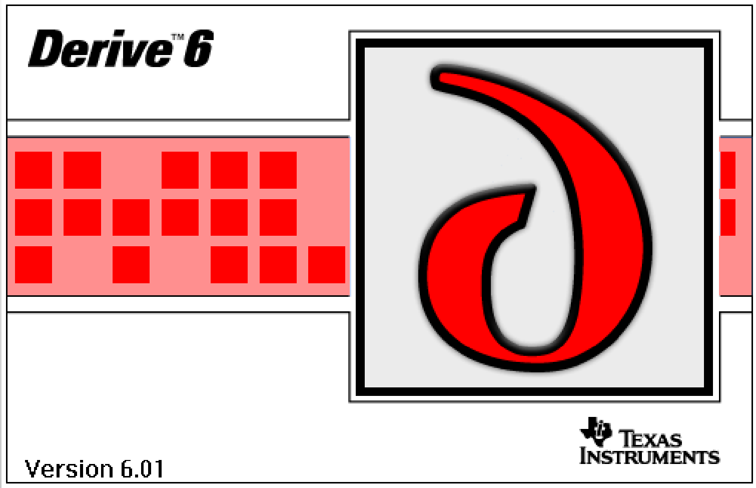


*Derive*6 and its Impact on Math Education

Josef Böhm, DERIVE Usergroup,
TIME 2004, Montreal

When *DERIVE* entered the market in the late 80s it was not designed as a tool for math education. Some enthusiastic and innovative teachers and members of school authorities – a major part of them from Austria – discovered its potential to change teaching mathematics on secondary school level.

So by and by – influenced by teachers and educators – *DERIVE* changed to a PeCAS (Pedagogical Computeralgebra System) and what is very important: without losing its mathematical power.

DERIVE 6 is another big step into this direction. It offers a bundle of new abilities which support modern points of view how to proceed in math education in our electronic and computer influenced era.

It is my big hope that conferences like this will help to improve the software and will also help to improve how to use it in the best possible way. Remembering the outcome of former ACDCA, *DERIVE* & TI-92 Conferences I am convinced that all these events gave a lot of input to the software producers and the users as well.

Load d6_1.dfw

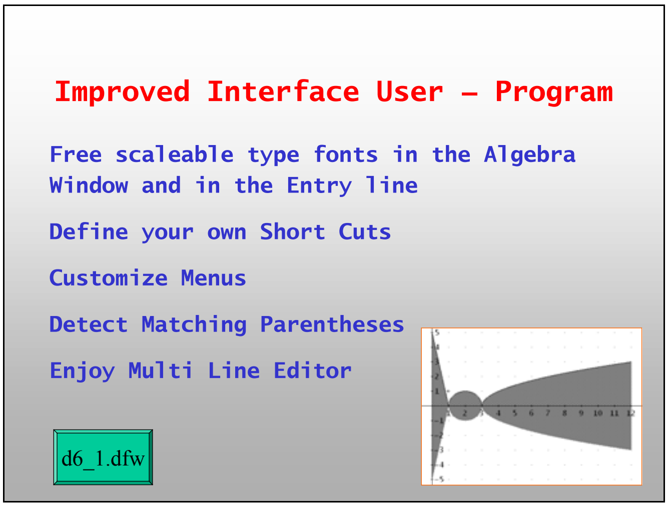
Change Font Size

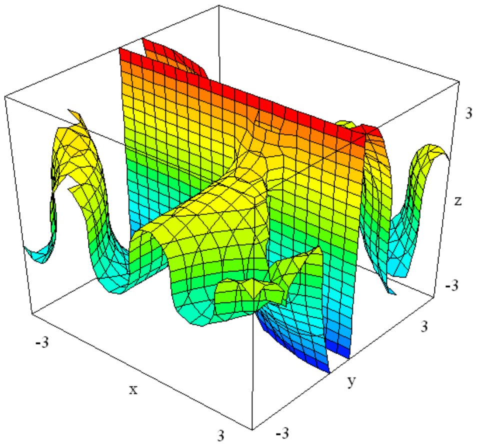
Try to plot

$$z^3 y^2 = (3x + 4y - 2z)\sin(x\,y)$$

This doesn´t work in DERIVE 6 3D-Plot Window

Include the DPGraph Icon into the Menu Bar and then doubleclick in it having highlighted the implicit defined function from above (of course, you must have DPGRaph installed!)

