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COMBINING THE POSSIBILITIES OF DERIVE AND EXCEL WHILE STUDYING BASES OF COMPUTER SCIENCE

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Abstract

Our experience of teaching and learning of computer science on the faculty of mathematics and physics shows that not a simple acquaintance with a package Derive, but also its wide use in practice, is the important constituent of mathematical education. The acquaintance and use of the Derive package by the students of the faculty were performed within the frameworks of the basic course "Bases of computer science", special computer science courses, and during passing the required computing practice. The textbooks and the methodical instructions prepared by the specialists of our faculty were the theoretical basis of the course. As a result of an experimental work the course was developed, in which studying the Derive package and using Microsoft Excel package (with the elements of VBA programming) were combined. The course embraces four topics: calculation of roots of algebraic functions; calculation of integrals; solving the systems of linear algebraic equations and the tasks of linear programming; mathematical modeling and new theories of learning. The theoretical material and the problems were submitted in an electronic textbook and were accessible to the students during their laboratory works. The peculiarity of the course is in technology: the students try to estimate an opportunity of the decision of the problem in the Derive package, to carry out the verification of their decisions numerically in Excel, with using VBA, and on the contrary.

1. Introduction

The last decade was characterized by the changes which have affected a technique of teaching of all disciplines, including mathematics. There is an integration of on-line technologies with the traditional mathematical courses. The integration results in improvement of quality of learning. A choice of on-line technology in the mathematical course depends on its organization, on individuality and style of training of every teacher. The training with use of technologies necessarily requires preliminary preparation of teachers. The work considers the course of information technologies (Derive, Excel, etc.) for the students of physics-mathematics faculty of pedagogical university. The students are the future teachers of mathematics. During studying the course the students can use electronic copies of laboratory works. It is all is a part of innovation service of training, which refers to as e-learning.

2. The review of the references in a subject

Last years the computer mathematics packages, the programs of Microsoft Office package have begun to be applied in training the teacher of mathematics. Ukrainian teachers also began using Derive, Excel, and dynamic geometry packages. Though the wide review of information technologies programs were not included to training of the future teachers at pedagogical university. Until recently at training the students of pedagogical high schools in computing technologies (course of Calculus) the next tutorials were used [1-5]. Then the works [6-9] were published, and situation has changed.

At the same time the increasing attention were given to applying technologies to the mathematical education. We have used the materials of the international conference on training to mathematics ICTM2 (July 2002, Greece), where more than 100 works were devoted to the subject "Technologies - effective integration of computing technologies (Calculators, Computer Algebra Systems, WWW resources)". In 1999-2002 years were published papers in "Teaching Mathematics and its Applications" (Oxford) that have specified on expediency of introduction of new information technologies in training mathematics, partly, with the help of a course of mathematical modeling [17-20]. So, Nyman M.A. & Berry J. in their "Developing transferable skills in undergraduate mathematic student through mathematical modeling" [14] told about importance of mathematics in the school and university programs. They say about those elements of mathematical education, which should be included in the mathematical programs in the beginning of the twenty first century, because the discrepancy between the school program and requirements of university mathematics was observed: “Too often the mathematics curriculum at all levels is seen as a ‘body of knowledge’, which needs to be delivered in order to provide an ‘acceptable graduate in mathematics’. In this era of powerful software on hand-held and computer technologies we need to review the procedures and rules that have been the central focus of the mathematics curriculum for over one hundred years. That is not to say that we do not need some of the traditional skills so that students can make effective use of the technology. However, there are important generic skills that mathematics provides, and to the employer of our graduates these skills are often more important than the actual mathematics that they have learnt” [14].

3. The teaching method

We consider four topics: 1) finding roots of algebraic and transcendental functions (6 hours); 2) calculation of the definite integrals (6 hours); 3) solving systems of the linear algebraic equations and the tasks of linear programming (6 hours); 4) approximation of functions, and modeling tasks (8 hours).

The practical work is accompanied by review of appropriate theoretical material. The Derive package and the spreadsheet Excel with the elements of VBA are applied during solving the tasks. At first we estimate an opportunity of solving the problems in the Derive package, and, if it is impossible, the numerical methods are used, that are realized with using functions and elements of VBA in the Excel package.

Laboratory work 1. To find the roots of algebraic and of transcendental equations:

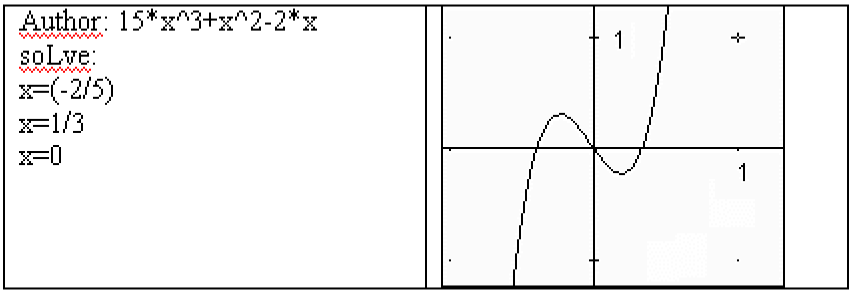
a) $15x^3 + x^2 - 2x = 0$	d) $4x^4 - 24x^3 + 57x^2 + 18x - 45 = 0$
b) $x^3 + 2x^4 + 4x^2 + 2 + x = 0$	e) $4\operatorname{arctg}(x^2 - 3x - 3) - \pi = 0$
c) $(1 + x^2)^2 - 4x(1 - x^2) = 0$	f) $8\cos(x) - x - 6 = 0$

The laboratory work is carried out by the stages: building the diagram, partition of roots, realization of an iterative method, and automation of calculations.

1) Building the diagram.

