrated.

[raylalagic.pdf](#)

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### ***Residue.Mth: Solving Problems of Integration Using the Residue Theorem***

In this paper we present the file RESIDUE.MTH, created to be used in subjects that deal with complex variable, aimed at Engineering students. Such file contains a series of macros which permit to solve integration problems using the residue theorem.

The macros contained in the file can be grouped within the following blocks:

- Compute of residues (of a given singularity, of the singularities of rational functions).
- Complex integrals using the residue theorem.
- Improper integrals of a rational function.
- Improper integrals of the form  $f(x)\sin(kx)$  or  $f(x)\cos(kx)$  where  $f(x)$  is a rational function.

We also show in the paper some examples of applications that have been carried out with our students of Telecommunication Engineering. The macros have been developed in order to be used as didactical

tools with explications of what the macros do step by step (using DERIVE as a PeCAS or as a white-box CAS).

Finally, we include the conclusions obtained after using this file with our students and also some future work on this subject.

[rodriguez.ppt](#)

[rodriguez.pdf](#)

[residues.dfw](#)

[residues.mth](#)

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### ***Significance of use of technology in mathematics in vocational education and some practical illustrations***

The role of mathematics in the field of vocational education is quite different from other levels of education. Students in vocational education are to a greater extent practically oriented therefore concrete objects are closer to their understanding. Classical mathematics very often does not make much sense to them, so they find it useless and hard to accept. Mathematics in vocational schools should be adjusted to the needs and possibilities of the students. It is very important to connect teaching of mathematics with student's vocation and their practical knowledge.

Due to a different role of mathematics also the use of technology in mathematic lessons of vocational education has a different role compared to other programs. In my presentation I will try to enlighten the above mentioned specialties of vocational education. Through a demonstration of a concrete case I will tackle the issue of choice of ICT and the proper way of using it in mathematics lessons in vocational schools.

[rojko.pdf](#)

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### ***Un échange de bons procédés entre maths et info***

J'ai pu vivre deux utilisations complètement distinctes de ces machines algébriques.

1- En enseignant l'info où j'ai souvent besoin d'objets ou de résultats des mathématiques qu'on n'enseigne pas au bac pour implémenter des objets informatiques pas vraiment simples. Je vous en donne deux exemples entre autres : le théorème chinois du reste -- utilisé pour introduire du bruit dans la composition de procédés en crypto -- et les extensions de corps finis en quotientant  $\mathbb{Z}_p[x]$  avec un polynôme primitif --pour construire des générateurs pseudo-aléatoire--. Dans le premier cas, je me sers de la TI en classe pour montrer comment inverser le théorème chinois. Dans le second, Maple me donne l'outil grâce auquel mes étudiants trouvent des polynômes primitifs nécessaires à une énumération du corps fini et je me permets de leur expliquer bien sommairement ces belles constructions algébriques. Évidemment, on ne retrouve pas d'appels aux logiciels Maple ou Derive



dans les programmes C-C++ --ce serait d'une lenteur inacceptable -- mais le support à la compréhension a priori que me donne ces machines est irremplaçable.

2-Lorsque j'enseigne les équations différentielles, je traite la machine de simple support -- essentiel vu la lourdeur de modèles cohérents et crédibles dans leurs solutions analytiques, itératives, en séries ou numériques --. Toutes les interventions que je consacre à la machine se font dans les travaux pratiques ; et ce que j'adore, c'est d'offrir une paramétrisation de fonctions possibles sans en fournir jamais le code... Les étudiants doivent saisir l'importance primordiale de cette paramétrisation -- le flot d'informations -- pour l'obtention des résultats voulus. Dieu sait que cette compréhension des flots d'information est essentielle à l'info.

Donc je pourrais donner une bien petite image de cette assistance mutuelle entre maths et info.

[saulnier.pdf](#)

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### ***The Constant Feature -- Spanning the Mathematics of Grades K - 12 and Beyond.***

The same simple but powerful idea can be used to teach important mathematics from counting through calculus. The common theme of recursion has applications to the curriculum including arithmetic, consumer mathematics, algebra, geometry, matrices, limits, numerical methods, and chaos theory. It can be approached beginning with four-function calculators on through hand-held and computer symbolic algebra systems. The presentation will expand on a recent article by the speaker in the NCTM's "Mathematics Teacher". In addition to a rich set of examples, the session provides an illustration of cohesion across the grades.

[schultz.pdf](#) (no link back because document is write protected)

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### ***Area Estimation and the TI-83: An Application for Economics***

The National Council of Teachers of Mathematics in its Standards (1989) suggested that all students, including the mathematically gifted, would benefit from a curriculum that used technology appropriately while developing in each student an appreciation and facility for estimation. Thus, mathematics teachers are challenged to find ways to apply the Standards in ways that will help stimulate student interest in mathematics and also prepare students for the demands of the Advanced Placement Calculus class.