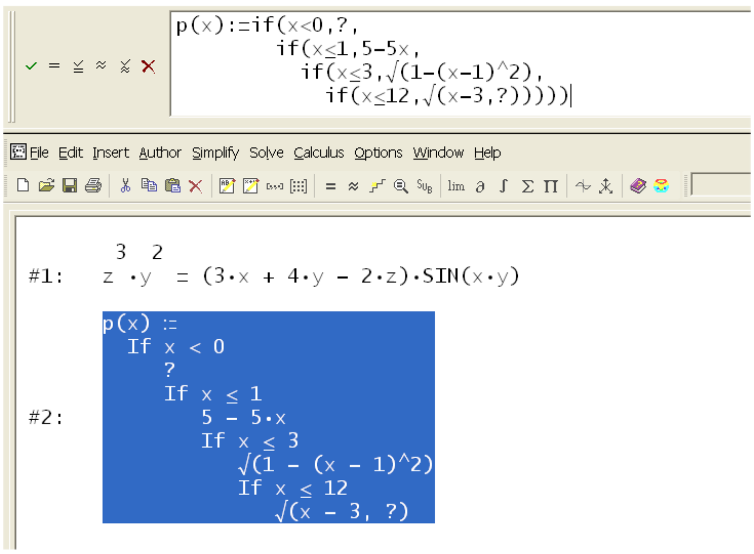




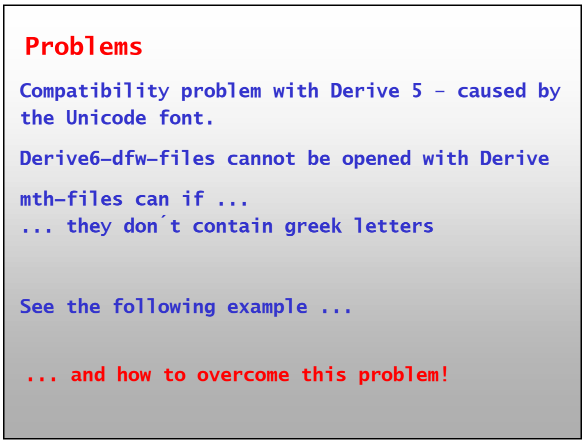
Customize your own DERIVE6 (eg introduce the short cut CTRL L for Multi Line Editing)

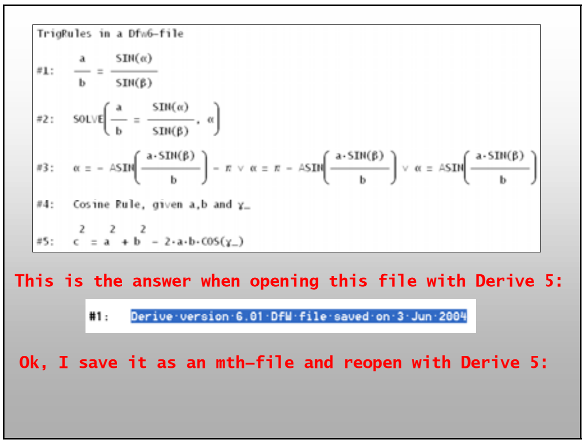
Use this short cut to demonstrate MLE by editing the following piecewise defined function giving the cross section of a goblet.

$$p(x) = \begin{cases} 5 - 5x \quad \text{for} \quad 0 \le x \le 1 \\ \sqrt{1-(x-2)^2} \quad \text{for } 1 \le x \le 3 \\ \sqrt{x-3} \quad \text{for } 3 \le x \le 12 \\ \text{else} \quad \text{undefined} \end{cases}$$

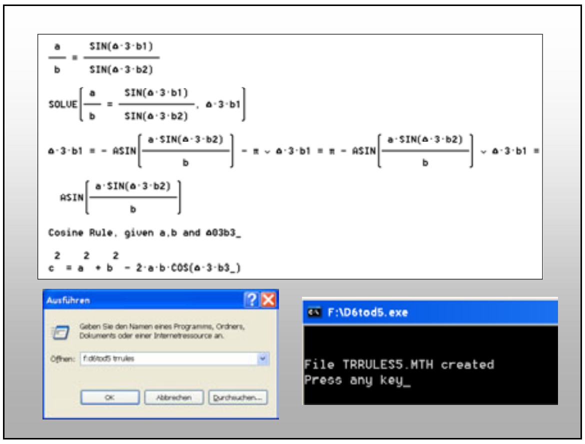


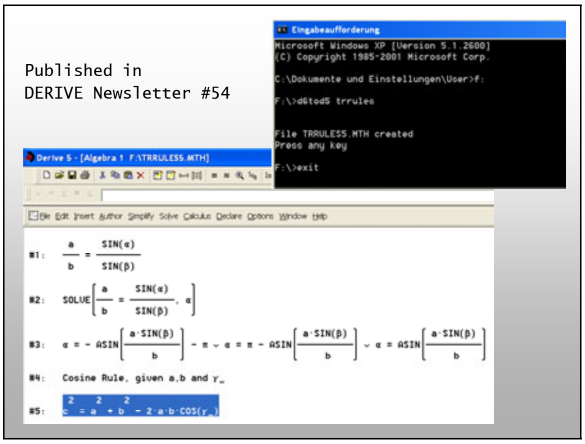
CONTS



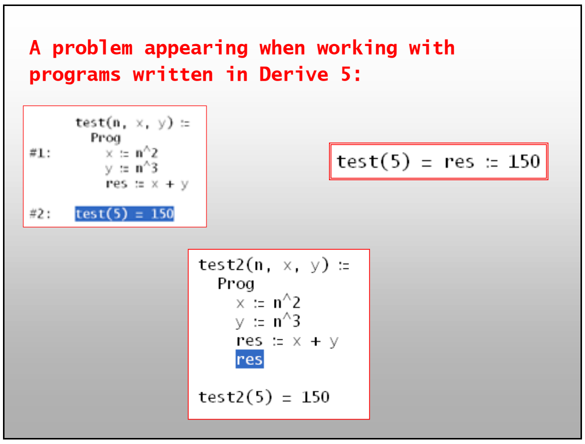


Receive a Derive5 file from anybody – make some changes, save (in Derive6) and send it back ....





I came across another problem executing Derive 5 programs with Derive 6:



I recommend to test your Derive 5 programs if using with Derive 6!!

CONTS

Now let´s stop complaining. The benefits of Derive 6 are much more than the problems, which all can be mastered as I showed just now – if you are aware of them!

Automatically Annotation of graphs

Plot 4 graphs y(x) = a*x^2 and identify the graphs.

Later we will introduce Slider Bars in order to animate the presentation.

part1

We can plot x^2, $2x^2$, $3x^2$, $-3x^2$ immediately using the Entry Line of the 2DPlot-Window and activate an option for automatically adding the annotations.

For the wishlist:

Plot commands in the Algebra Window (specify color, point size, ...)