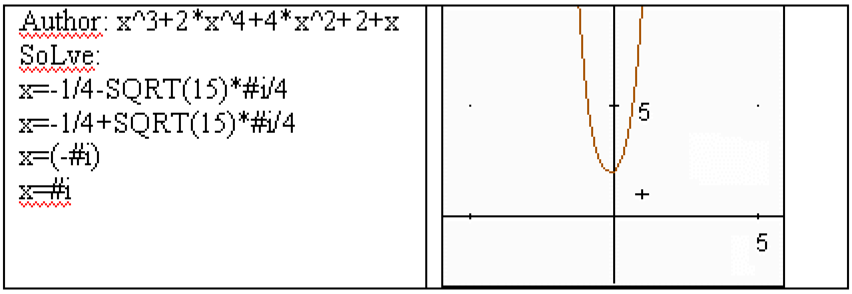
The Derive package is used for finding roots of functions a) -f).

a) The function $f(x) = 15x^3 + x^2 - 2x$ has three real roots.

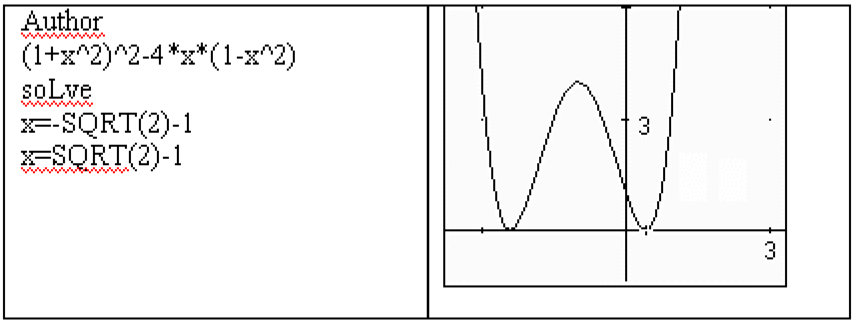


Really, $15x^3 + x^2 - 2x = x(15x^2 + x - 2) = 15x(x - \frac{1}{2})(x + \frac{2}{5})$.

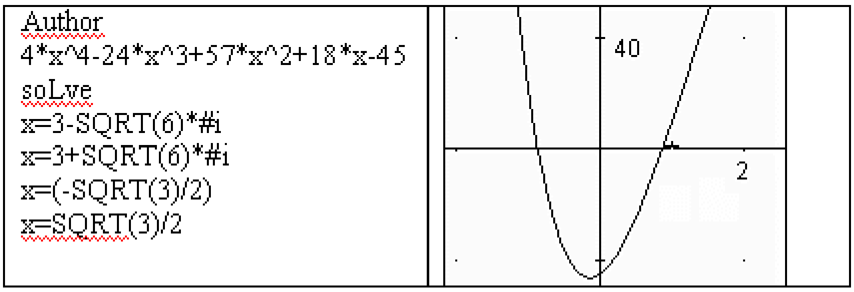
b) The function $f(x) = x^3 + 2x^4 + 4x^2 + 2 + x$ has no real roots.



c) The function $f(x) = (1 + x^2)^2 - 4x(1 - x^2)$ has two real roots.



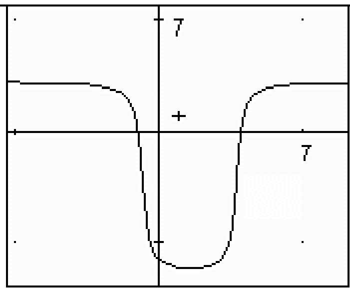
d) The function $f(x) = 4x^4 - 24x^3 + 57x^2 + 18x - 45$ has two real and two complex roots.



Let's consider transcendental functions: $y(x) = 4\mathrm{arctg}(x^2 - 3x - 3) - \pi$,
$y(x) = 8\cos(x) - x - 6$.

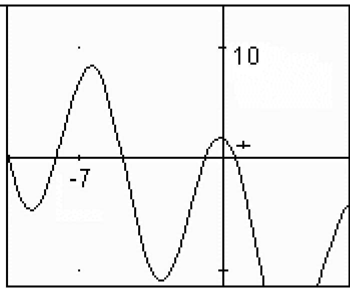
e) The function $f(x) = 4\text{arctg}(x^2 - 3x - 3) - \pi$ has two real roots.

Author:
4*ATAN(x^2-3*x-3)-PI
SoLve:
x=(-1)
x=4



f) For the function $f(x) = 8\cos(x) - x - 6$ Derive's soLve does not give an answer.

Author: 8*COS(x)-x-6
soLve
?
We see, that the function has 5 real roots.



Hence, the system Derive has not found values of roots of function $y(x) = 8\cos(x) - x - 6$. For solving the problem we shall use the numerical methods - the iterative method and the secant method by using the Excel package.

2) Partition of roots.

The first stage of finding numerical roots is a partition of roots that is revealing of intervals, each interval covers only one of possible roots.

Let's consider a stage of partition of roots on the example of finding roots of function $y(x) = x^2 - \ln(x) - 2$. From the diagram constructed in Derive (Fig. 1), we see that the roots of function are on the interval [0, 2].

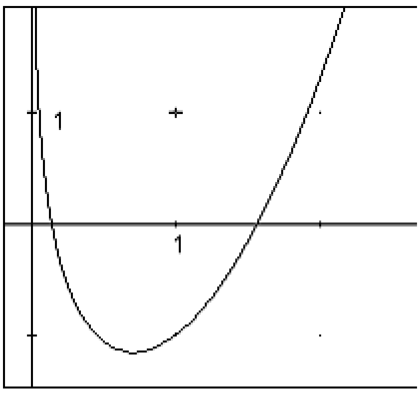


Fig. 1. The diagram of function $y(x) = x^2 - \ln(x) - 2$

The function is defined when $x > 0$. Let's construct in the Excel package the table of values of function $y(x) = x^2 - \ln(x) - 2$ on the interval [0.1, 2.1] with the step=0.1.

Let's fill the table, as shown in a Fig. 2. Into the cell A1 we write the text "The Table of values of function", and into cells A2, B2 – the texts "X" and "Y". Into the cell A3 we write the number 0.1, into the cell A4 - 0.2. We select a range of the cells A3:A4 and place the mouse in the right bottom corner of this range. Let's stretch the cursor of the mouse down into the cell A23, and the number 2.1 will appear in it. Into the cell B3 we shall write formula "=A3^2-ln (A3) -2", the value of function in the point 0.1 will appear in the cell B3; further we shall move the cursor of the mouse into the right bottom corner of the cell B3 and we shall stretch the cursor of the mouse down to the cell B23, and we shall see in it the value of function in the point 2.1 according to the formula "=A23^2-ln (A23) -2". The resulted table looks like in a fig. 2.