Introducing students to the study of interesting "real-world" applications of mathematics is a tool that teachers can use to accomplish these goals. Asking students to find the area of a region bounded by two curves is one such "real-world" question that can require students to utilize technology in order to answer a "real-world" problem via estimation methods.

Specifically, students will be shown how to measure how equitably income is distributed in a population. This will be accomplished by introducing students to two concepts from the field of economics, the Lorenz Curve and the Gini coefficient. The TI-83/TI-83 plus calculator will be used to demonstrate the application.

[selitto.pdf](#)

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### ***DGS and CAS as tools supplementing each other in an inquiry task***

The presentation/paper deals with the project "Locus curves" which was carried out as a part of the first university level geometry course for mathematics teacher students. The students were given the following task as a 4x3-table (rows R1-R4 refer to the rows and C1-C3 to the columns): "What kind of locus does the point P draw IF the distances d1 and d2 are interpreted in one of the following ways: (C1) d1 and d2 are the Euclidean distances of the point P from two fixed points P1 and P2, (C2) d1 is the Euclidean distance of the point P from a fixed point P1 and d2 is the Euclidean distance of the same point P from the line L, (C3) d1 and d2 are the Euclidean distances of the point P from two fixed intersecting lines L1 and L2, AND IF the type of the constancy is one of the following ones: (R1) the sum of the distances is constant, (R2) the difference of the distances is constant, (R3) the product of the distances is constant or (R4) the ratio of the distances is constant. " Each group of two or three students was asked to solve two of the 12 different tasks from the table. The students were first asked to construct the curves geometrically as locus curves determined by the geometrical properties of the curves using the dynamic geometry programme GEONExT and then to present the solutions as applets locally or in net. In addition to this, the features of curves were asked to be examined analytically with the CAS-software QuickMath (<http://www.quickmath.com/>) which can be run freely through the net.

In the presentation I will report what curves the students with these tools were able to find from this rich family of curves (including well known cone sections, Cassinian curves, lemniscate of Bernoulli, circle of Apollonius etc.) and give some examples of how they performed the tasks. In the presentation I will also discuss how the different points of view offered by DGS- and CAS-approach supplement each other in solving these kinds of inquiry tasks.

[silfverberg.pdf](#)

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### ***Combining the possibilities of Derive and Excel while studying bases of computer science***

Our experience of teaching and learning of computer science at the Kharkov G.S.Skovoroda Pedagogical University shows that not a simple acquaintance with a package Derive, but also its wide use in practice, is the important constituent of a course "Bases of computer science" for the students of speciality "Mathematics" of physics-mathematics faculty. In the last decade, undoubtedly, there were various changes in the program of preparation of the teachers of mathematics, both deep, and insignificant. The acquaintance and use of the Derive package by the students were performed within the frameworks of the basic course "Bases of computer science", special computer science disciplines, and during passing the obligatory computing practice. A theoretical basis of a course was the textbooks and the methodical instructions prepared by the teachers of our faculty. As a result of the experimental work the course was developed, in which studying the Derive package and using Excel (with the elements of VBA programming) were combined. The course includes study of the following topics: the calculation of roots of algebraic functions; the calculation of integrals; and decision of the systems of linear algebraic equations and the tasks of linear programming; the mathematical modeling and new theories of learning. Theoretical material and the problems were submitted in an electronic textbook and were accessible to the students during performance by them of laboratory works. The peculiarity of the course is in technology: the students study to estimate an opportunity of the decision of the problem in the Derive package, and to carry out the verification of the received decisions numerically in Excel, with use VBA.

[stolyarevska.pdf](#)

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### ***TI-Based Learning Environments: Developing Conceptual Understandings of Functions-related Concept***

The study occurred in three graphing calculators based Precalculus classes. Two intertwined aspects were considered. The first aspect dealt with enabling students to build a stronger conceptual understanding of functions and related concepts and the second was to study what conditions provide for a successful learning environment utilizing graphing calculators. The key factors identified and applied were: long-term exposure to ill-structured problems; writing about the concepts; the teacher answering questions with appropriate questions/prompts to provide for scaffolding; cooperative learning; and the teacher's proficiency with graphing calculator. The students developed a deeper understanding of the concepts and they were more willing to attempt complex problems. Their communication skills improved. The study indicates that problem-based learning in a technology oriented environment provides appropriate conditions for developing critical thinking and

communication skills. This paper reports on advantages and challenges encountered in (a) “covering curriculum” and (b) building students’ confidence in this technology-based learning environment.

