

# *Derive 6*

-

## A System for Teaching Students and Learning Mathematics

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**ACDCA**

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# Goal of CAS

CAS supports mathematicians in  
doing mathematics.

Proper use of CAS requires proper mathematical knowledge.

# PeCAS = Pedagogical CAS

CAS supports mathematicians in  
doing mathematics.

Proper use of CAS requires proper mathematical knowledge.

PeCAS supports teachers and students in  
teaching, learning, and doing mathematics.

Proper use of PeCAS requires minimal mathematical knowledge.

# ... what makes a CAS a PeCAS:

W Shakespeare:

"Nothing is either good or bad - only thinking makes it so."

→ "Computers are neither good nor bad teaching tools - only using makes them so."

V Kokol-Voljc:

"It is not the TOOL, but the USE OF THE TOOL which is or is not pedagogical."

Pragmatic definition:

A PeCAS is a CAS which makes the pedagogical use easier.

# Why we need PeCAS (i):

Change from DOS to Windows

caused many more people to use computers,  
because less expert knowledge was required.

Change from CAS to PeCAS

will cause more teachers and students to use computer maths.

# Casanova & Don Juan

**Giacomo Girolamo Casanova**

Italian adventurer & writer  
1752-1798

**Don Juan**

Spanish legend, used as hero  
in opera, play, and fiction

# Casanova & Don Juan

## Similarity:

Both were famous womanizers

## Difference:

Casanova wanted women's pleasure

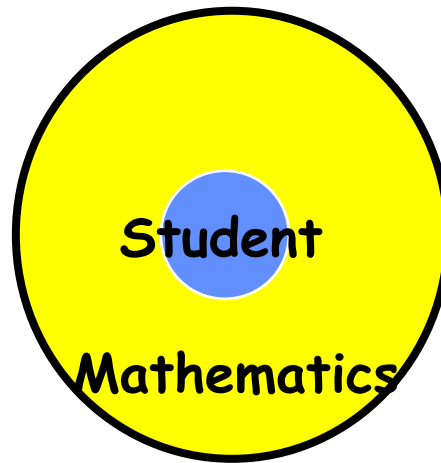
Don Juan wanted his pleasure

# There are two types of teachers:

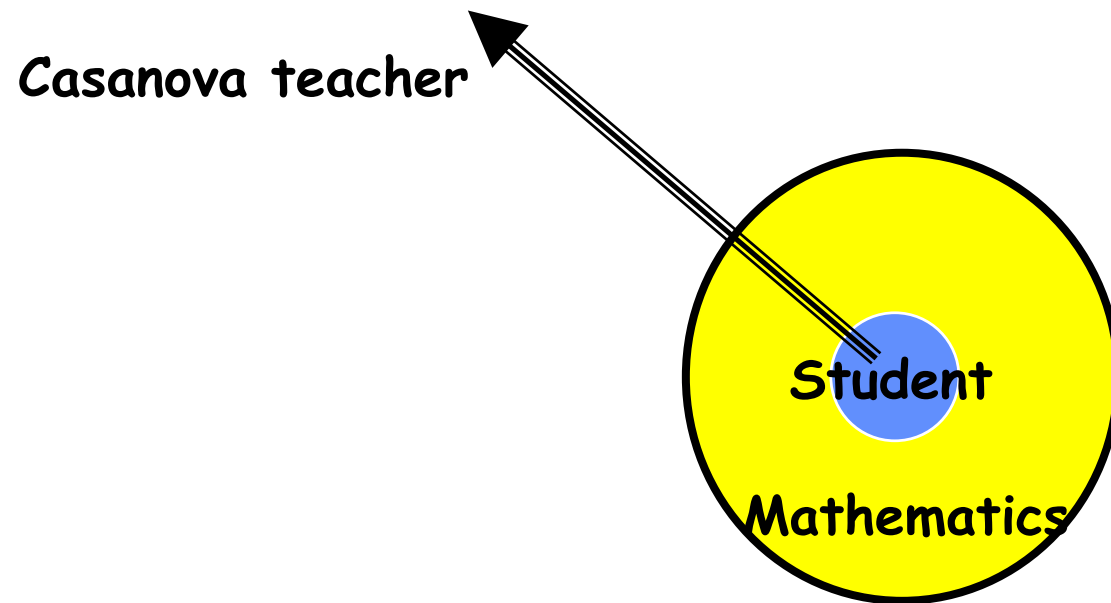
- Don Juan type teachers
- Casanova type teachers



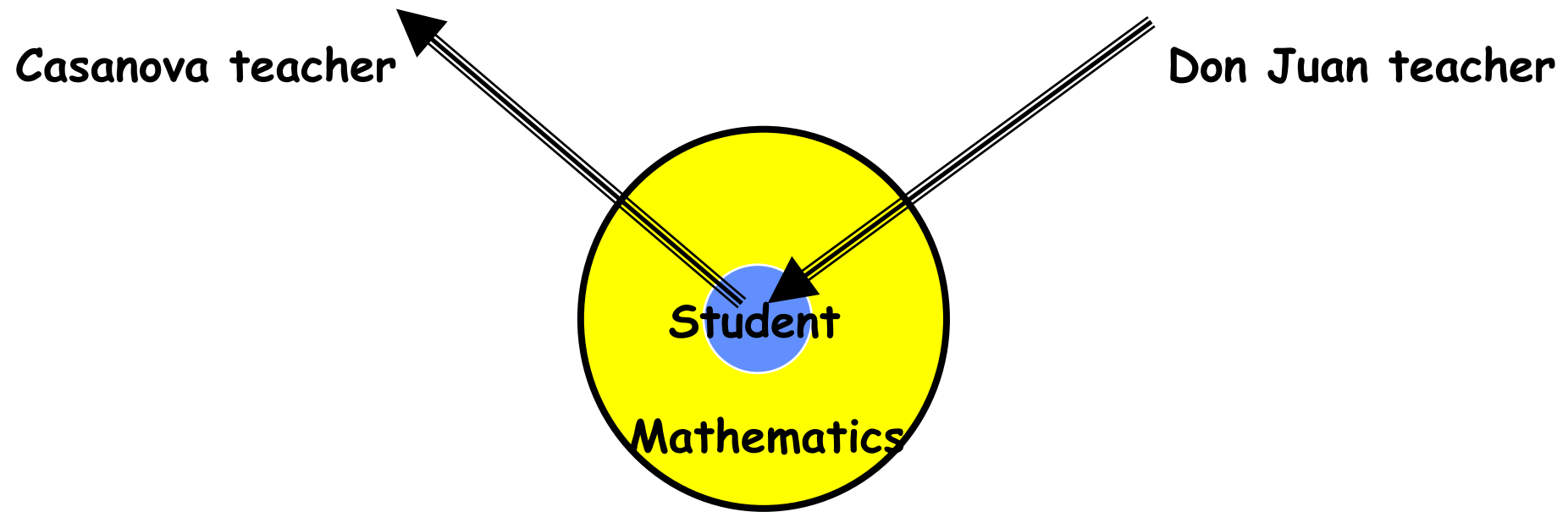
# Teaching ...