Further we shall allocate a range of cells A2:B23 for construction of the diagram. Let's call the Diagram Wizard for building the diagram and after five steps we shall receive the diagram (Fig. 3).

	A	B
1	***The table of values***	
2	**X**	**Y**
3	0,1	0,312585
4	0,2	-0,35056
5	0,3	-0,70603
6	0,4	-0,92371
7	0,5	-1,05685
8	0,6	-1,12917
9	0,7	-1,15333
10	0,8	-1,13686
11	0,9	-1,08464
12	1	-1
13	1,1	-0,88531
14	1,2	-0,74232
15	1,3	-0,57236
16	1,4	-0,37647
17	1,5	-0,15547
18	1,6	0,089996
19	1,7	0,359372
20	1,8	0,652213
21	1,9	0,968146
22	2	1,306853
23	2,1	1,668063

Fig.2. Realization of partition of roots

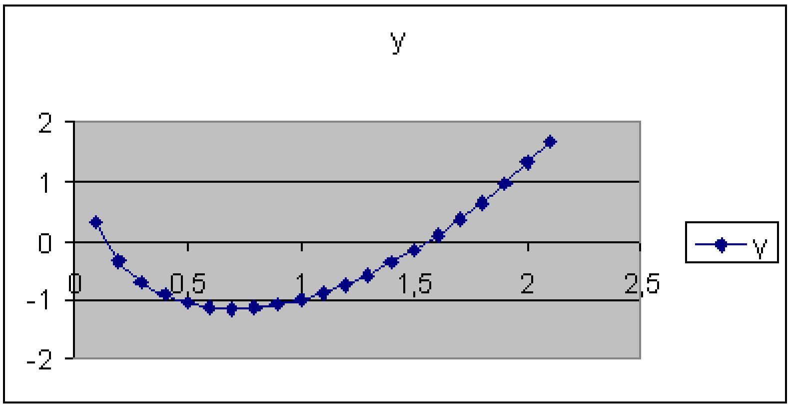


Fig. 3. The graph of function $y(x) = x^2 - \ln(x) - 2$ on the interval [0.1, 2.1]

From the cells of the range A2:B23 of the table (Fig. 4) we see, that the roots of function are on the intervals [0.1, 0.2], [1.5, 1.6]. Let's specify intervals for finding roots. For this purpose in the cells C3:C13 we shall write values of argument X with the step 0.01 on the interval [0.1, 0.2]; in the cells D3:D13 the values of function $y(x) = x^2 - \ln(x) - 2$ are counted up. According to it in the cells C15:C23 we shall write values X with the step 0.01 on the interval [1.5, 1.58]; in the cells D15:D23 the values of function $y(x) = x^2 - \ln(x) - 2$ are calculated.

	A	B	C	D
1	***The table of values***			
2	X	Y	X	Y
3	0,1	0,312585	0,1	0,312585
4	0,2	-0,35056	0,11	0,219375
5	0,3	-0,70603	0,12	0,134664
6	0,4	-0,92371	0,13	0,057121
7	0,5	-1,05685	0,14	-0,01429
8	0,6	-1,12917	0,15	-0,08038
9	0,7	-1,15333	0,16	-0,14182
10	0,8	-1,13686	0,17	-0,19914
11	0,9	-1,08464	0,18	-0,2528
12	1	-1	0,19	-0,30317
13	1,1	-0,88531	0,2	-0,35056
14	1,2	-0,74232	X	Y
15	1,3	-0,57236	1,5	-0,15547
16	1,4	-0,37647	1,51	-0,13201
17	1,5	-0,15547	1,52	-0,10831
18	1,6	0,089996	1,53	-0,08437
19	1,7	0,359372	1,54	-0,06018
20	1,8	0,652213	1,55	-0,03575
21	1,9	0,968146	1,56	-0,01109
22	2	1,306853	1,57	0,013824
23	2,1	1,668063	1,58	0,038975

Fig. 4. Data for specification of the values of roots

We see, that the first root is on the interval [0.13, 0.14], and another root - on the interval [1.56, 1.57]. These intervals are colored in the table in a Fig. 4.

3) Realization of an iterative method.

Let's consider the realization of an iterative method in the Excel package. The idea of an iterative method consists in the following. Let's replace the equation $y(x)=0$ by the equivalent equation $x=f(x)$. Let ξ is a root of the equation $x=f(x)$, and x_0 is some approximation to a root. Let's substitute x_0 in the right part of the equation $x=f(x)$, we shall receive the number $x_1=f(x_0)$. Let's do the same with x_1, we shall receive number $x_2=f(x_1)$ and so on. After these steps we shall receive a numerical sequence $x_0, x_1, x_2, \ldots, x_n, \ldots$ It is an iterative sequence.

The function $x^2-\ln(x)-2=0$ has two forms of view ($x=\exp(x^2-2)$ - form 1) and ($x=\sqrt{\ln(x)+2}$ - form 2). There is a question, which form of view we must choose for two intervals. And we use the theorem.

The theorem. Let equation $x=f(x)$ has one root on an interval [a, b] and the conditions are executed:

1) Function $f(x)$ is defined and is differentiated on [a, b];
2) There is such real value q, that $|f'(x)|\le q<1$ for all $x\in$[a, b].

Then the iterative sequence $x_n=f(x_{n-1})$ (n=1, 2, …) converges at any initial value $x_0\in$[a, b].