These commands should be "progammable"

Bold style for plotting lines and curves

Comfortable setting of equal scalings on both axes in the full screen 2D-plot window

The first great new feature

The first great new feature:

Stepwise Simplification

show steps

Some examples

$\mathrm{LN}(x^3)$

$\mathrm{SIN}\left(\frac{10\cdot\pi}{3}\right)$

$\mathrm{SIN}(3\cdot x)\cdot\mathrm{COS}(5\cdot x)$

$\frac{d}{dx}(x^2\cdot\mathrm{SIN}(3\cdot x))$

$\int \frac{x^3+2\cdot x}{\sqrt{x}}\,dx$

$\int_2^5 \frac{x^3+2\cdot x}{\sqrt{x}}\,dx$

$\int e^{3\cdot x}\cdot\mathrm{SIN}(x)\,dx$

$$\#28:\quad \frac{d}{dx}\left(x^2\cdot \mathrm{SIN}(3\cdot x)\right)$$

Switch Display Rules off - let the students explain the steps
Which rule is applied at each single step.

$$\#29:\quad x^2\cdot\frac{d}{dx}\mathrm{SIN}(3\cdot x)+\left(\frac{d}{dx}x^2\right)\cdot \mathrm{SIN}(3\cdot x)$$

$$\#30:\quad x^2\cdot \mathrm{COS}(3\cdot x)\cdot\frac{d}{dx}(3\cdot x)+\left(\frac{d}{dx}x^2\right)\cdot \mathrm{SIN}(3\cdot x)$$

$$\#31:\quad x^2\cdot \mathrm{COS}(3\cdot x)\cdot 3\cdot\frac{d}{dx}x+\left(\frac{d}{dx}x^2\right)\cdot \mathrm{SIN}(3\cdot x)$$

$$\#32:\quad x^2\cdot \mathrm{COS}(3\cdot x)\cdot 3+\left(\frac{d}{dx}x^2\right)\cdot \mathrm{SIN}(3\cdot x)$$

$$\#35:\quad \int e^{3\cdot x}\cdot \mathrm{SIN}(x)\,dx$$

$$z^w \rightarrow e^{w\cdot \mathrm{LN}(z)}$$

$$\#36:\quad \int e^{3\cdot x\cdot \mathrm{LN}(e)}\cdot \mathrm{SIN}(x)\,dx$$

$$\int e^{a\cdot x+b}\cdot \mathrm{SIN}(c\cdot x+d)\,dx \rightarrow \frac{e^{a\cdot x+b}\cdot(a\cdot \mathrm{SIN}(c\cdot x+d)-c\cdot \mathrm{COS}(c\cdot x+d))}{a^2+c^2}$$

$$\#37:\quad \frac{e^{3\cdot x\cdot \mathrm{LN}(e)}\cdot(3\cdot \mathrm{LN}(e)\cdot \mathrm{SIN}(x)-\mathrm{COS}(x))}{(3\cdot \mathrm{LN}(e))^2+1}$$

$$\#38:\quad \frac{e^{3\cdot x}\cdot(3\cdot \mathrm{LN}(e)\cdot \mathrm{SIN}(x)-\mathrm{COS}(x))}{9\cdot \mathrm{LN}(e)^2+1}$$

This opens a wide field of didactic opportunities:

- Let the students explain the displayed rules in their own words
- Simplify stepwise without displaying the rules, ask about the applied rules
- Compare the traditional rules with the "Derive Rules"

$$\int x^n\,dx \rightarrow \frac{x^{n+1}}{n+1}$$

$$\#7:\quad \mathrm{SUBST\_DIFF}\left(\frac{2\cdot x^{7/2}}{7}+\frac{4\cdot x^{3/2}}{3},\ x,\ 2,\ 5\right)$$

$$\mathrm{SUBST\_DIFF}(F(x),\ x,\ a,\ b) \rightarrow F(b)-F(a)$$

$$\#8:\quad \sqrt{\left(\frac{3960500}{441}\right)}-\sqrt{\left(\frac{21632}{441}\right)}$$

$$\#9:\quad \frac{890\cdot\sqrt{5}}{21}-\frac{104\cdot\sqrt{2}}{21}$$

- Evaluating an integral stepwise we encounter a function which is neither not mentioned in the manual nor in the online-help. Explain its purpose!
  Can you explain the transformation from #8 to #9?

All stepwise simplified examples are in the file steps.dfw

An interesting question appeared in the Derive news recently.

Stepwise simplification makes Derive to a math teacher:

Derive 6 a teacher?

$$\int \frac{x^3}{e^x - 1}\,dx$$ has no closed form result, but

$$\int_0^{\infty} \frac{x^3}{e^x - 1}\,dx = \frac{\pi^4}{15}$$ **How is this possible?**

Let´s ask Derive! (or Albert Rich & Co)

$$\#56: \int_0^{\infty} \frac{x^3}{e^x - 1}\,dx$$

$$e^0 \rightarrow 1$$

If a>0 and p>-1,

$$\int_0^{\infty} \frac{x^p}{c\cdot e^{a\cdot x} - c}\,dx \rightarrow \frac{\dfrac{p!\cdot\zeta(p + 1)}{a^{p + 1}}}{c}$$

$$\#57: \quad 3!\cdot\zeta(4)$$

If n>=0 is an integer,

$$n! \rightarrow 1*2*3*...*n$$

$$\#58: \quad 6\cdot\zeta(4)$$

If n>0 is even,

$$\zeta(n) \rightarrow \frac{2^{n - 1}\cdot\pi^n}{n!}\cdot|\mathrm{BERNOULLI}(n)|$$

$$\#59: \quad \frac{4\cdot6\cdot\pi^4}{15\cdot4!}$$

If n>=0 is an integer,

$$n! \rightarrow 1*2*3*...*n$$

$$\#60: \quad \frac{\pi^4}{15}$$

Derive is our assistant – DERIVE, A Mathematical Assistant.

