CONTS

MODELLING AND DYNAMIC GEOMETRY IN TEACHING OF MATHEMATICS

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ABSTRACT

Mathematical modelling as the process of finding and validating a mathematical model that explains a phenomenon you want to study with the main purpose of understand it, and apply this understanding in a more general way, is an excellent vehicle for the teaching of mathematics from Upper Middle School (pupils between 15 and 17 years old) to College since it gives meaning to the mathematical content we are teaching, covering the gap between the mathematical knowledge acquired by our students in non school environments and the formal knowledge we want to teach in school. Mathematical modelling also awakes the student interest on branches of human knowledge other than mathematics. Now, the use of Dynamical Geometry software like The Geometer's Sketchpad (V4.0) puts on an additional attraction in the students perspective, an attraction that you can not achieve with others computerized media.

The main objective of this paper is to reflect on the pros and cons of this approach through the development of a modelling activity, and to take into account the potentialities of DG software in the teaching of mathematics while commenting some results the author has gotten with his own students.

> "Nor the mathematical information nor the demonstration before a class foster a real mathematical knowledge in our students. They learn by heart the algorithms and the equations with the only purpose of get a good score in a test, or to please their teacher and be rewarded with a good note."

Introduction

It is known, since ancient times, that in order to learn some trade it is mandatory that the learner goes into a workshop where that trade is practiced, and learn that trade in a practical way since the begining. Writters made themselves writting; soccer players learn playing soccer; dancers form themselves dancing; shoemakers learn to make shoes making shoes and observing the experts making shoes. Then, why not learn Mathematics doing mathematics?

Mathematics were born as a problem solving tool, therefore, it is logic to think that, if we want to teach Mathematics, we must face the student with situations were they must to develop a solving strategy and, at the same time, put into play the mathematical knowledge she posses and acquire new knowledge. In this sense, our math classrooms should be a workshop where the student learns to solve problems in an effective way, using Mathematics correctly.

People use Mathematics in their daily lives and, in most cases, they do not notice that they are using them. At school, in rare ocassions teachers refer to the ways in which we use math in daily life, and in others fields of human knowledge as Physics, Chemistry, Engineering, Biology, and so forth. In these fields, mathematical models play a fundamental role in problem solving, and in the explanation and forecasting of phenomena. So, the adaptation and use of mathematical models in the classroom can be an excellent vehicle by which our students apply their mathematical knowldege and acquire new one.

Mathematical modelling and the teaching of Mathematics

This teaching proposal is based on the development of mathematical models (analytical or geometrical) to solve certain problemic situations. It is the student who develops such models and uses her knowledge to explain the accuracy and limitations of the models. This developing process is what I call **Mathematical Modelling.** This proposal in not centered in teaching to model phenomena; it is the search of mathematical models to explain and forecast phenomena that makes the student put in action her mathematical knowledge and foster new one.

According to Dreyer (1993), the mathematical modelling is a process that can be divided in different stages:

- **Identification.** The question to answer must be clearly posed. The underlying mechanism in the phenomena must be identify as accurately possible. The problem must be formulated in words ans the relevant data must be documented.

- **Assumptions.** The problem must be analysed in order to decide which factors are important and which factors must be ignored, so that we can state realistic assumptions.
- **Construction.** The problem must be translated into a mathematical language. In general, this language appears as a collection of tables, graphs, and functions. The word problem is transformed, after the proper identification of variables, into an abstract mathematical problem.
- **Analiysis.** The mathematical problem is solved so that the unknown variables are expressed in terms of known quantities. These quantities are analysed to obtain information about parameter.
- **Interpretation.** The solution to the mathematical problem must be compared to the original word problem to see if it makes sense in the real situation.
- **Validation.** Verify if the solution agrees with the data of the real problem. If the results of this verification were not satisfactory, return to the word problem and re-analyse it. If it is the case, modify the assumptions or add new ones, and construct a new model.
- **Instrumentation.** If the solution, at a reasonable extent, agrees with the data, then the model can be used to predict future events, to help planning course actions, etc.

In the teaching of Mathematics, each of those modelling stages are of great help to the development of the mathematical knowledge and skills we want our students acquire and practice. When we face our students to situations in which they have to consider a phenomenon (natural or not), to construct assumptions about it, and engage in the construction of a mathematical model that, reasonably, replicate the phenomenon, we are making them put into action the proper mathematical knowledge that they already have; and, as far as this knowledge is not sufficient to find a satisfactory answer to their quest, we are motivating our students to acquire new mathematical knowledge, regardless the fact that students are practicing their mathematical skills and mental abilities.