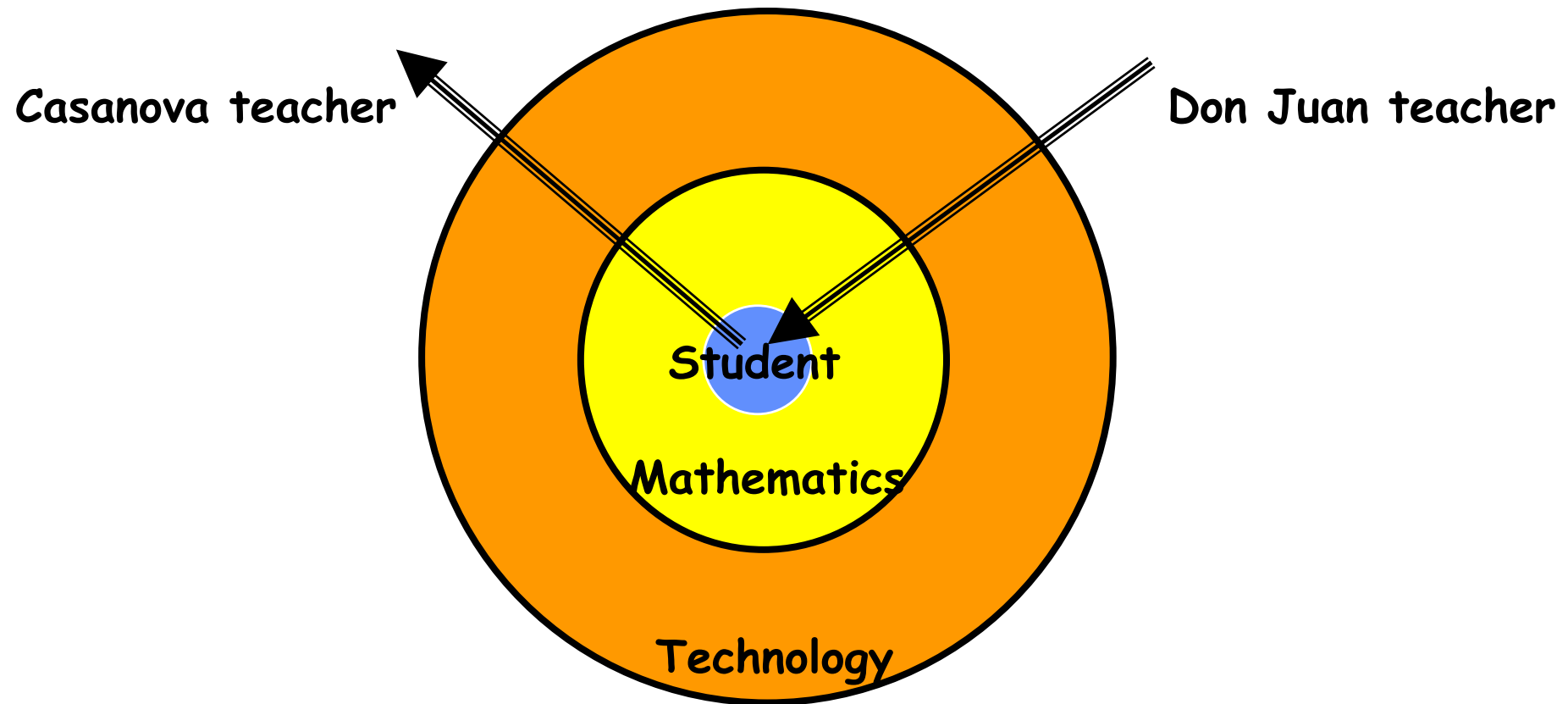
# Teaching ...



# Teaching ...



# Teaching ...



# Conclusion:

Don Juan teacher teaches mathematics  
(... some teach technology).

Casanova teacher teaches students.

I believe we should teach students.  
(i.e. we should be Casanovas).

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Don Juan teacher teaches mathematics  
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I believe we should teach students.  
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# Sheikh Zayed Bin Sultan Al Nahyan

President of the United Arab Emirates



*“Youth is the wealth of a nation.”*

# Star – The people's paper

(Monday 17 May 2004)

*“Teachers ... help the  
country to develop human  
capital, ...”*



# (1)

Look at how Derive simplifies  $\sin(10\pi/3)$ .

Look at how Derive differentiates  $5ax^3$ .

**(1) ... about accepting black boxes:**

Such a “black-box expert” may be frightening for many teachers.

“Must I believe all of this ?” 

(1) ... making black boxes white:

Ask Derive6 to explain how it obtained the result!

"Now I feel comfortable !" 

# (1) ... Display Steps:

Derive6's new Display Steps feature has a lot of good uses:

- user wants to look into the black box
- user wants to study subtleties of simplification
- student studies topic by observing an "expert"
- student studies topic by recognizing the steps an "expert" did  
(with display of rules switched off)

(2) ... your students use handhelds:

Connect Derive6 with TI-89 or Voyage200 calculators  
and exchange data.

Combine the best of two worlds, e.g.  
mobility with computational power & powerful user interface

**(3)** Again we start with an example:

Look at  $y = a \cdot x^2$ . How does  $a$  affect the graph?

### (3) ... in a paper&pencil environment:

Look at  $y=a.x^2$ . How does  $a$  affect the graph?

Student draws graph for  $a=1$ .

Student draws graph for  $a=2$ .

Student draws graph for  $a=...$

...

This takes a lot of time ...

... and for many students

this turns into a graph drawing exercise.

“What was this all about???”



(3) ... in a CAS (or GC) environment:

Look at  $y=a.x^2$ . How does  $a$  affect the graph?

Ask CAS to plot graph for  $a=1$ .

Ask CAS to plot graph for  $a=2$ .

Ask CAS to plot graph for  $a=...$

...

This takes only seconds. Or, even better:

Ask CAS to plot graph and use slider bar to change  $a$ .

"How easy to understand !"





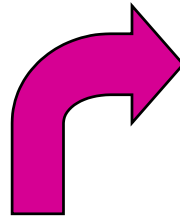
## (3) ... analysis (i):

How do we mostly learn?

- by making experiments
- by trial and error
- by doing and observing (J W Goethe)