The function $y(x)=x^2-\ln x-2$ is defined on the [0.1, 2.1]. It can be differentiated on the [0.1, 2.1]. Let's execute in Derive checking the other condition of the theorem. Let's check up the form of veiw (1) on the interval [0.13, 0.14]:

```
Author: EXP(x^2-2)
Calculus, Differentiate, x,1: DIF(EXP(x^2-2),x)
Simplify: 2*x*#e^(x^2-2)
Manage, Substitute, x, 0.13: 2*(0.13)*#e^((0.13)^2-2)
Simplify: 13*#e^(-19831/10000)/50
approXimately: 933/26071
```

From the fact, that the value of module by derivative from function $\exp(x^2-2)$ on the interval is less than 1, we conclude that for the interval [0.13, 0.14] it is possible to choose the form of view 1.

Let's check up in Derive the form of view 2 on the interval [1.56, 1.57].

```
Author: SQRT (LN (x) +2)
Calculus, Differentiate, x, 1: DIF (SQRT (LN (x) +2), x)
Simplify: 1 / (2*x*SQRT (LN (x) +2))
Manage, Substisute, x, 1.56: 1 / (2* (1.56) *SQRT (LN (1.56) +2))
approXimately: 4469/21801
```

The form of view 2 is chosen for the interval [1.56, 1.57] of another root as the value of module by derivative from function $\sqrt{\ln x+2}$ on this interval is less than 1.

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In view of these facts we shall fill up the table. In the cells E2, F2 we shall enter the texts "X0", "X1". Into the cell E3 we shall enter the number 0.13, and into the cell F3 the formula "=exp (E3^2-2)", in which we shall receive the value of function $y(x) = \exp(x^2 - 2)$ in the point 0.13.

The algorithm of iterative method

The algorithm consists of two actions.

Action 1. Let's make a copy of value of the current cell (F_i) and through function of the menu Edit a Special insert, Value we shall copy a value from a cell F_i into a cell E_{i+1}.

Action 2. Let's copy the formula "=exp (x^2-2)" from a cell F_i into a cell F_{i+1} dragging the mouse.

	A	B	C	D	E	F	G
1	*The table of values*				*Iterative method*		
2	X	Y	X	Y	X0	X1	*Checking*
3	0,1	0,312585	0,1	0,312585	0,13	0,137642	0
4	0,2	-0,35056	0,11	0,219375	0,137642	0,137924	0
5	0,3	-0,70603	0,12	0,134664	0,137924	0,137934	0
6	0,4	-0,92371	0,13	0,057121	0,137934	0,137935	0
7	0,5	-1,05685	0,14	-0,01429	0,137935	0,137935	1
8	0,6	-1,12917	0,15	-0,08038			
9	0,7	-1,15333	0,16	-0,14182	X0	X1	
10	0,8	-1,13686	0,17	-0,19914	1,56	1,563549	0
11	0,9	-1,08464	0,18	-0,2528	1,563549	1,564276	0
12	1	-1	0,19	-0,30317	1,564276	1,564424	0
13	1,1	-0,88531	0,2	-0,35056	1,564424	1,564454	0
14	1,2	-0,74232	X	Y	1,564454	1,564461	0
15	1,3	-0,57236	1,5	-0,15547	1,564461	1,564462	0
16	1,4	-0,37647	1,51	-0,13201	1,564462	1,564462	1
17	1,5	-0,15547	1,52	-0,10831			
18	1,6	0,089996	1,53	-0,08437			
19	1,7	0,359372	1,54	-0,06018			
20	1,8	0,652213	1,55	-0,03575			
21	1,9	0,968146	1,56	-0,01109			
22	2	1,306853	1,57	0,013824			
23	2,1	1,668063	1,58	0,038975			

Fig. 5. The iterative method on a sheet

At first i=3. We repeat actions 1-2 while values in cells E_i and F_i do not become "identical". How to check up this identifiability?

We use the definition of a limit. Into the cell G3 we shall enter the formula "=if (abs (E3-F3) < 0.0001; 1; 0)". Let's copy the formula in the cell G4 and further, together with a copy of a cell Fi (action 2). Then the value "1" in a cell G_i will be a signal of the end of an iterative process.

According to these in the cells E9, F9 we shall enter the texts "X0", "X1". Into the cell E10 we shall enter the number 1.56, and in the cell F10 – the value of function $y(x) = \sqrt{\ln x + 2}$ according to the formula "=sqrt (log(E10) +2))". The algorithm from two specified actions is carried out. Also into the cell G10 we shall enter the formula "=if (abs (E10-F10) < 0.0001; 1;

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0)”, we shall copy it downwards. The value "1" in some cell G_i will be a signal of the end of an iterative process.

Thus, we have received the numerical values of roots: x_1=0, 137935, x_2 = 1.564462 (Fig. 5). But we see that at realization of calculations of iterative method many actions need to be carried out manually. Is it possible to automate process of calculations?

4) Automation of calculations

Let's consider a way of automation of calculations on the example of using a secant method for finding roots of algebraic functions.

The basic idea of the secant method. Let on an interval [a, b] there is a root of function f(x), and x_0, x_1 are the different points on this interval. The formula is applied recurrent to finding the following approach to a root:

$$x_i = x_{i-1} - y(x_{i-1})\frac{x_{i-1} - x_0}{y(x_{i-1}) - y(x_0)}, \text{ where } i=2, 3, \ldots.$$

Let's write down the program of a secant method in VBA language. We consider, that the reference value of a root is in the cell C21 (1.56), and the result is in the cell F21. The variable TOL will determine accuracy of calculations. The program is written down as procedure, which is connected to user object CmdSek (Command Button).

```
Private Sub CmdSek_Click ()
TOL=0.01
x = [c21]
If Abs (y (x)) < TOL Then
Root=x
Else
If x=0 Then h=TOL Else h=x*TOL
x0=x: x1=x+h
Do
Root=x1 - y (x1)*(x1 - x0) / (y (x1) - y (x0))
x0=x1: x1=Root
Loop Until Abs (y (x1)) < = TOL
End If
[f21] =Root
End Sub
```

The function y (x) is used in the procedure. It is necessary to write the module in VBA language:

```
Function y (x)
y=x ^ 2 - Log (x) - 2
End Function
```

Then, to execute procedure CmdSec_Click (), we shall press the button “Secant Method”. We see the result in the cell F21. It is a root on the interval [1.56, 1.57]. For calculating a root on the interval [0.13, 0.14] it is necessary to modify the program: to enter the line x = [c7] instead of line x = [c21] and the line [f20] =Root instead of line [f21] =Root and to cause the procedure once again.