Besides, as teachers, we cannot make aside the use of technology, especially that designed specifically for the teaching of mathematics such as CAS calculators and Dynamic

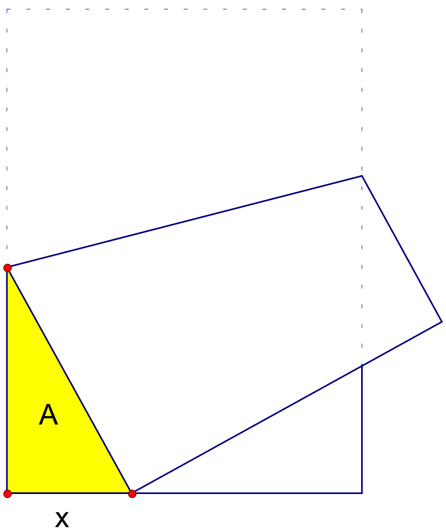
Geometry software packages. In particular, the use of the Dynamic Geometry software, *The Geometer's Sketchpad,* allows the student the development of the capacity to visualize the phenomenon, propose better mathematical models, and have several representations of the same model –graphical, algebraic, tabular, and pictorial– in the same window.

An example

The following activity was posed in a fourth semester mathematics course (on Algebra and Geometry) to students (16-17 years old) at the Colegio de Ciencias y Humanidades in Mexico City. This activity is part of the unit called "Polynomial Functions with Grade Upper Than Two", and its main objective is that students have an example of a situation in which this kind of functions emerge. First, the activity was developed whitout the aid of technology, except for an ordinary cientific calculator; then we used *Sketchpad* in the construction of a pictorial model that served as a starting point in the analysis of the situation and the verification of the previous results.

The bigest area triangle (Adapted from Murdock, Kamischke, and Kamischke; 1998)

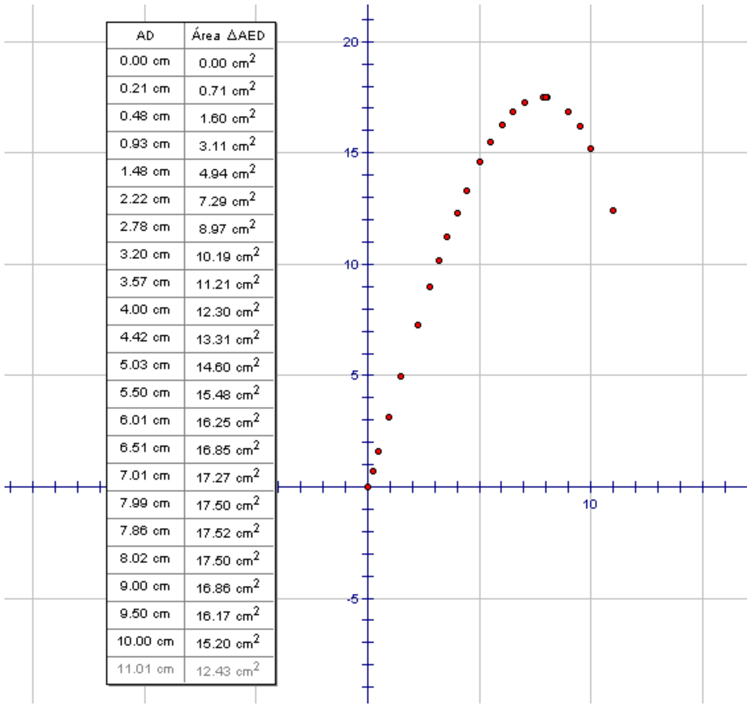
Take a paper sheet and fold it so the upper left corner touches the lower edge of the sheet (look at the figure below). Find the area of the triangle in the lower left corner of the folded sheet (the yellow right triangle in the figure).



- ¿Where, in the lower edge of the sheet, you must put the upper left corner in order that the right triangle has the maximum area?
- ¿Find the relationship between distance x, from the left edge to the right triangle vertex, and the area of the triangle, A?

Students solution

After the construction of the geometrical model of the sheet (that implies certain degree of geometrical knowledge), the students starts measuring distance x and calculating the triangle area, A. With these measurements they build a table (about 20 entries). Next, they plot the values on the table and, depending on the trend of the plotted points, they propose a function as an analytical model of the situation. The figure at right shows the table generated with *Sketchpad*, and the corresponding plot (assuming a 11×13.5 paper sheet).