**(3)** ... analysis (ii):

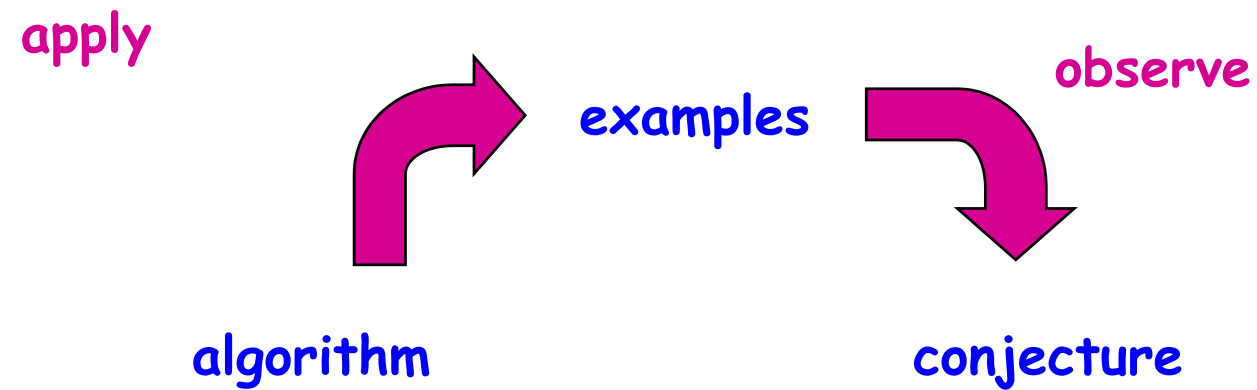
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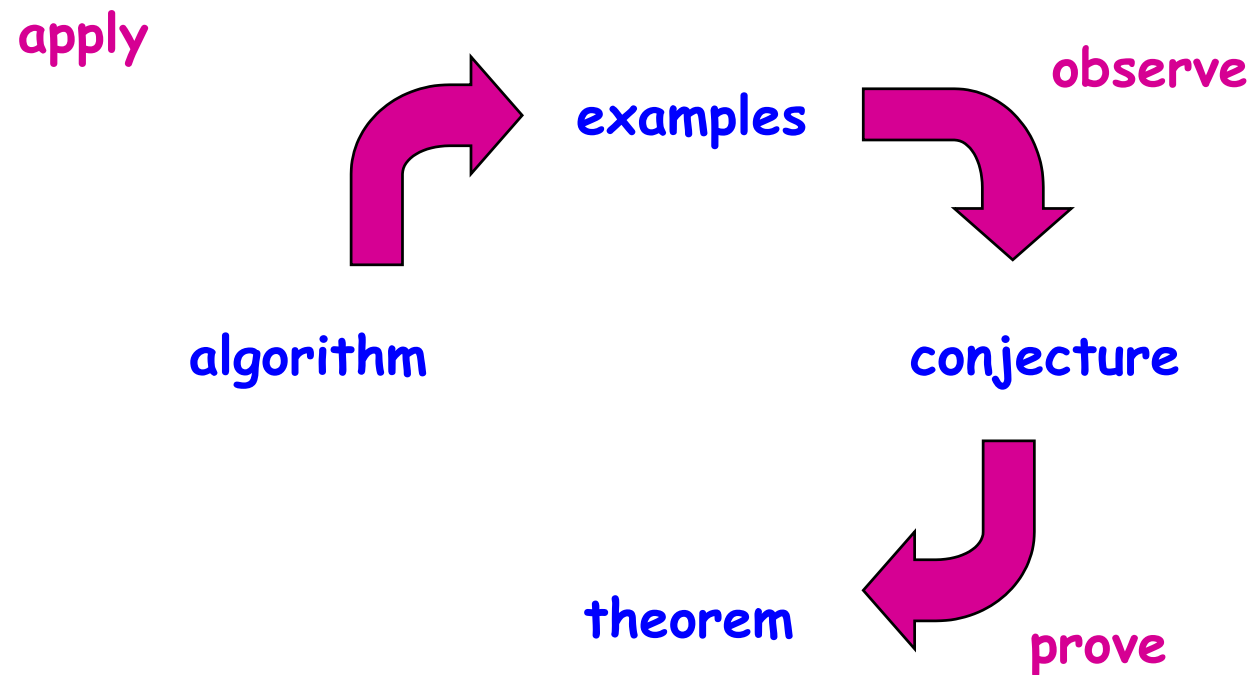
examples

algorithm

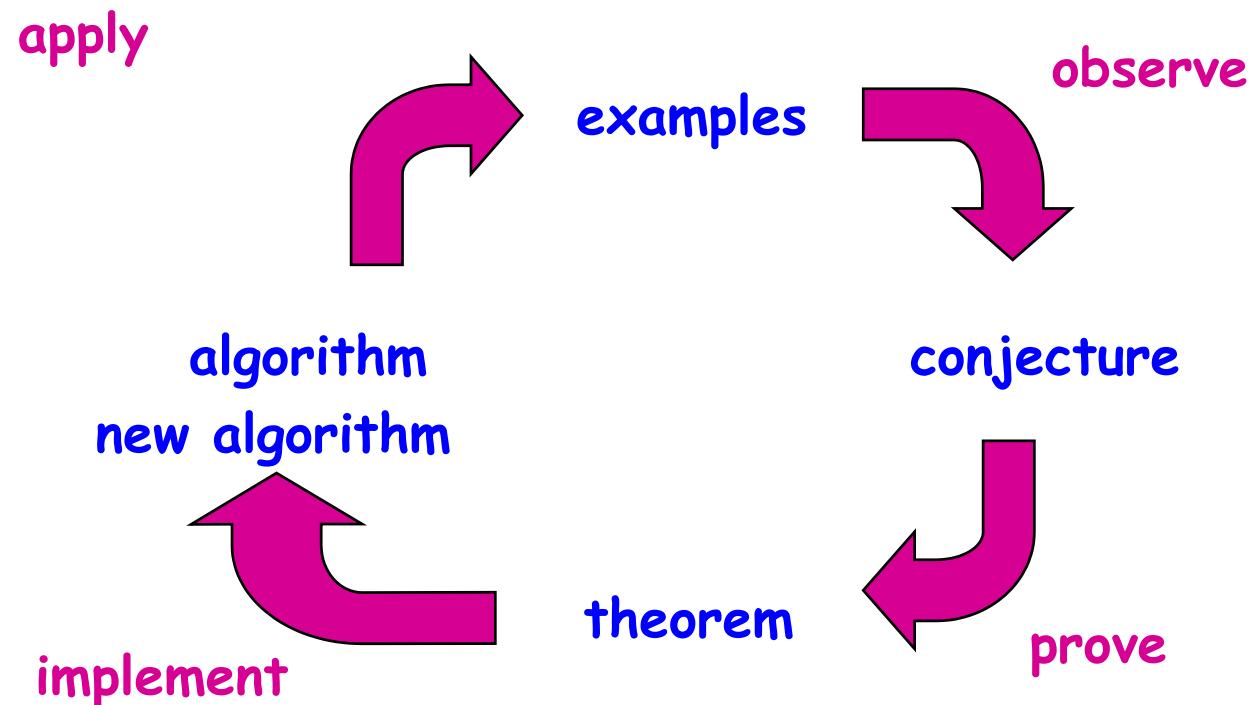
(3) ... analysis (ii):



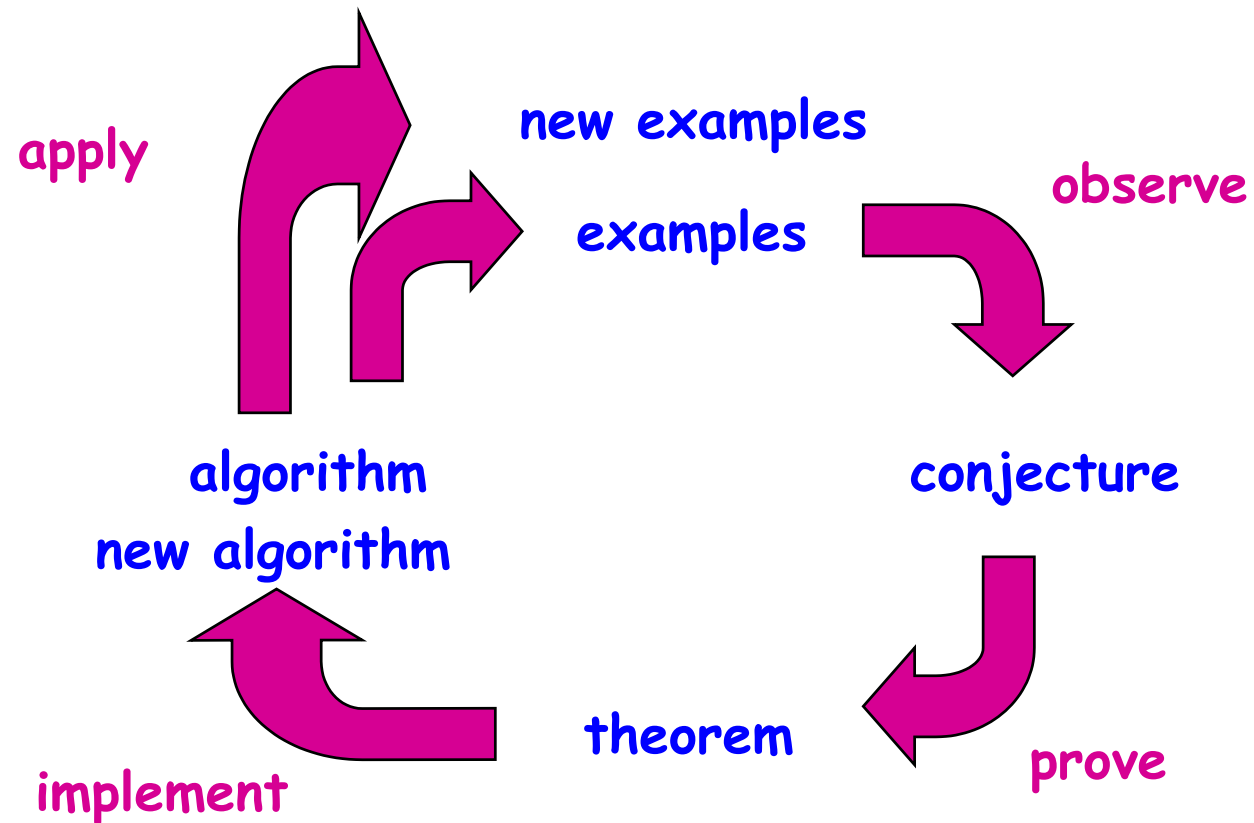
### (3) ... analysis (ii):