[thielalagic.pdf](#)

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### ***The Correctness, Completeness and Compactness Standards of Computer Algebra Systems and of School Mathematics***

In many cases when solving a school algebra problem (e.g. an equation) using a computer algebra system (e.g. Derive, Maple, Mathematica, MuPAD) we get the answer that is perfectly suitable for both the teacher and the student as well as others. Nevertheless, one may encounter answers having some qualities that are disturbing when used at school, such as the answer being valid on certain conditions only, solving is not brought to an end, the answer containing elements unknown at the specific school level, etc. The qualities can be represented as deficiencies in relation to correctness, completeness and compactness. Based on the smoothing of disturbing qualities, the answers offered by computer algebra systems may conditionally be divided as follows: applicable with the help of extra explanations provided to students; adaptable using the resources of the same computer algebra system; unsuitable. The paper also provides examples of smoothing possibilities. The problems (or rather answers) treated in this paper concern division by zero – from calculating  $1/0$  to literal equations and inequalities. An analysis of textbooks reveals that the standards vary. There are various conventions, e.g. assume that variables are restricted, check solutions in the end of solving process only, and such like. In this paper the different standards of computer algebra systems and of school mathematics have been compared.

[tonisson.pdf](#)

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### ***Using CAS in Traditional and Alternative Assessment Models***

The use of technology is changing how we teach and as a consequence how we assess. Although technology is rapidly becoming indispensable in enhancing understanding of concepts and analytical structures, the challenge remains in using technology to design meaningful tools to assess the new learning objectives in our changing curricula.

Since assessment should emerge from established curriculum objectives and classroom practice, assessment with technology should come as a natural by-product of the classroom experience. There are fears in the community of practitioners that the use of technology in assessment will either trivialize what is being assessed or significantly increase its level of difficulty. Both scenarios are unacceptable, particularly the latter as it places increased burden on student success especially for the

weak student. Our attempts at raising levels of understanding should not come at the price of creating yet more stressful examination situations for our students.

In this presentation examples will be shown of traditional and alternative assessment models incorporating the use of computer algebra systems in grades 6 through 12, including its use in the International Baccalaureate diploma program.

[torressk.pdf](#)

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### ***Why DO We Teach Theorems in Calculus?***

Why is it that we teach theorems to our students in Calculus? And how can technology, especially Derive and a web-based discussion tool, help this endeavor? The speaker aims to convince you that the study of key theorems is important to student development of reasoning, problem solving, logic and language skills. In this presentation, we will examine some theorems from College Algebra and Trigonometry and contrast them with the types and usage of theorems in Calculus. It is proposed that the student interaction with theorems in calculus undergoes significant development, from a first interface with the Intermediate Value Theorem, through the Fundamental Theorems, to more advanced notions such as applying the Mean Value Theorem to definite integrals or the interaction between orthogonality and dot product. The distinctions between “if  $p$  then  $q$ ” and its converse or contrapositive, and the full implications of an “if and only if” theorem can be developed gradually in the context of a calculus sequence, with the student making significant progress in just one semester of study. Course software, especially graphing utilities and computer algebra systems, can aid in exploration of the theorems, or be utilized as experimental tools to gather data in the development of a conjecture prior to proof. And a web-based discussion tool can be helpful in fomenting further discussion of related topics, such as interpretations of a recent theorem in context.

[townsley.pdf](#)

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### ***Some Security Issues of Public Key Cryptosystems using DERIVE***

The paper deals with the proper generation of primes for RSA and other cryptographic purposes. It turns out that there are a number of pitfalls which should be avoided. In particular, the randomness and the unpredictability of the generated primes is definitely a key issue when it comes to the proper implementation of a public key cryptosystem based on them. Furthermore, primes with certain special properties pose a security problem. It is shown how “safe” primes can be generated using *DERIVE*.

[wiesenbauer.pdf](#)

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### ***Can CAS improve the Mathematical Abilities of Pupils?***

In Belgian secondary schools the number of pupils is going down in the sections with eight mathematics lessons a week compared to those pupils in the sections with six mathematics lessons a week. To avoid a detrimental influence on engineering, mathematics, physics and IT education, a radical reform of the curriculum of the six-lesson sections should take place. These pupils aren't lacking in knowledge, but their understanding of mathematics, skills in problem solving and mathematical creativity need greater attention.

Using Computer Algebra Systems (CAS) can increase improvement in these three areas in the mathematics lessons. To prove this thesis with an experiment, two groups of pupils from six-lesson sections (an Intervention group and a Control group) will be compared.

In the first phase (September 2003) the mathematical skill level of these groups has been determined with respect to mathematical understanding, problem solving skills and mathematical creativity. Over a period of several weeks, the mathematics lessons of the Intervention group will be extensively supported with CAS, while the lessons of the Control group remain traditional (November 2003 – May 2004). In the last phase (May 2004) both groups will be tested again, and it is expected that the mathematical level of the Intervention group will increase more than the mathematical level of the Test group.