We shall receive the table with the results in the cells F20:F21.

The secant method	
x1=	0,137727
x2=	1,564438

The homework problems. To solve the equations analytically in the Derive package. To execute the partition of roots in Excel and use numerical methods - iterative and secant - for finding roots.

1.1) $f(x) = 0{,}5^x - (x-2)^2 + 1$

1.2) $f(x) = (x+2)\cdot \log_2(-x) + 1$

1.3) $f(x) = 2x^3 - 9x^2 - 60x + 1$

1.4) $f(x) = (x-2)^2 2^x - 1$

1.5) $f(x) = x^2 - 20\cdot \sin(x)$

1.6) $f(x) = 3x^4 + 4x^3 - 12x^2 - 5$

Laboratory work 2. To calculate the definite integrals:

$$\int_{0.8}^{1.6} \frac{dx}{\sqrt{2x^2+1}} dx$$

$$\int_{1.2}^{2} \frac{\lg(x+2)}{x} dx$$

1) Use of the Derive package

a) In a Derive package is feasible the following actions for a finding of indefinite integral for the first function:

Author:
1/SQRT (2*x^2+1)
Calculus, Integrate, x
INT (1/SQRT (2*x^2+1), x)
Simplify:
SQRT (2) *LN (SQRT (2*x^2+1) +SQRT (2) *x)/2

Thus, the analytical formula is found $\sqrt{2}\,\dfrac{\ln(\sqrt{2x^2+1}+\sqrt{2}x)}{2}$

b) Now we shall find the definite integral:

Author:
1/SQRT (2*x^2+1)
Calculus, Integrate, x, 0.8, 1.6
INT (1/SQRT (2*x^2+1), x, 0.8, 1.6)
Simplify:
SQRT (2) *LN (-12*SQRT (34)/25+8*SQRT (114)/25+3*SQRT (969)/25-64/25)/2
approXimately:
20484/49601

Thus, for the first integral the Derive package gives an answer in analytical and numerical forms.

The next expression gives the decision:

$$\sqrt{2}\ln(-12\frac{\sqrt{34}}{25}+8\frac{\sqrt{114}}{25}+3\frac{\sqrt{969}}{25}-\frac{64}{25})/2$$

And its numerical value: 0.412976

c) For the second integral Derive gives the answer only in numerical form.

Author:
LOG(x+2)/x
Calculus, Integrate, x, 1.2, 2
INT(LOG(x+2)/x,x,1.2,2)
Simplify:
INT(5*LN(x+16/5)/(5*x+6),x,0,4/5)
approXimately: 17323/26715
The numerical value of integral: 0,648437

2) Use of the Excel package. It is known, that the value of integral is the area S of a curvilinear trapezoid, which is limited by the lines y=0, x=a, x=b (a < b) and by the graph of function $y = f(x)$ on an interval [a, b]; the function has on this interval only non-negative values. In our example the function $y(x) = \lg(x+2)/x$ is considered, its diagram is presented in a fig. 6.

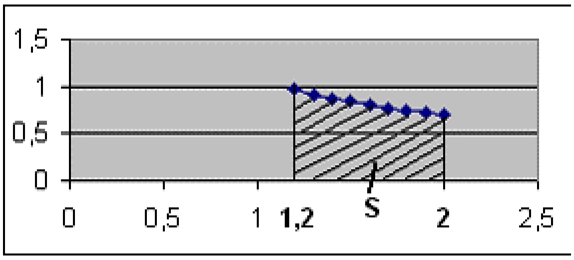

Fig. 6. The geometrical interpretation of definite integral

The diagram of function $y(x) = \lg(x+2)/x$ is constructed in Excel according to the table (fig. 7).

	A	B
1		
2	X	Y
3	1,2	0,969292
4	1,3	0,918402
5	1,4	0,874125
6	1,5	0,835175
7	1,6	0,800584
8	1,7	0,769608
9	1,8	0,741667
10	1,9	0,716303
11	2	0,693147

Fig. 7. The table of values of the function $y(x) = \lg(x+2)/x$

If we draw in the trapezoid (fig. 6) the stepped figure that consists of rectangular trapezoids with heights $[x_i, x_{i+1}]$ and bases y_i, y_{i+1} (fig. 8), then the area S of a trapezoid will be equal to the sum of the areas of these step figures.

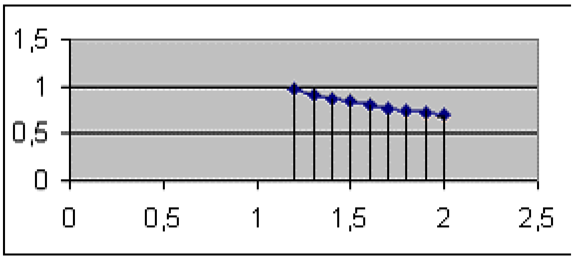

Fig. 8. The stepped figure

The calculation of the area of a curvilinear trapezoid is the basis of a trapezoid rule for finding the definite integrals.

3) The trapezoid method. Let's consider idea of the method. Let interval of integration [a, b] for function f (x) is broken on **n** parts with a step $h = \frac{b-a}{n}$. Then

$$\int_a^b f(x)dx \approx h(\sum_{i=1}^{n-1} f(x_i) + (f(x_0) + f(x_n)/2), \text{ where } x_i = a + ih, (i = 0,1,2,...,n).$$

Thus, for the table in a fig. 7 we find the sum (in the cell B12) of values from the cells B4:B10, then we add to them the sum (in the cell B13)of values from the cells B3 and B11, and we find the product (in the cell B14)of the value from the cell B13 cell and value h. The results are in a fig. 9.