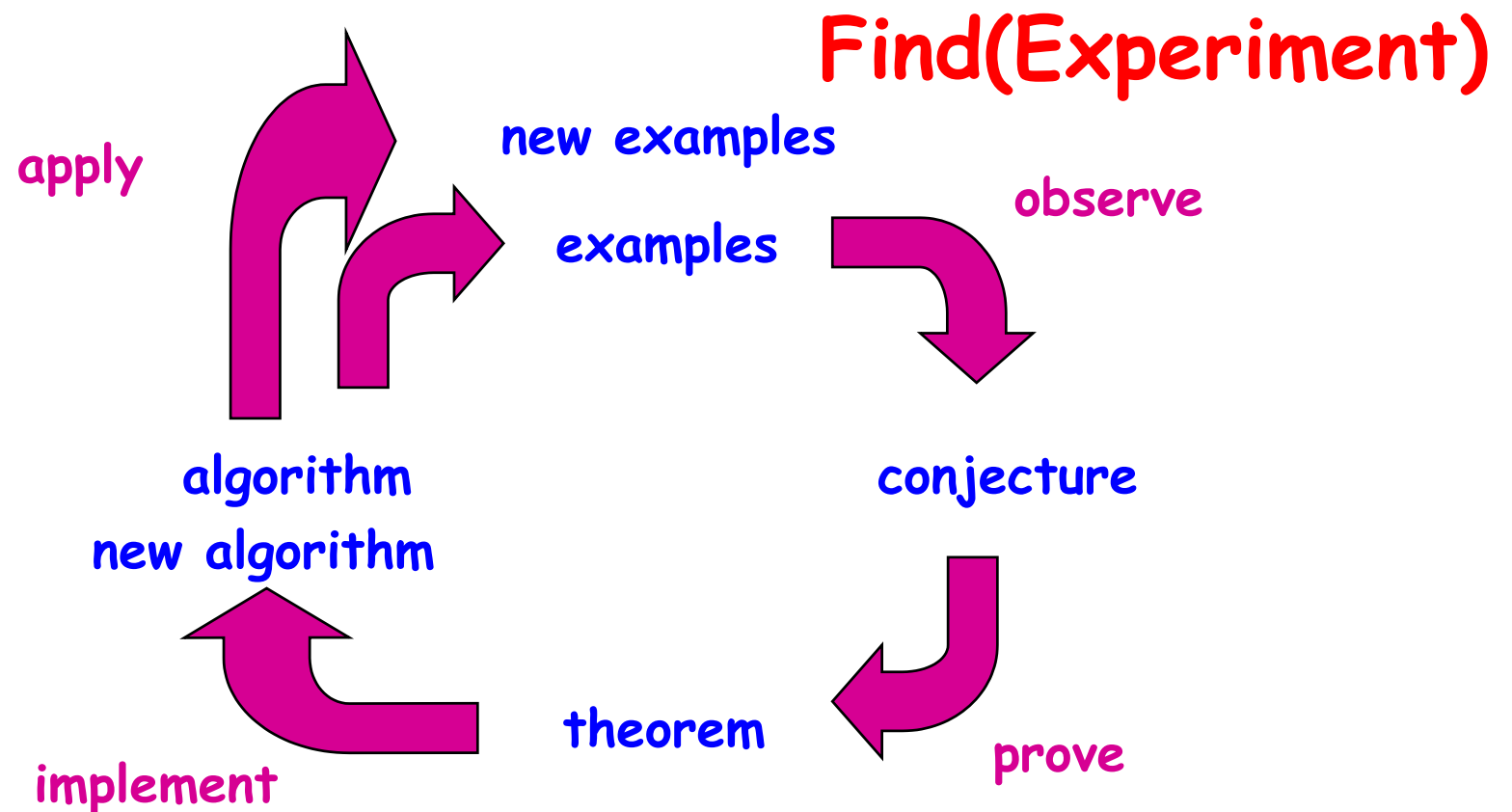
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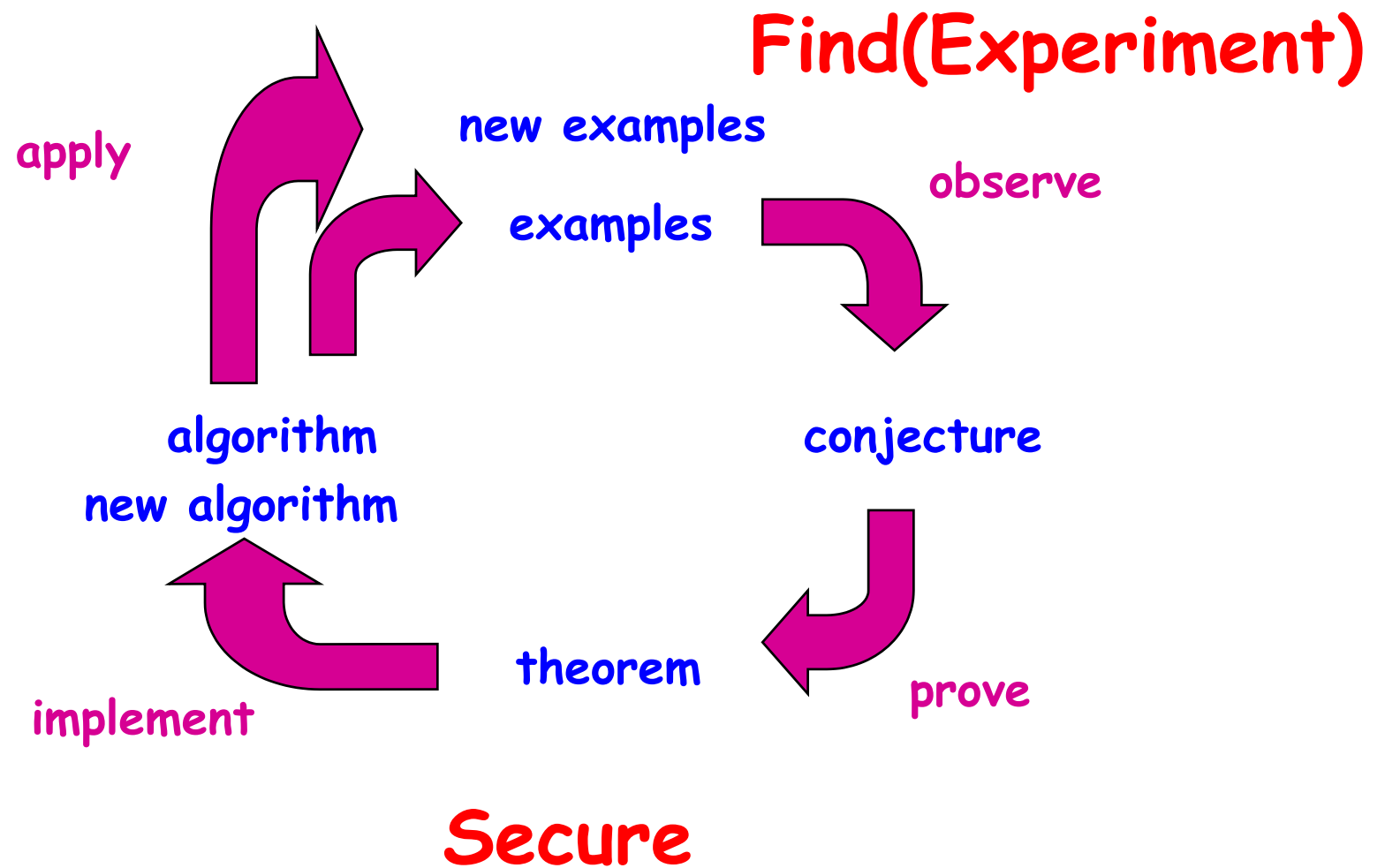
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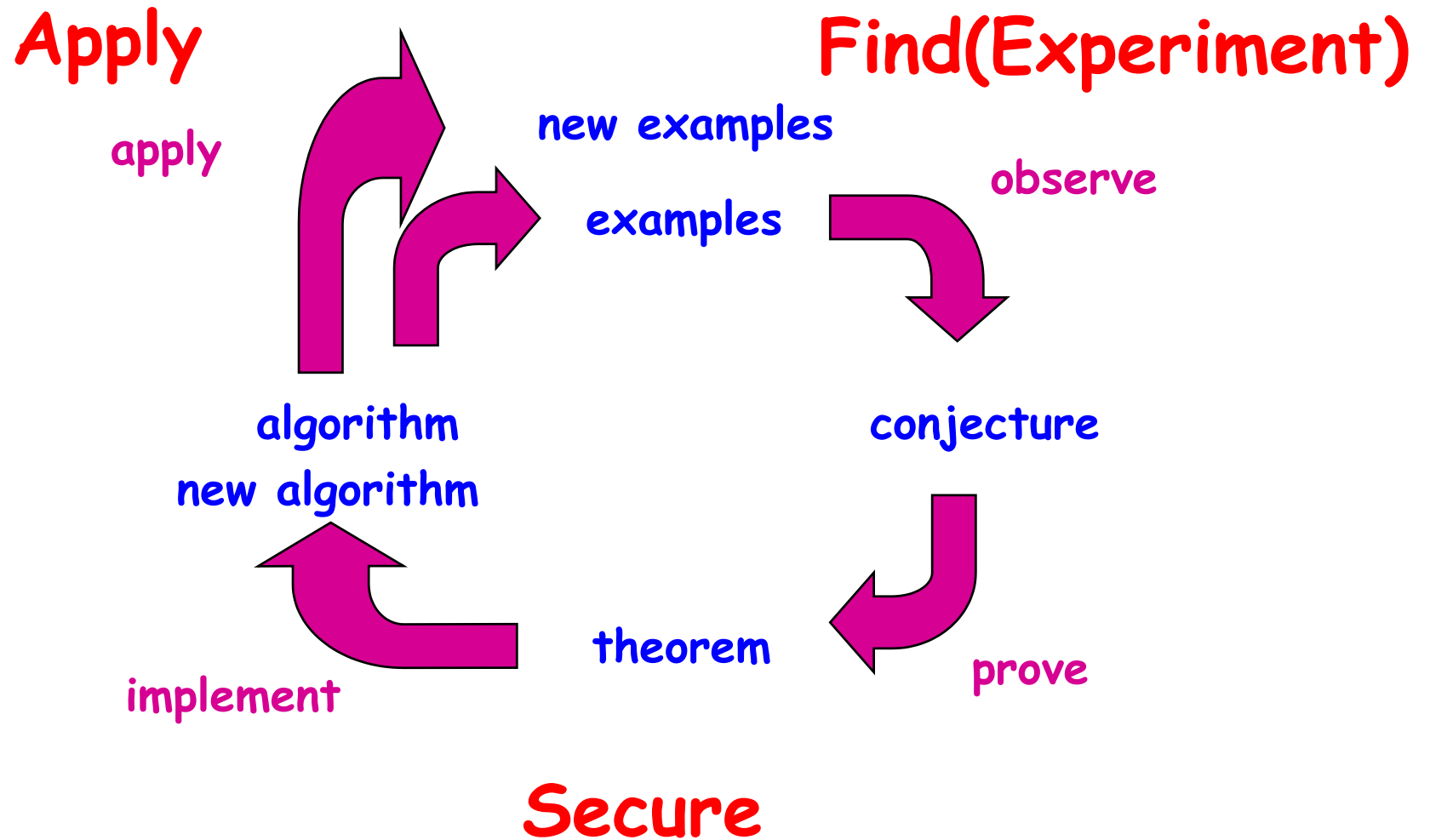