During the lecture, I would like to talk about this experiment and give the (first) results.

[Wonterghem.pdf](#)

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### ***New Models in Assessment in Computer integrated Mathematical Instruction - First Results of the Austrain CA-Projects***

Since 1985 more & more teachers in Austria have been in a position to allow students to use electronic calculators in tests, if they had enough practice in class. Teachers of Mathematics at schools which are preparing students for university can choose and formulate their own test problems. Consequently their examples were gradually adapted to the new demands of computer algebra systems (DERIVE, MATHCAD, MATHEMATICA). Assessment, however, was still based on the traditional rules of test writing. In the Austrian CAS II Project (1997/98) the students used TI-92 in Mathematics in their lessons and in their tests. Based on these experiences made by the research teachers new models of assessment have been worked out:

(1) short tests – problem solving tests, (2) project work, (3) cross curriculum tests, (4) written group tests.

I chose this following variant in form 11: a) Short tests, e.g. 25 minutes, to check reproductive skills or reproductive knowledge (possibly without CAS too).

b) One longer test per term, e. g. 100 minutes, to check problem solving skills.

c) Each student should work on a short chapter of mathematics and present it to the other students.

I am going to explain the four different models and to report about the different reactions of the students and of the research teachers.

[wurnig.pdf](#)

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<http://163.25.178.88/lii/basicgeometry/index.html>

**Chun-Yi Lee**

### ***Designing instructional tools by Flash MX ActionScript-some examples to teach basic geometric concepts***

As we all know, Flash has been a useful tool to create motion pictures. However, integrating these functions into mathematical learning was little. With its strong functions, Flash MX ActionScript could be used to create highly interactive activities to motivate students' mathematics learning. We developed a series of instructional tools on the web

(<http://163.25.178.88/lii/basicgeometry/index.html>)

to teach junior high school students basic geometric concepts, they are "exploring exterior angles of polygons", "exploring interior angles of polygons", "exploring properties of parallel lines". Each package includes a discussion the object and designing ideas of the package, questions to explore, an on line test, electronic tools, and media illustrations of the package.

An experimental design was used to test the effect of the applications of these electronic tools. It was found that students in the experimental group performed better on questions of the basic concept level. Seventy percent of the students agreed that integrating these FLASH tools into instruction help their mathematics learning, and eighty-six percent of them like to have similar curriculum in the future.

[yuan1.pdf](#)



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### ***Integrating writing and technology into mathematical learning***

The purpose of this study was to design writing materials and if appropriate manipulating software were developed to help high school students explore the writing works. An experimental design was also used to study its effect on high school students' understanding of mathematics. A virtual classroom was created on the web ([inforscience.nctu.edu.tw](http://inforscience.nctu.edu.tw)) so that students in the experimental group could type their answers and submitted their works to. When students completed submitting their works to the destined site, the teacher gave feedbacks and graded the assignments on the web. Ten writing works were assigned to the students in the experimental group during the study semester.

The experimental group did perform better than the control group on the achievement test at a significant level (\* $P < 0.05$ ). More discussions occurred among students. However, the amounts written by students were small. Students prefer to solve the problems by writing short answers to respond to the writing works, and few used their own words to explain their reasoning process. The researcher also suggested that having preservice teachers participate in this writing activity may help them understand students' reasoning processes as well as students' learning difficulties. School teachers with this help may be more willing to the use of writing as a learning tool in their mathematical classrooms.

[yuan2.pdf](#)

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### ***The Combinatorial Matrix Approach on Symbolic Polynomial Systems***

A new approach(The Combinatorial Matrix Approach) to eliminate variables on symbolic polynomial systems will be presented. It is proved that in the cases of a polynomial systems with bi-variable two degrees or with bi-variable three degrees it is more efficient than other methods such as the Wu's Elimination and the Grobner Bases Approach even the Dixon approach.

We know that, in computer algebra, to efficiently eliminate the variables in a symbolic polynomial system is very crucial to a computer algorithm and a software to solve the realistic problems. For example, in machine proving and in computer automated reasoning, there are a bunch of symbolic polynomial systems in which the variables needed to be eliminated. Some of the problems are difficult to be solved with the existed eliminating methods such as Wu's elimination and the Grobner Bases Approach.

The Combinatorial Matrix Approach focuses on how to derive a linear symbolic system from a nonlinear polynomial system. Then solve the equivalent linear system instead of the nonlinear system. If an algebraic software is implemented with the Combinatorial Matrix Approach to the variable elimination algorithm, it will significantly improve the performance of the software. It will play an important role in automated reasoning and in math educational software products.

[zhou.pdf](#)