Derive is now much more than a calculation tool,
much much more than a black box.

CONTS

The second great new feature

The second great new feature:

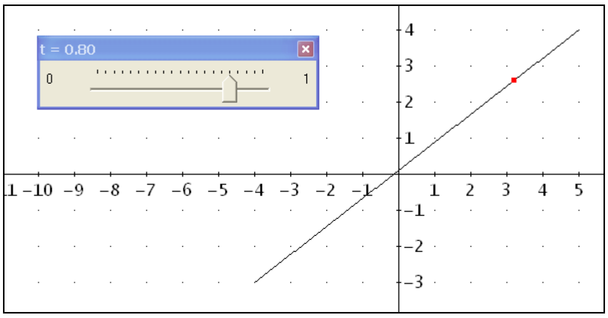
Slider Bars in 2D– and 3D–Window

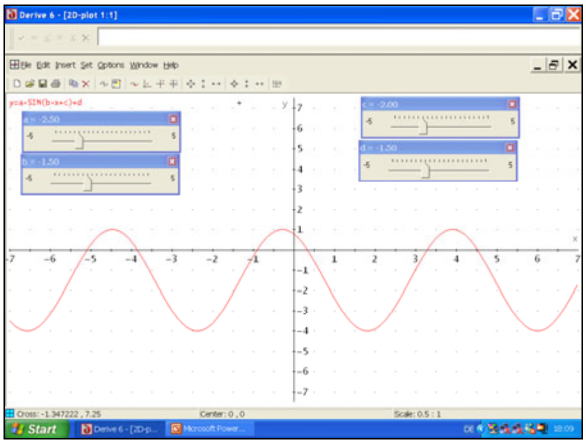
Show the importance of parameter t in the parameter form of a line $x = a + t(b-a)$.

We show the influence of controlpoints on the form of a Bézier curve.

let´s slide .....

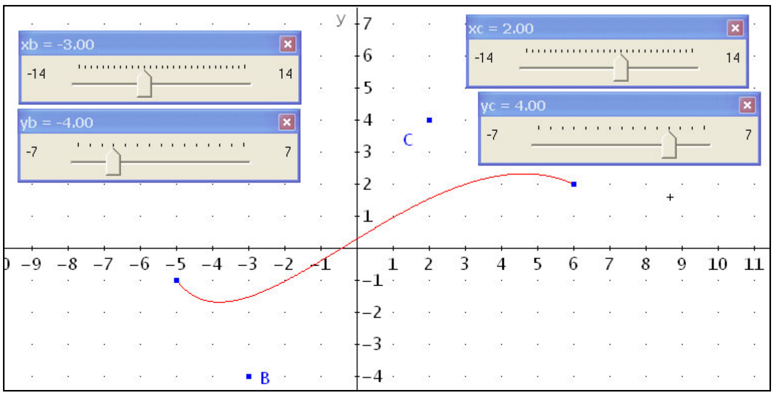
Visualize the influence of parameters a, b, c and d in the generalized sine–function $a*\sin(b*x+c)+d$.





line.dfw

sine.dfw

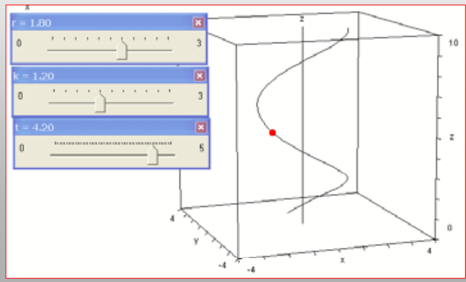


Load bezier.dfw and follow the advice

Let´s change into 3D space:

Represent a helix and let a point ”walk” on our DNS.

```
#2:   [r·SIN(t), r·COS(t), k·t]
```



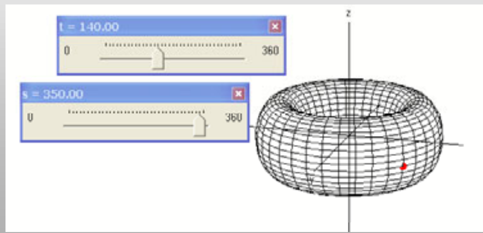
We move along the helix

Load helix.dfw and follow the instructions.

dann –7 <= t <= 7; 28 Steps, Insert Plot Point Size MAX, Trace Plot Farbe wählen, Punkt auf der Helix wandern lassen

Plot the wire grid of a torus and show the position of any point on the bodie´s surface.

```
[(a + b·COS(s°))·COS(t°), (a + b·COS(s°))·SIN(t°), b·SIN(s°)]
```



Wire grid of a surface + point on the grid to identify and study the meaning of parameters s and t.
I worked with degrees, because I cannot use multiples of PI as boundaries for the slider bars. 0 to 360 is possible!! I´ll demonstrate this later.