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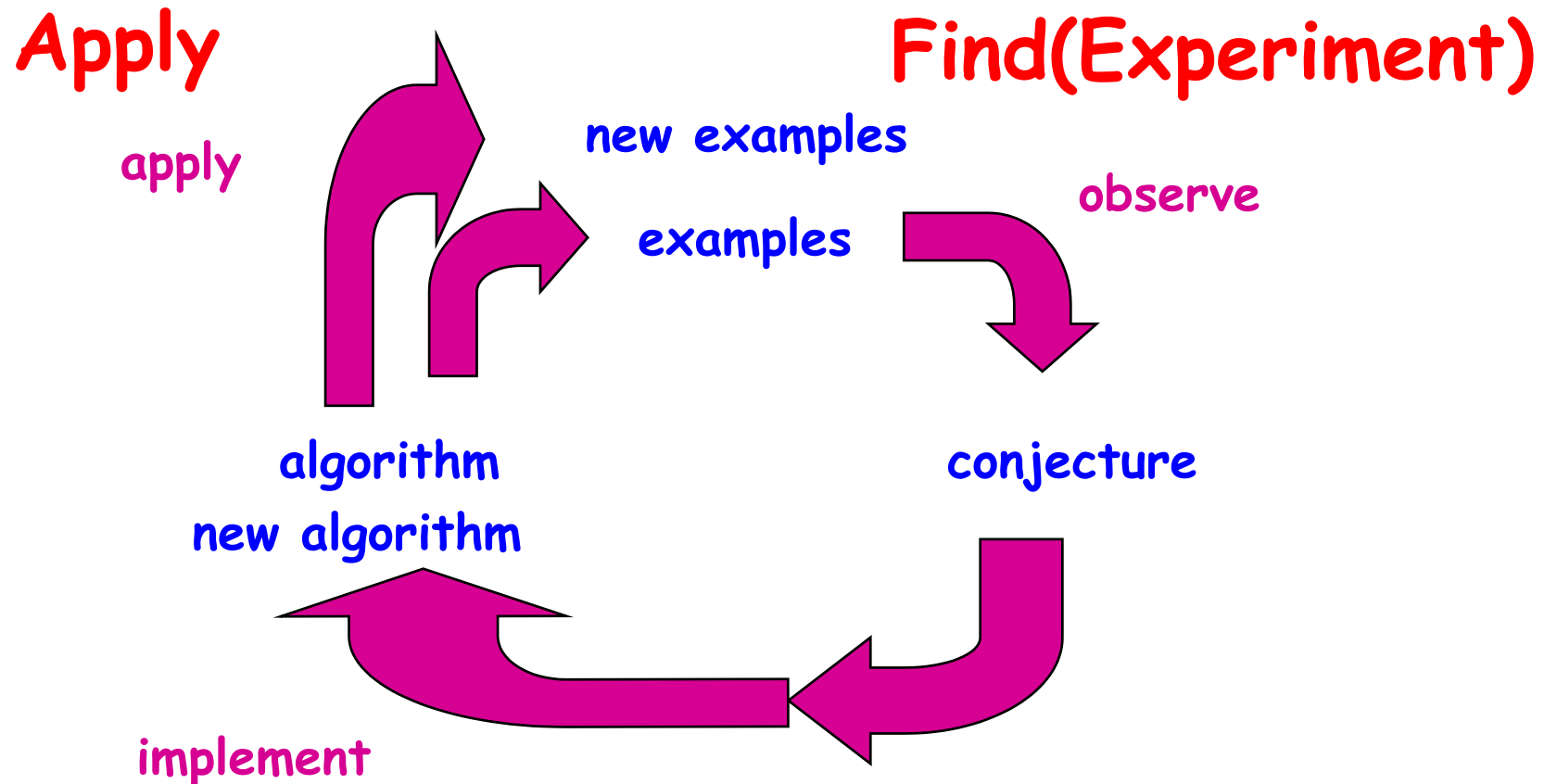