	A	B
1	**Trapezoid method**	
2	**X**	**Y**
3	1,2	0,969292
4	1,3	0,918402
5	1,4	0,874125
6	1,5	0,835175
7	1,6	0,800584
8	1,7	0,769608
9	1,8	0,741667
10	1,9	0,716303
11	2	0,693147
12	B12=Sum(B4:B10)	5,655864
13	B13=B12+(B3+B11)/2	6,487084
14	B14=B13*0,1	0,648708

Fig. 9. The results of calculations by the trapezoid method

The result value is in the cell B14. It is equal to 0.648708. This value differs from the exact value 0.648437, which was received in the Derive package. If we shall count up the value of integral with a step of calculation 0.05, we shall receive the value 0.648505. The calculation of the value of the integral with the step 0.05 may be executed independently in the cells C3: D21.

4) **Calculation of an error**. The error of the trapezoid method is equal to

$$R_n(f) = -\frac{(b-a)^3}{12n^2} f''(\varsigma), a \le \varsigma \le b$$

For calculation of an error value it is necessary to perform an investigation of the function $f''(x)$. In the Derive package is feasible the following calculations:

Author:
LOG(x+2)/x
Calculus, Differentiate, x, 2:
DIF(LOG(x+2)/x,x,2)
Simplify:
2*LN(x+2)/x^3-(3*x+4)/(x^2*(x+2)^2)

Manage, Substitute, 1.2
2*LN(1.2+2)/(1.2)^3-(3*(1.2)+4)/((1.2)^2*(1.2+2)^2)
approXimately:
29001/34906
Manage, Substitute, 2
2*LN(2+2)/2^3-(3*2+4)/(2^2*(2+2)^2)
approXimately:
2435/12794

From the diagram of function $f''(x)$ on the interval [1.2, 2] we see (fig. 10) that the function decreases monotonously.

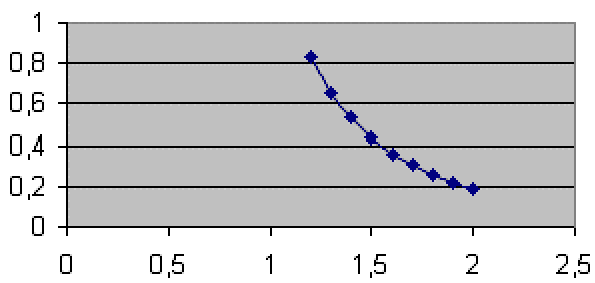

Fig. 10. The diagram of function $f''(x)$

The diagram is constructed by the data from the table in a fig. 11. The values of function $f''(x)$ change from 0.830831 up to 0.190324.

x	y''
1,2	0,830831
1,3	0,657614
1,4	0,530055
1,5	0,433988
1,6	0,360217
1,7	0,302594
1,8	0,256903
1,9	0,220185
2	0,190324

Fig. 11. The table of values of function $f''(x)$

Let's count up in the cells on a sheet the error values for n=9 and n=18:

N	Error
9	0,000438
18	0,000109

Fig. 12. The error values

Let's compare the results of calculation of integral, which were received in Excel, with the results, which were received in Derive.

N	Derive	Excel	Error
9	0,648437	0,648708	0,000271
18	0,648437	0,648505	6,8E-05

Fig. 13. The calculated values of errors

From the data on fig. 12-13 we judge correctness of results of calculations. The average accuracy of calculations on a trapezoid method is equal to 0.648708-0.648505≈0.0002 (the accuracy with three decimal points).

Thus, the work was executed through the following stages:

1. Calculation of definite integral in the Derive package;
2. Construction of the table of values of function and its diagram in the Excel package;
3. Calculation of definite integral by a trapezoid method;
4. Finding an error of calculations.

The homework problems: To calculate the integrals under the formula of trapezoid with three decimal points:

$2.1)\ \int_{0,2}^{1} \frac{tg(x^2)}{x^2+1}dx$ $\quad$ $2.3)\ \int_{0,4}^{1,2} \frac{\cos(x^2)}{x+1}dx$ $\quad$ $2.5)\ \int_{0,2}^{1.0} (x+1)\cos(x^2)dx$

$2.2)\ \int_{0,8}^{1,2} \frac{\sin(2x)}{x^2}dx$ $\quad$ $2.4)\ \int_{0,15}^{0,63} \sqrt{x+1}\lg(x+3)dx$ $\quad$ $2.6)\ \int_{1,6}^{3,2} \frac{x}{2}\lg(\frac{x^2}{2})dx$

Laboratory work 3. To solve the system of linear algebraic equations and the linear programming task.

1) To find a solution of the system of linear algebraic equations.

It is necessary to find a solution of the system, where $A = \lfloor a_{ij} \rfloor$ - the matrix $(m * m), \det(A) \neq 0$. The method of solving belongs to a class, that gives the exact solution. It is the Gauss method of exception. We shall consider one possible realization of the method.

The Udodov's problem (after the novel of A.Chekhov "The Coach"). The merchant has bought 138 arshins (=28 inches) of black and dark blue cloth for 540 rubles. How many arshins he has bought of that and other cloth, if dark blue costs 5 rubles for arshin, and black - 3 rubles for arshin?

In the Derive package a way of solution is the follows:

```
Author
[x+y=138, 5*x+3*y=540]
soLve
[x=63, y=75]
```

The result: the merchant has bought x=63 of black cloth and y=75 of dark blue cloth.

Let's consider the possibilities of the Excel package for solution of the problem. On a sheet of a spreadsheet (see fig. 14) is entered: in the cells B4:B5 – the values 5 and 3 (cost of dark blue and black cloth for arshin), and in the cells B6:B7 – the values 540 and 138. The decision will be appeared in the cells B9:B10.