For the wishlist:

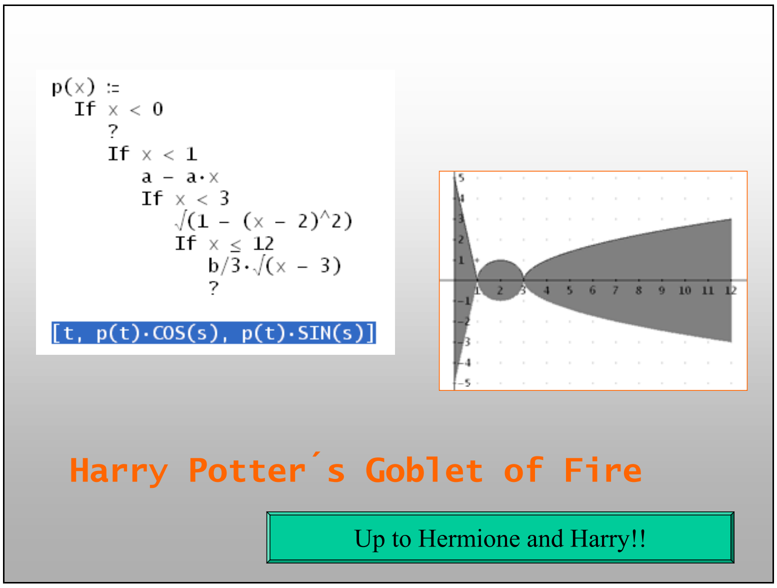
Decimal numbers as boundaries (especially π) and requesting increment instead of number of steps

No flickering of the screen when sliding

Very important:
The bars should remain in embedded graphs!

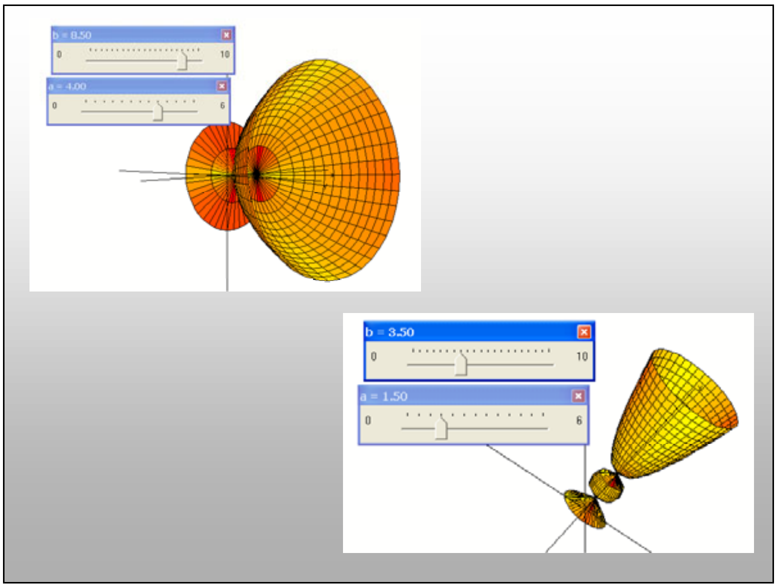
The bars should not appear in the Algebra Window!

dfw-files containing pasted and/or embedded graphs tend to get huge – compressing makes them tiny.
Why not inplement the compressing procedure into Derive?
(also for 3D graphs!)

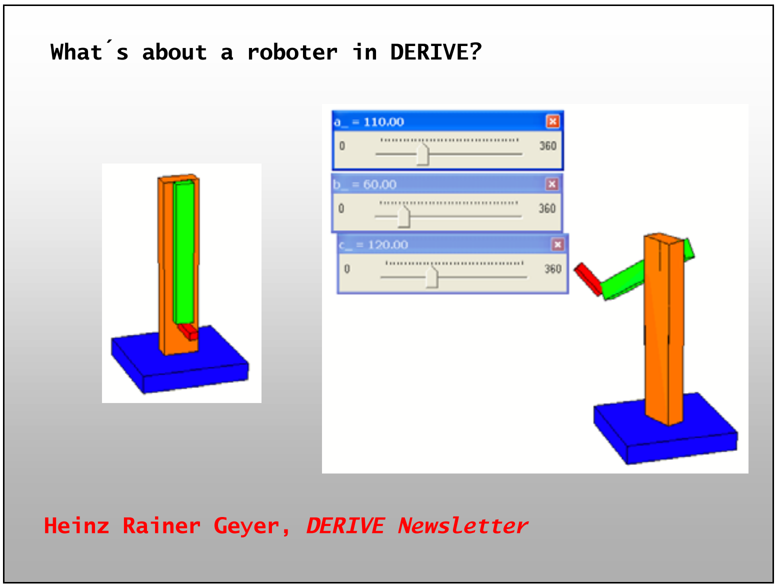


We generalize the crosssection of the cup from above – nice job for students – introducing one or more parameters.
What else could be made variable??
Load potter.dfw and follow the instructions
Work first in two dimensions and then make an upgrade into the 3rd dimension
Take care for the dimensions of the box

Wire Grid presentation might be useful to demonstrate the position of a point on the surface depending on the parameter values s and t.



Some years ago my Austrian friends Lechner, Klinger & Co wrote a chapter on roboters in their first book on the use of CAS in school. Now we can close the circle and show a roboter in real time!



robot.dfw

Heinz Rainer Geyer produced with his students a nice roboter. Try his little man working with slider bars for variables a_, b_ and c_ .(0° – 360°)

For the wishlist:

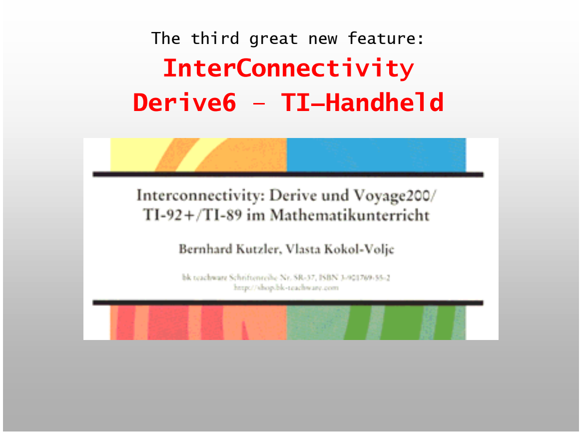
Improve 3D graphic (compare with DPGraph!!)