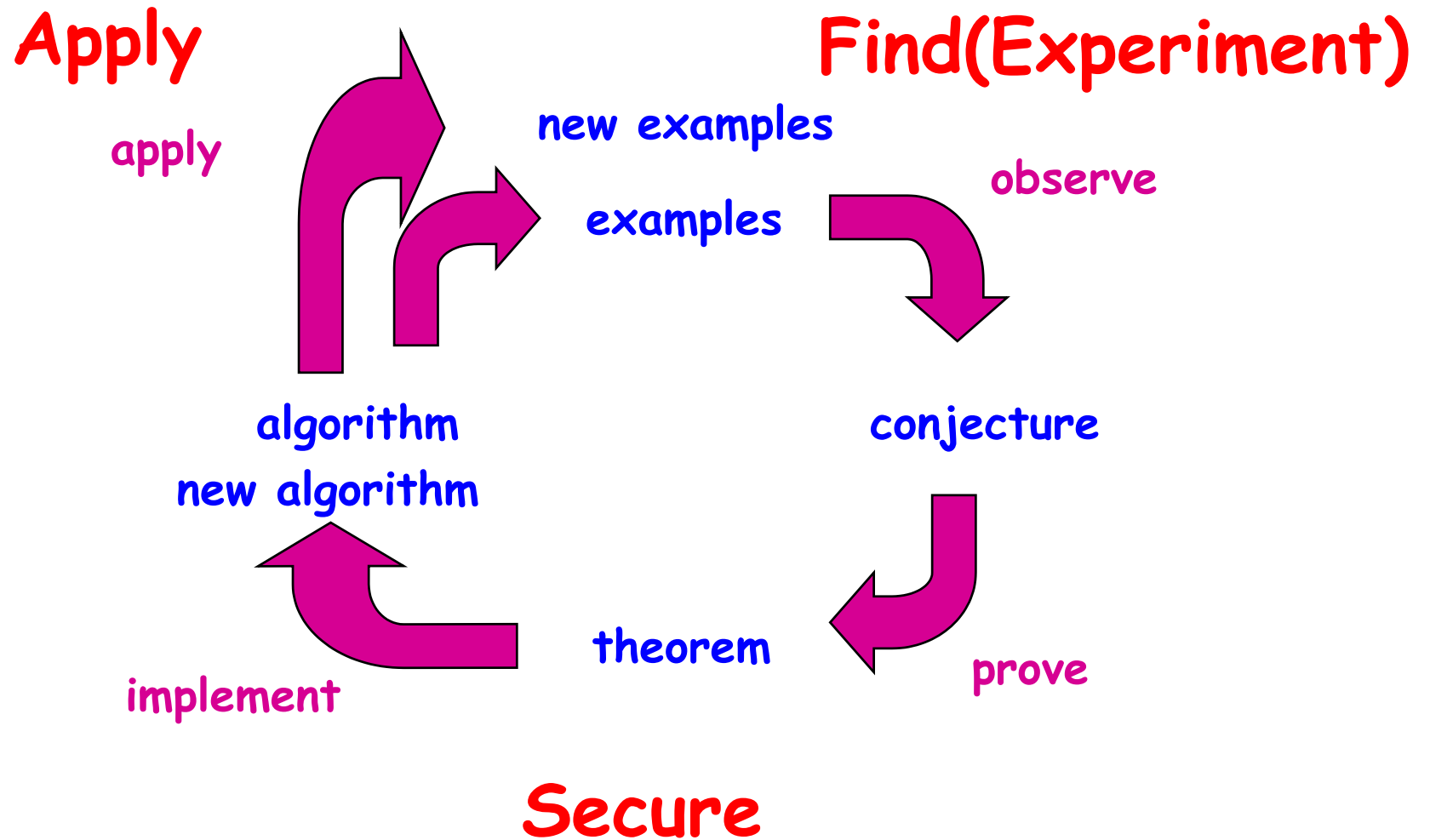
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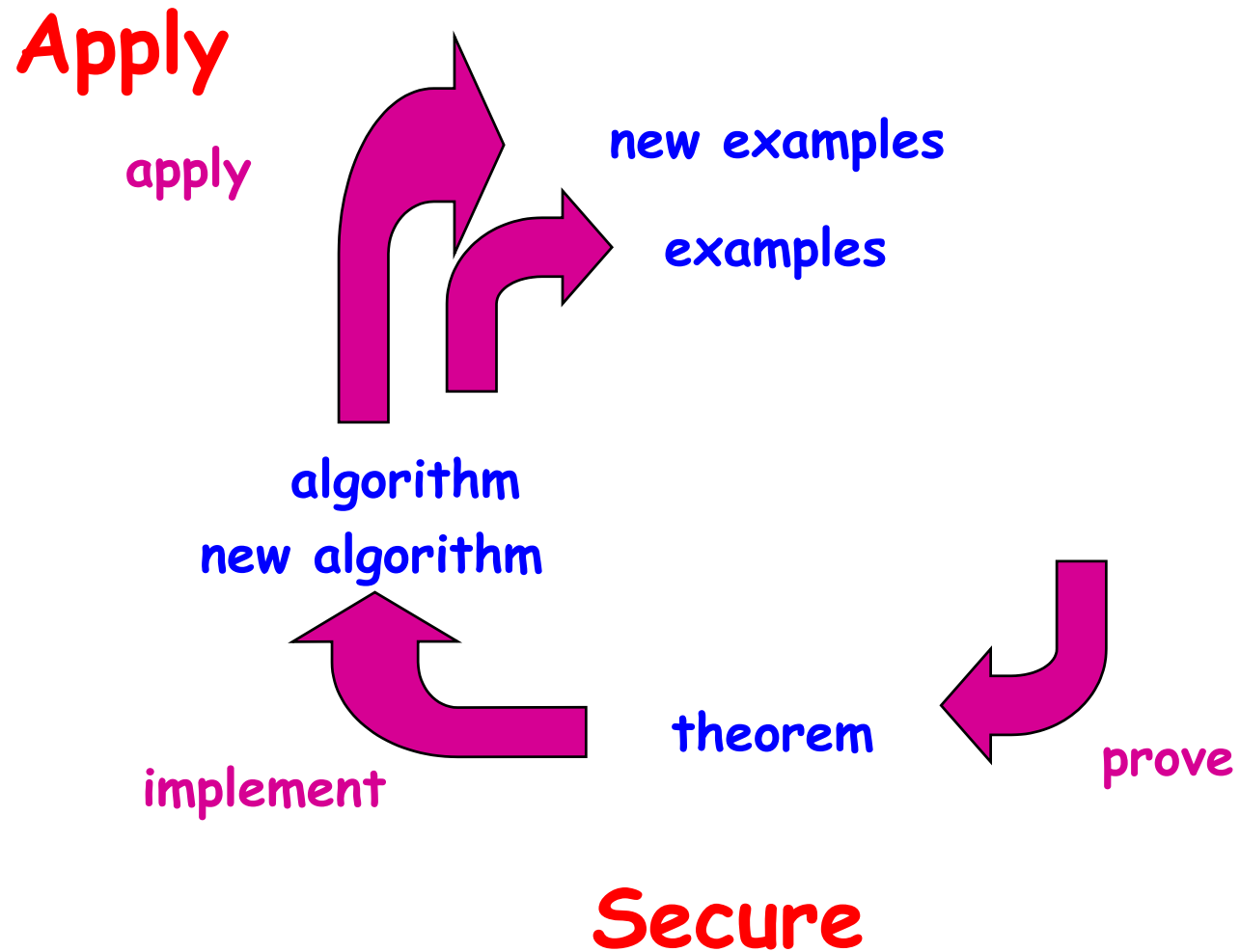
(3) ... analysis (ii):