	A	B	C
1		Example 1	
2			
3		The Udodov's problem	
4	Blue cloth	5	Rubles/Arshin
5	Black cloth	3	Rubles/Arshin
6	Total amount	540	Rubles
7	Total cloth	138	Arshin
8		An answer	
9	Has bought blue cloth		Arshin
10	Has bought black cloth		Arshin

Fig. 14. Representation of the problem in Excel

Now we need to solve the system of linear equations: B6=B4*B9+B5*B10 (Total amount); B7=B9+B10 (Amount of cloth), where the values in the cells B9, B10 are unknown.
Let's choose the Solver - a built-in procedure in Excel (by the command Service, Solver).
In the Solver dialog box we shall enter four restrictions B6=B4*B9+B5*B10, B7=B9+B10, B9:B10=integer, B9:B10>=0. Variables, that are stipulated solution, there are in two cells B9:B10. They are specified in the item Changing cells.
The restrictions for the program Solver are entered in the special window Restriction.
Except for restrictions, in a dialog box the Search of solution is necessary to enter criterion function and to specify what value is desirable for receiving in the target cell - maximal, minimal or zero. The author [15] recommends to enter fictitious criterion function "=B9*B10*1E-100", and we shall write down it in the cell B15 (Fig. 15). We indicate, that is necessary to receive the least value of this fictitious criterion function. Further, after running the command Execute, on a sheet Excel in the cells B9:B10 we shall receive the solution the value 4,725E-97. Thus, [B9]=75, [B10]=63.

	A	B
11		The criterion function is not present
12		Restrictions
13		b7=b9+b10
14		b6=b4*b9+b5*b10
15	Fictitious criterion function	4,725E-97

Fig. 15. The additional data

It is possible to prepare the report according to the results of calculations. The report enables to check up the target data (contents of cells, restriction, and the value of criterion function received).

2) To solve the task of linear programming

Typical problem of linear programming is the next: to find a maximum of linear function $\sum_{j=1}^{n} c_j x_j$ under certain conditions $\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,\dots,m, x_j \ge 0, j = 1,\dots,n,$ where c_j, a_{ij}, b_i are the numbers given. The problems of linear programming are the mathematical models of many problems of the technical and economic contents, including the transport's problems. For 2D-space the domain of search and interpretation of a method of search is possible to represent directly by well-known simplex method.

Let's consider an idea of a simplex method and its geometrical interpretation for the decision of the problem.

The manufacture problem (L.Laurier [12]). The workshop on manufacture of non-standard furniture has received simultaneously two orders. The manufacturing one complete set of the first type needs 11 samples and 3 working hours, for the second type needs 4 samples and 5 working hours. At disposal of the foreman there are 46 samples and 32 hours of time. Besides he knows that in the first order he will receive 14 francs for each thing, and in the second - only 7 francs. How he should organize his work to receive the maximal income?

If x and y are the quantities of subjects of furniture made on each of the orders, we receive three conditions:

on furniture - 11x+4y <=46

on time - 3x+5y <=32

for maximum - Z=14x+7y (the criterion function).

Let’s use the auxiliary variables t and u, with the help of which our system can be written down in the following view:

11x+4y+t=46

3x+5y+u=32

MAX [14x+7y]

The idea is, that, proceeding from the obvious decision x=0, y=0, t=46, u=32, to improve it at the expense of the greatest possible increase of values by the variables, which gives the greatest increase of the income function z.

In the Derive package a way of solution is the follows:

Author:

[11*x+4*y+t=46, 3*x+5*y+u=32]

soLve :

[x=(-(5*t-2*(2*u+51))/43), y=(3*t-11*u+214)/43]

We can’t find such t and v, that give MAX [14x+7y].

The students use Solver (the Excel package) for solution of the problem. They find the answer: x=3, y=3.

The homework problem is the transport’s problem.

A city has two warehouses of TV sets and two shops. It is necessary daily to take out from the first warehouse 50 TV sets, and from another - 70. The first shop thus receives 40 TV sets, and another - 80. It is a question, how it is necessary to organize work of transport, that the expenses for transportations would be the minimal. The cost of transportations: from a warehouse 1 into shop 1 - 1200 rubles; from a warehouse 1 into shop 2 - 1600 rubles; from a warehouse 2 into shop 1 - 800 rubles; from a warehouse 2 into shop 2 - 1000 rubles.

Let's try to solve the problem in the Derive package:

Author:

[x+y=50, z+t=70, x+z=20, y+t=100]

soLve:

[x=@1, y=50-@1, z=20-@1, t=@1+50]

Thus, we have found the decision with the parameter @1. The presence of parameter is caused by that we have uncertainty such as 0/0: the determinants of system are equal to zero. Really, the determinant of system in the Derive package is calculated as following:

Author:
DET ([[1, 1, 0, 0], [0, 0, 1, 1], [1, 0, 1, 0], [0, 1, 0, 1]])
Simplify:
0

It is equal to zero.

The determinant at unknown x is equal to zero:

Author:
DET ([[50, 1, 0, 0], [70, 0, 1, 1], [20, 0, 1, 0], [100, 1, 0, 1]])
Simplify:
0

If we shall make check and for other two determinants, we shall be convinced, that they too are equal to zero.

How to find the decision of the system [x=@1, y=50-@1, z=20-@1, t=@1+50] such, that it provided the least cost of transportations? The Derive package has no the built-in function for solving the problem. Let's take advantage of Excel. The students use already known procedure Solver and receive an answer.

Laboratory work 4. To solve the mathematical modeling problems.
The problems for this laboratory work are the traditional by their contents for a course of Calculus. The students use the problems from the manual [6]. They also consider the problems from the papers [11, 13, 16]. The essence of the problems: 1) about revealing the optimum sponsor's help to the students [13]; 2) about the greenhouse effect [16]; 3) about the growth of the population on the globe from the beginning of XX century till its end [11], etc.
In the **first problem** students are presented with the following scenario.

A generous benefactor has agreed to sponsor them for the first two years of their studies. The benefactor has proposed two sponsorship schemes and the student must choose between them. *Scheme A.* In the first month the sponsorship amount is 500 rubles and this amount rises by 500 rubles each month, so in the second month the amount is 1000 rubles, in the third month 1500 rubles and so on. *Scheme B.* In the first month the sponsorship amount is 0,01 ruble and this amount is doubled each month, so in the second month the amount is 0,02 rubles, in the third month – 0,04 rubles and so on.