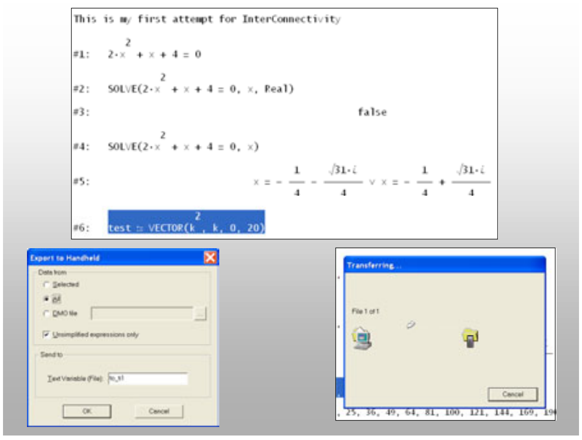
Not only embedded graphs can be transferred via rtf-file to a htm-file - make it directly and without restrictions. This is very important in online-times. (also for 2D graphs)

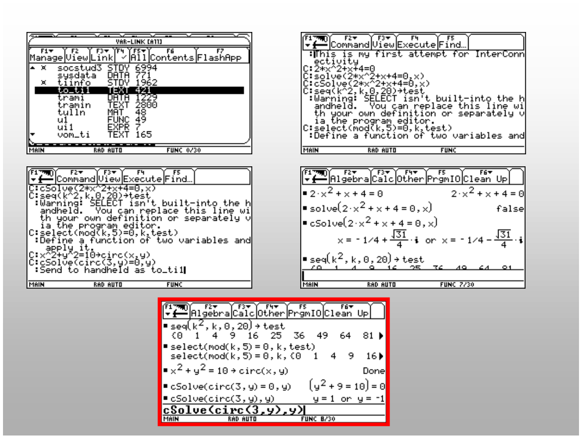
The 3rd great new feature

Handheld and PC can communicate!!

Important for the whole TI-CAS family!







It is very easy to transfer data from handheld to PC and vice versa thanks TI-Connect, which is automatically installed when you install Derve 6.

Let´s assume that we would like to store a Derive session on the V200.

As you might know, VECTOR is not a TI-command. How will the TI overcome this problem?

The file is blown by the wind from one device to the other ……

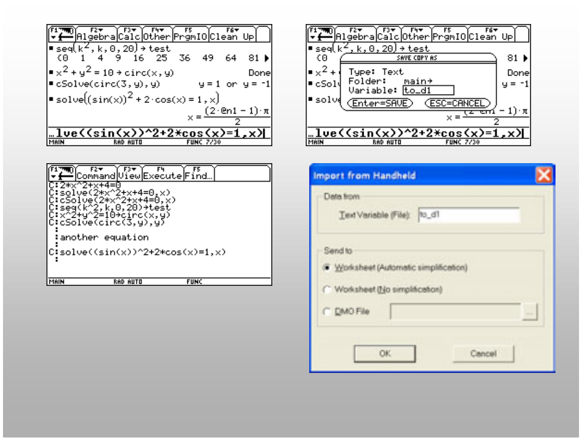
... .and it arrives as a text file.

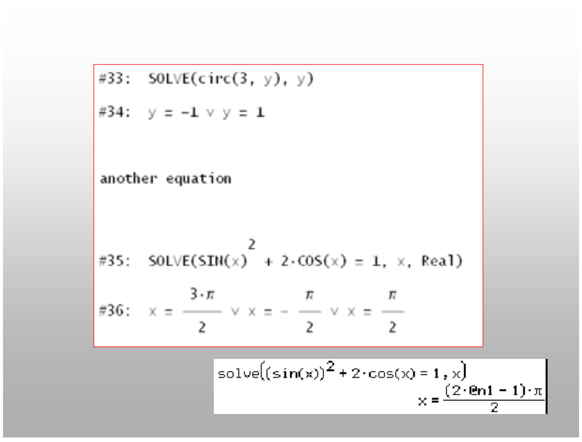
We open the file and recognize the commands as executable lines (C: and the comments as not executable lines)

VECTOR has been translated into the equivalanet seq() command.

In my Derive session I used also SELECT, but SELECT is unknown by the TI.

Now I run the session on the TI and proceed by defining a function circ(x,y) and want to solve an equation!





and another equation involving trig functions.

The first step is saving the session as a text file (to_d1), go back to Derive and try to import this file (choosing one of the three options).

csolve → solve

comment → comment in a text box

The solution of the trig equation doesn´t look the same? Explain!!! Task for students!!

CONTS

A wonderful feature indeed with plenty of didactical possibilities (I strongly recommend Bernhard and Vlasta´s booklet).

But from my point of view there is still a wide field for improvements.

Let´s follow a very ordinary session of school mathematics.

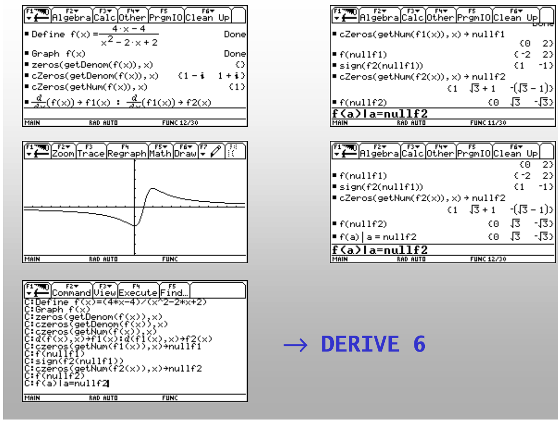
(Many thanks for Matthias Haslauer and Josef Lechner providing this classroom example – and experience.)