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(3) ... analysis (ii):



(3) ... analysis (ii):

Bourbakism  
was made for

- *inner-mathematical documentation*

**Apply**

apply

new examples

examples

algorithm

new algorithm

theorem

implement

prove

**Secure**

(3) ... analysis (ii):

Bourbakism  
was made for

- inner-mathematical documentation
- inner-mathematical communication

Apply

apply

new examples

examples

algorithm

new algorithm

theorem

implement

prove

Secure

(3) ... analysis (ii):

Bourbakism

was made for

- inner-mathematical documentation
  - inner-mathematical communication
- but not for

- teaching (but we do it !!! ☹)

Apply

apply

new examples

examples

algorithm

new algorithm

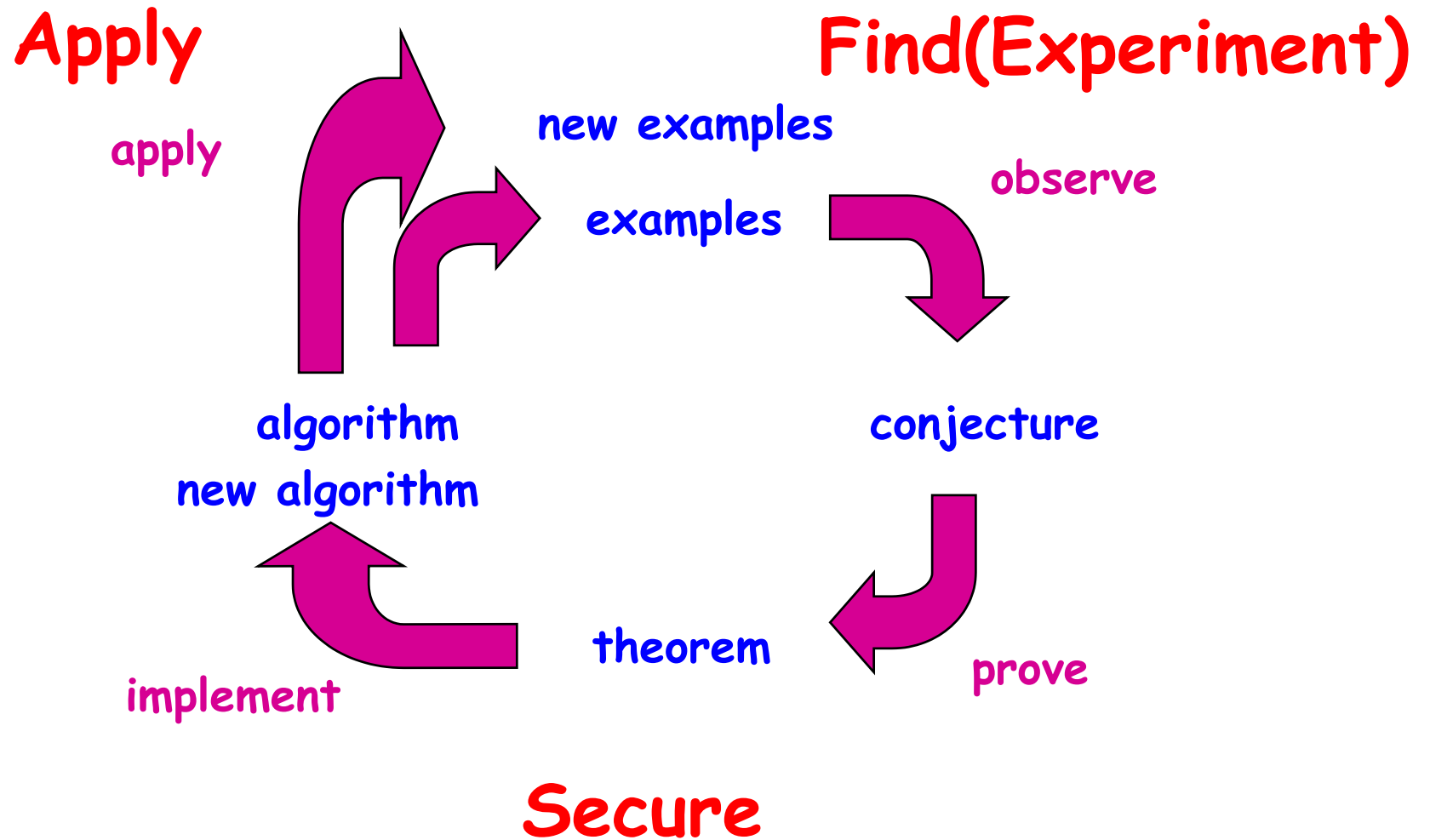
theorem