It is necessary to reveal, what of ways of the help will be best for two years of sponsorship.
Students try to solve the problem in the Derive package: they know that the sum of the members of an arithmetic progression is equal to $S_A = \frac{n(a_1 + a_n)}{2}$, where $a_n = a_1 + (n-1)d$, d is the difference: the sum of the members of a geometrical progression is equal to $S_B = \frac{b_1(q^n - 1)}{q-1}$, where q is the denominator of a progression. As a_1=500, d=500, b_1=0.01, q=2, we shall receive the following.

Author:
a=12*(2*x+23*x) the sum of an arithmetic progression, x is the difference
Simplify:
a=300*x
Author:
b=0.01*(2^24-1) the sum of the geometrical progression
Simplify:
b=3355443/20
Build:
3355443/20=300*x
solve:
x=1118481/2000
approXimately:
x=559,24 The sponsor's help will be identical, if the difference of an arithmetic progression will be equal to 559.24.

The problem "Modelling the Greenhouse Effect".

The greenhouse effect refers to the gradual global warming due to an increased concentration of certain gases in the atmosphere. The data are represented in the table in a fig. 16.

Year	Raising
1860	0,00
1880	0,01
1896	0,02
1900	0,03
1910	0,04
1920	0,06
1930	0,08
1940	0,10
1950	0,13
1960	0,18
1970	0,24
1980	0,31
1988	0,40

Fig. 16. The data for the problem

Let's search for approximation function in one of the following representations: $F(x,a,m) = ax^m, F(x,a,m) = a\exp(mx), F(x,a,b) = a\ln x + b$. The search of approximation as first two functions does not result in success. Let's try to test logarithmic function as approximation. Let's receive the result. The same result turns out, when we shall try in Excel to construct linear trend (fig. 17):

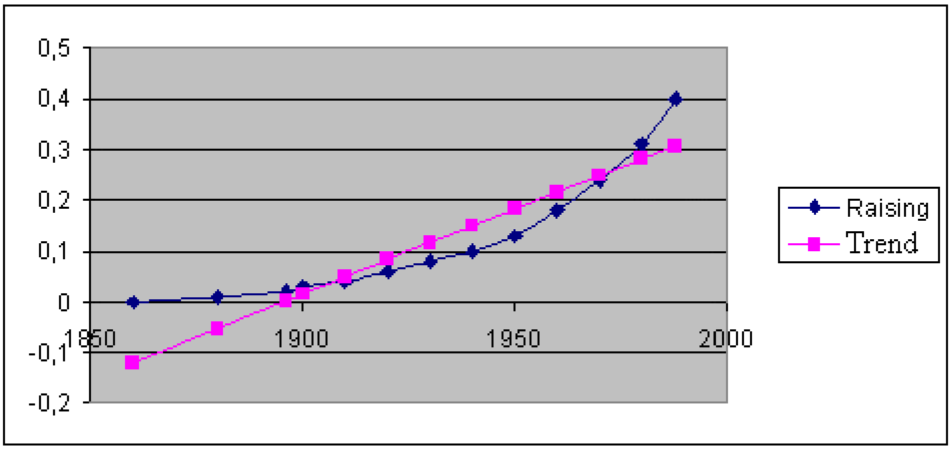

Fig. 17. The linear trend line

But a polynomial trend is most suitable to our situation (see fig. 18).

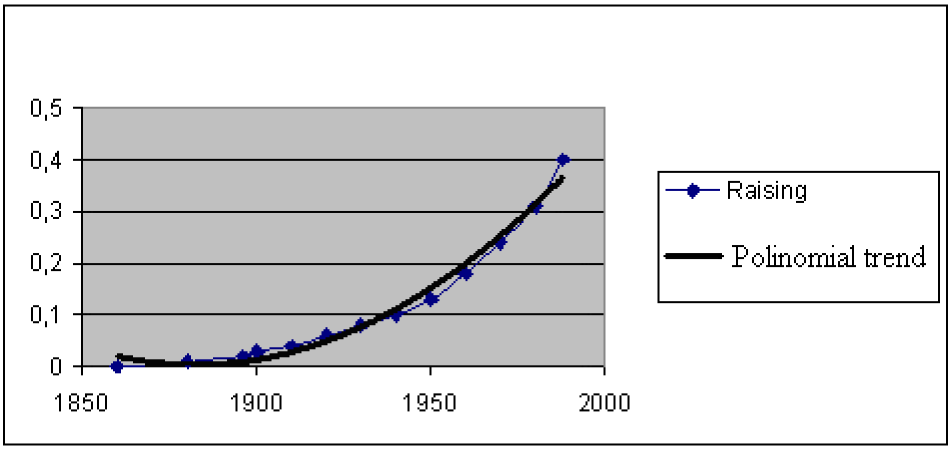

Fig. 18. The polynomial trend line

The homework problems: The students' task is to create mathematical model and use it to predict:

(a) the increase in the Earth's average temperature by the year 2060 since its 1860 value.

(b) when the Earth's temperature will be 8^0 C above its 1860 value.

4. Technologies of training

The preparation of the students was carried out at the faculty of physics and mathematics in the Kharkov G.S.Skovoroda Pedagogical University. The training was carried out in different groups of the students by the specialties: "Mathematics - Informatics" (4-th year of training), "Mathematics - English language" (3-rd and 4-th years of training) "Informatics - English language" (5-th year of training) during 2001-2002 - 2003-2004 academic years. The students had only about 30 hours on the course of training. Some difficulties were observed at mastering by means of programming by language VBA and use of numerical methods by the students of a specialty "Mathematics - English language".

Totally, the analysis of the mentioned above four subject it is possible to consider sufficient, that in a deadline of training to carry out real steps on ways of mastering by the students by modern information technologies. Each of sections the students study from one about four weeks. The studying begins with the compressed review of a theoretical material and analysis of examples. Then the students study in a computer class, where they use the calculator, the

Derive package, Microsoft Excel with the elements of VBA. The students work in groups. They need to execute, at least, part of each task from home work in a class. The work in groups allows the students to discuss problems among themselves and to use the help of the teacher if necessary.

5. Conclusions

As a whole at use of new information technologies at study of mathematics the students get ability to reflect, habits to reveal persistence and interest, conviction that they further can successfully solve such difficult questions, as the problem solving.

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