In our lesson we perform an easy to do investigation of a rational function – without using any specific trick.

At home we would like to transfer this investigation to Derive in order to produce a fine paper including the graph of the function.

No special task,

very traditional, basic routines applied



→ DERIVE 6

This doesn´t look strange. Or ......

Interesting and very userfriendly you are asked to plot the graph (on the TI this we had graph f(x))

Now let me „simply“ simplify (calculate)!

```
#1:  f(x) := (4·x - 4)/(x^2 - 2·x + 2)
Highlight, then press appropriate toolbar graph button:
#2:  f(x)
#3:  zeros(a, b) :=
#4:  zeros(DENOMINATOR(FACTOR(f(x), Trivial)), x)
#5:  SOLUTIONS(DENOMINATOR(FACTOR(f(x), Trivial)), x)
#6:  SOLUTIONS(NUMERATOR(FACTOR(f(x), Trivial)), x)
#7:  PROG(f1(x) := d/dx f(x), f2(x) := d/dx (f1·x))
#8:  null1 := SOLUTIONS(NUMERATOR(FACTOR(f1·x, Trivial)), x)
```

The unsimplified import looks quite ok!
Or do you expect any problems?

On the next page we will have the simplified expressions

```
#6:  SOLUTIONS(NUMERATOR(FACTOR(f(x), Trivial)), x)
#7:  f1(x) := d/dx f(x)
#8:  f2(x) := d/dx f1(x)
#9:  null1 := SOLUTIONS(NUMERATOR(FACTOR(f1(x), Trivial)), x)
#10: f(null1)
#11: SIGN(f2(null1))
#12: null2 := SOLUTIONS(NUMERATOR(FACTOR(f2(x), Trivial)), x)
#13: f(null2)
#14: SUBST(f(a), a, null2)
```

Look for the simplified expressions!

A new function zeros is defined by Derive ??

```
#1:  f(x) := (4·x - 4)/(x^2 - 2·x + 2)
#2:  f(x) := 4·(x - 1)/(x^2 - 2·x + 2)
Highlight, then press appropriate toolbar graph button:
#3:  f(x)
#4:  4·(x - 1)/(x^2 - 2·x + 2)
#5:  zeros(a, b) :=
#6:  zeros(a, b) :=
```

We discover a function called “zeros“??

We see that Derive does not recognize zeros, but czeros which is applied in connection with providing complex numbers which might confuse students when working only with real numbers.

```
#7:  zeros(DENOMINATOR(FACTOR(f(x), Trivial)), x)
#8:  zeros(x^2 - 2·x + 2, x)
#9:  SOLUTIONS(DENOMINATOR(FACTOR(f(x), Trivial)), x)
#10: [1 + i, 1 - i]
#11: SOLUTIONS(NUMERATOR(FACTOR(f(x), Trivial)), x)
#12: [1]
#13: f1(x) := d/dx f(x)
#14: f1(x) := 4·x·(2 - x)/(x^2 - 2·x + 2)^2
```

zeros is not translated, czeros() is translated as SOLUTIONS – with complex solutions!

Now the zeros of the second derivative are calculated correctly – only reals solutions appear - but we face new problems by finding the function values. The list cannot be used for substituting .....

#15: $f2(x) := \frac{d}{dx} f1(x)$

#16: $f2(x) := \dfrac{8\cdot(x-1)\cdot(x^2-2\cdot x-2)}{(x^2-2\cdot x+2)^3}$

#17: nullf1 := SOLUTIONS(NUMERATOR(FACTOR(f1(x), Trivial)), x)

#18: nullf1 := [0, 2]

#19: f(nullf1)

#20: $([0, -4] + 6)^{-1}\cdot([0, 8] - 4)$

The list of solutions cannot be used as argument for a function – the handheld allows this!

#21: SIGN(f2(nullf1))

#22: $SIGN(([0, -4] + 2)\cdot(([0, -4] + 6)^{-3}\cdot([0, 16] - 8)))$

#23: nullf2 := SOLUTIONS(NUMERATOR(FACTOR(f2(x), Trivial)), x)

#24: nullf2 := $[1, \sqrt{3} + 1, 1 - \sqrt{3}]$

#25: f(nullf2)

#26: $([-2, -2\cdot\sqrt{3} - 2, 2\cdot\sqrt{3} - 2] + 11)^{-1}\cdot([4, 4\cdot\sqrt{3} + 4, 4 - 4\cdot\sqrt{3}] - 4)$

#27: SUBST(f(a), a, nullf2)

#28: $([-2, -2\cdot\sqrt{3} - 2, 2\cdot\sqrt{3} - 2] + 11)^{-1}\cdot([4, 4\cdot\sqrt{3} + 4, 4 - 4\cdot\sqrt{3}] - 4)$

#29: VECTOR(f(a), a, nullf2)

#30: $[0, \sqrt{3}, -\sqrt{3}]$

Maybe that translating the with-operator might be a better solution??

Our – highly appreciated |–operator is not translated with a list, because of using the SUBST–command in Derive. VECTOR would do the job!

My (personal) recommendations?

Take special care that the elementary functions and activities are mutually understandable – students and teachers are the customers.

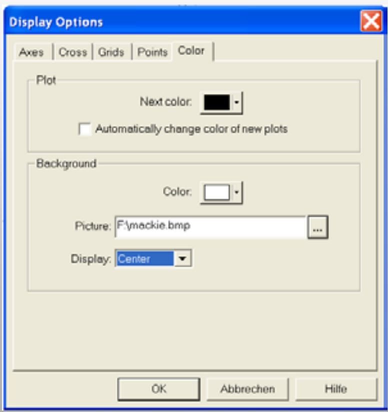
Include the possibilities and benefits of **InterConnectivity** in both manuals (Derive and Handheld) to make it as popular as it deserves.