implement

prove

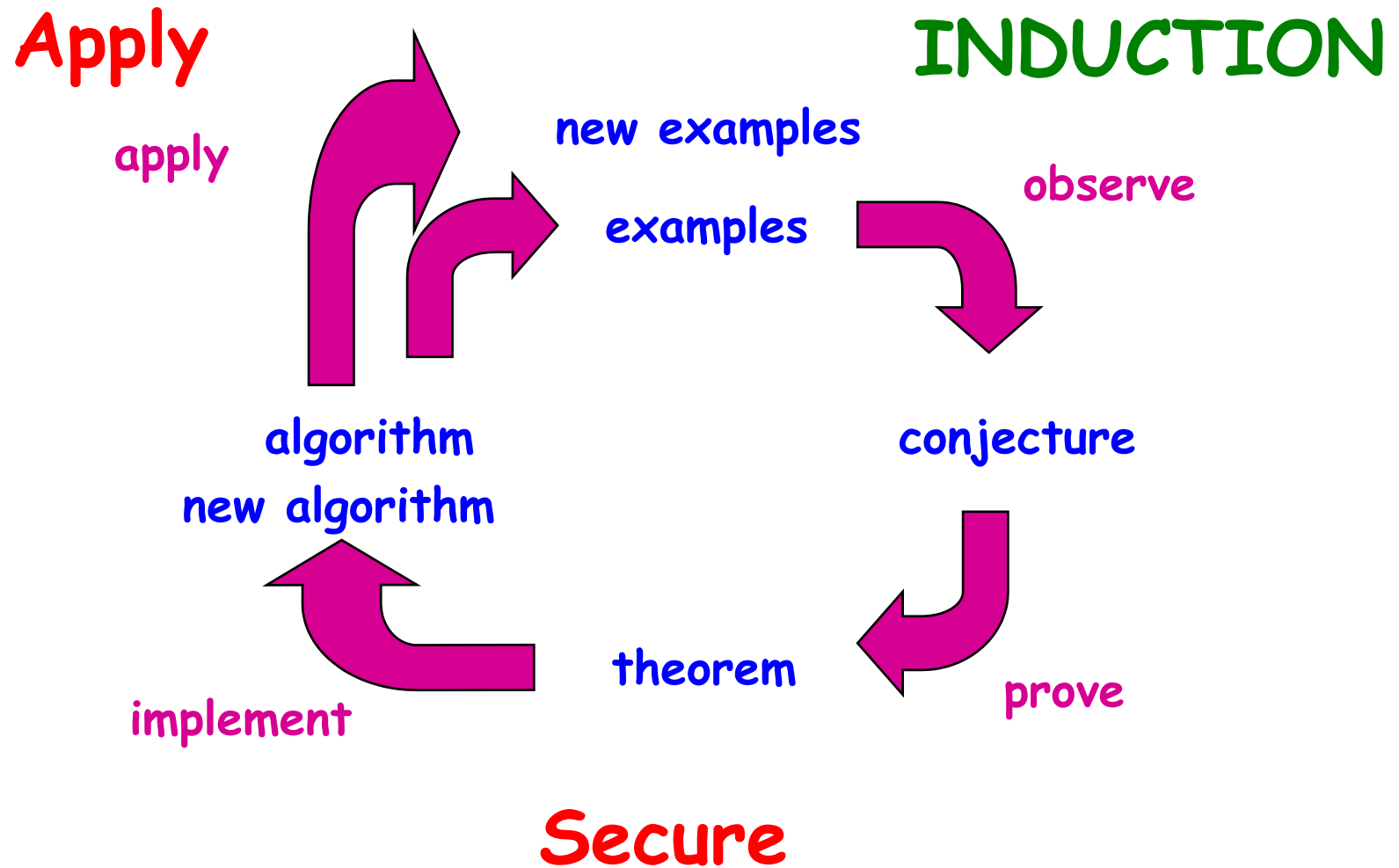
Secure

(3) ... analysis (ii):

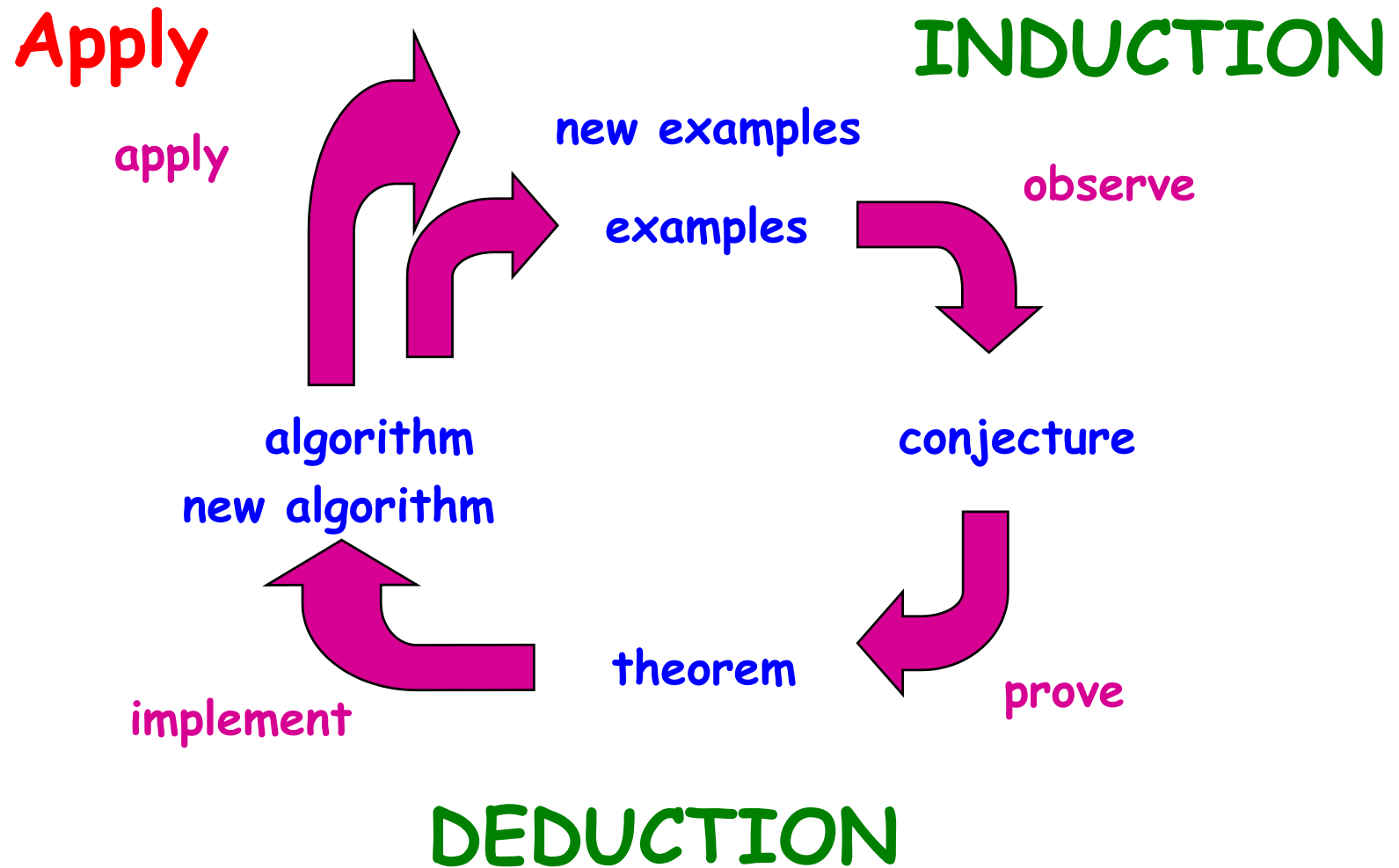




(3) ... analysis (ii):



(3) ... analysis (ii):



(3) ... analysis (ii):

PRODUCTION

INDUCTION

apply

new examples

observe

examples

algorithm

conjecture

new algorithm

theorem

prove

implement

DEDUCTION



# T · E · A · C · H

"GIVE A PERSON A FISH AND YOU FEED THEM FOR A DAY.  
TEACH A PERSON TO FISH AND YOU FEED THEM FOR A LIFETIME." - CONFUCIUS

(2)

Confuzius (adapted to mathematics teaching):

Give your students some mathematics and  
you feed them for the next exam.

Teach your students how to fish for  
mathematics and you feed them for a  
lifetime.

Derive 6

is a

PeCAS!

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