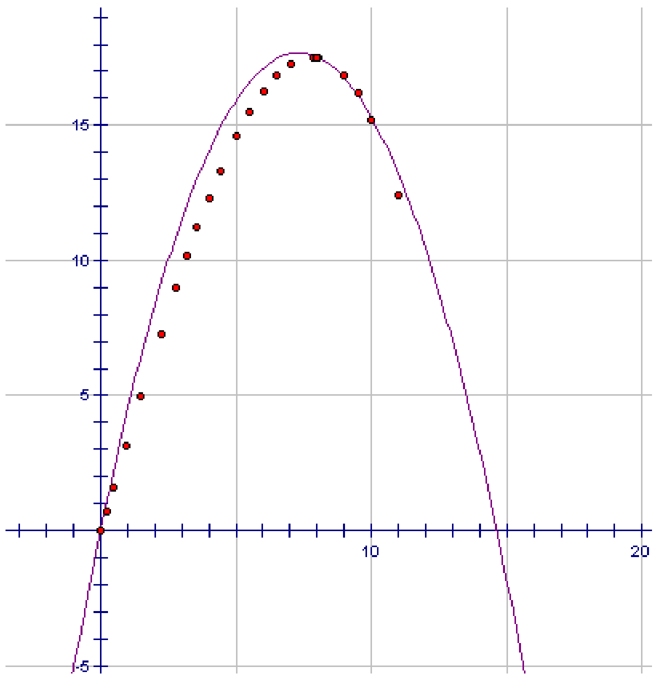
AD	Área ΔAED
0.00 cm	0.00 cm^2
0.21 cm	0.71 cm^2
0.48 cm	1.60 cm^2
0.93 cm	3.11 cm^2
1.48 cm	4.94 cm^2
2.22 cm	7.29 cm^2
2.78 cm	8.97 cm^2
3.20 cm	10.19 cm^2
3.57 cm	11.21 cm^2
4.00 cm	12.30 cm^2
4.42 cm	13.31 cm^2
5.03 cm	14.60 cm^2
5.50 cm	15.48 cm^2
6.01 cm	16.25 cm^2
6.51 cm	16.85 cm^2
7.01 cm	17.27 cm^2
7.99 cm	17.50 cm^2
7.86 cm	17.52 cm^2
8.02 cm	17.50 cm^2
9.00 cm	16.86 cm^2
9.50 cm	16.17 cm^2
10.00 cm	15.20 cm^2
11.01 cm	12.43 cm^2

From the analysis of the plot the students jumps into the conclusion that the x value for which the area is a maximum is about 7.86, with a corresponding area of 17.52.

In this case, students proposed a quadratic model for the relationship between area and x-distance, because the trend in the plot is similar to a parabola. In order to get a quadratic model, the students take three points from the plot, or the table, (I always recomend to take a point from the first part of the table, other from the middle part, and the last one from the last part of the table), and substitute them in the quadratic function equation ($y = ax^2 + bx + c$). They get a system of three linear equations with three unknowns whose solution may be found by any of the known methods (matrices and determinants included). The quadratic models students find have aproximately the following form

$$A = -.33x^2 + 4.83x$$

They plot this function in the same coordinate system as the table points, and get the following.

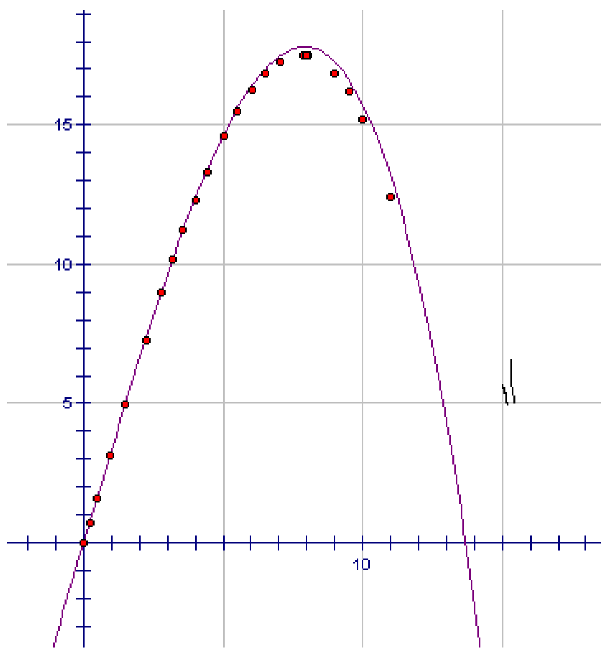


Students realize that the quadratic model is not a good one.

By teacher suggestion or by their own initiative, students try next a cubic model. To find it, they need to solve a 4×4 linear system. The function they find has an equation similar to the next one (the parameter values depend on the points taken and the number of significant digits they consider):

$$A = -0.018x^3 + 3.375x$$

Now, the plot has the form shown at the right; this form is a much better aproximation that the previous one (in fact, most of students fitted a more accurate curve because they considered four significant digits instead of three).



Conclusions

Besides the geometrical knowledge that students must put into action in constructing the geometrical Sketchpad model, the process that concludes with the response of the two questions in the activity implies that the student needs to know about polynomial functions, solving linear systems, curve fitting, plotting functions, etc. If one student doesn't know these mathematic concepts, she must learn them, becasue she has the need to do so. Not being the main learning object in the activity, these concepts become a working tool that most students should learn in order to find an answer to the questions. In my experience, students do not question why they must learn this or that concept, they just learn it so they can answer properly the questions. If they do not know what to do or how to use a concept, they ask the teacher or another classmate to explain.

With this teaching approach, the student involves herself in the problem solving activities in such a way that the teacher acquires the only roll of monitor and coordinator of the activities. With this approach, those teacher's demostrations before the class, where the dialogs and discussions have as unique interlocutor the blackboard, while students where assleep, or thinking in everything except mathematics, have become part of the past.

References

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