Include the list of mutual understandable functions and commands in both manuals (or at least in the DERIVE Online-Help).

Fortunately things are in progress!! Fortunately!!

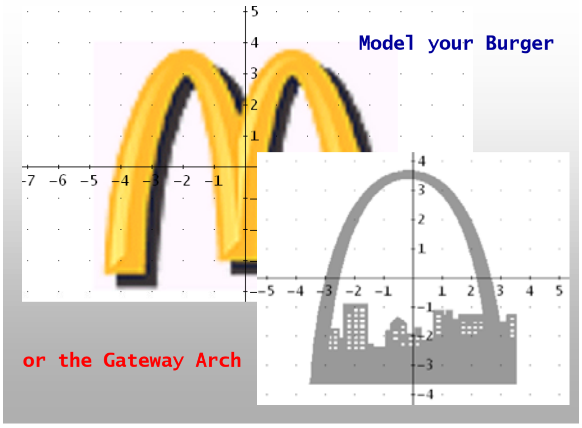
What are your recommendations?

Finally, an underestimated and widely unknown feature which is a great resource of inspiring ideas for teaching.

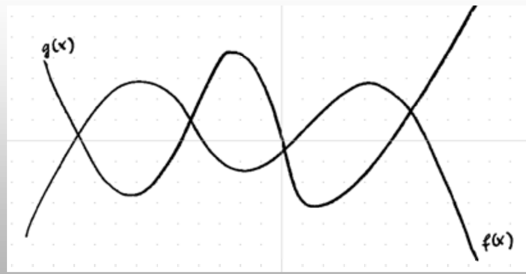


What´s about a Burger?

We browse the Display Options in the 2D-Plot Window!



I put a sketch on the scanner.



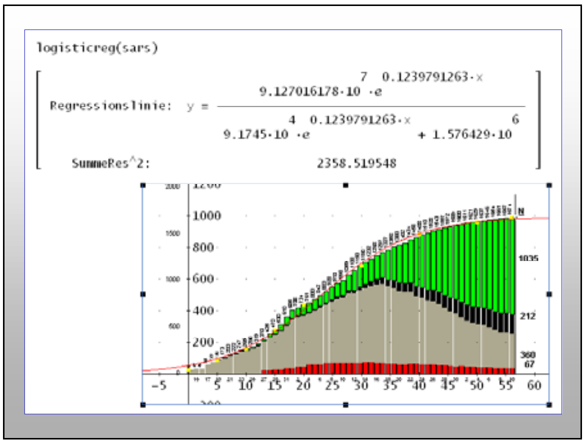
Who of you will find the best modelling function?

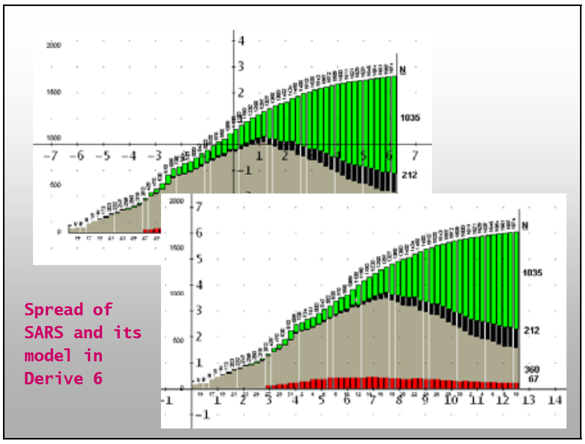
… Numerous ideas to provoke mathematical modeling using functions ….

Which one is the best function? How to decide?

My colleague Tania Koller went shopping and produced some pictures.
Use splines or other functions to model Cinderella´s shoe.

Applying Cubic Splines the students tried successfully to model Cinderella´s shoe!!



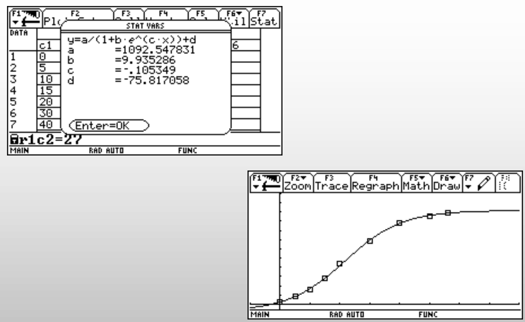


Take any real data diagram from the news and find a suitable mathematic model!
(Many thanks to my friend Peter Hofbauer for providing the data.)

sars.bmp and sars.dfw

A demanding task is waiting for students and teachers.

The model function is valid for the DERIVE coordinates. Now we have to transform it into „world coordinates“.
I applied a logistic regression routine provided by Phillip MacDonald for the DERIVE Newsletter



Interconnectivitiy makes possible to transfer the data easily to the handheld and then perform the regression with the built-in routine. The result could be sent back to the PC.

and compared with the TI´s built in regression tool.
Excel – DERIVE – TI-Handheld can cooperate!!!

CONTS

That´s the good news for Derive 6.1

- In the next release of Derive 6, the change in behavior of simplified assignment statements within function definitions will be clearly stated in the online help.
- The maximum number of intervals allowed for slider bars will increase from 50 to 1000.
- Decimal values will be allowed for slider bar min/max values.
- The graphing calculator function zeros will be translated properly.
- The list of interconnectivity functions and their mappings will be available via the Derive 6 online help.

To learn more about DERIVE and CAS TI

join the

International DERIVE & CAS-TI Usergroup

http://www.austromath.at/dug/