Make New from Old

Example 1

Original version (end examination task)

A polynomial function $f(x)$ of degree 4 which is symmetric wrt y-axis intersects function

$$g(x) = \frac{1}{16k}x - \frac{1}{32k}x^2$$

under a right angle in P(2|0) and additionally in point Q(0|0).

a) Show that $f(x)$ has form $f(x) = kx^4 - 4kx^2$.

b) Find the zeros, turning and inflection points of the graphs of both functions for $k = \frac{1}{4}$.

c) Graphs of f and g form an area for $x \geq 0$. (Intersection points are P and Q from above). Show that the area is given by

$$A(k) = \frac{1}{24k} + \frac{64k}{15}.$$

d) For which value of k do you obtain the minimal area?

Proposal for a New Version

Given is a family of functions $g_k(x) = \frac{1}{16k}x - \frac{1}{32k}x^2$.

a) Which is the form of all curves of the family?

b) What are the common properties of all graphs? Give reasons!

c) What is the influence of parameter k on form and position of the graphs. Present your findings using an appropriate survey.

d) Find a "partner family" $f_k(x)$ so that each g_k is intersecting the corresponding f_k orthogonally.

e) Show for any k that condition d) is fulfilled.

f) What is the common area enclosed by two "partners"? Shade this area on your device for any appropriate k. Explain how you can achieve this.

g) Which value of k makes this area extremal? Is it a Maximum or a Minimum?

Example 2

Original version (end examination task)

Given is a family of curves $y_a(x) = -\frac{x^4}{4a} + \frac{x^2}{2} + \frac{3a}{4}$; $a > 0$.

a) Investigate the family in general on symmetry, zeros, maximum- and minimum values and inflection points.

b) Plot the graph for $a = 1$ considering the results from a) on an appropriate system of coordinates. If no general results are available, then perform the special investigation of the function.

c) Calculate generally the area formed by the graph and the x-axis. Which value for a yields an area between the graph and the x-axis $A = 100$? (2 decimal digits).

d) Suppose there is another general function $y_b(x)$ with $b > 0$ and $a \neq b$. Give reasons why both graphs can never have real intersection points.

Proposal for a New Version

Given is a family of curves $y_a(x) = -\frac{x^4}{4a} + \frac{x^2}{2} + \frac{3a}{4}$; $a > 0$.

a) Investigate the family in general on symmetry, zeros, maximum- and minimum values and inflection points. Find the locus of extremal values and inflection points.
What is the difference between this family and the family with $a < 0$?
Give a sketch of your findings.

b) Which value for a yields an area between the graph and the x-axis $A = 100$?

c) The line passing both inflection points of the curves forms three distinct areas with the graph.
What is the ratio of the measures of the areas? (For all curves of the whole family)

d) Which is the distance from the x-axis to lay a horizontal line that the three segments between the intersection points with the graph are of equal length?

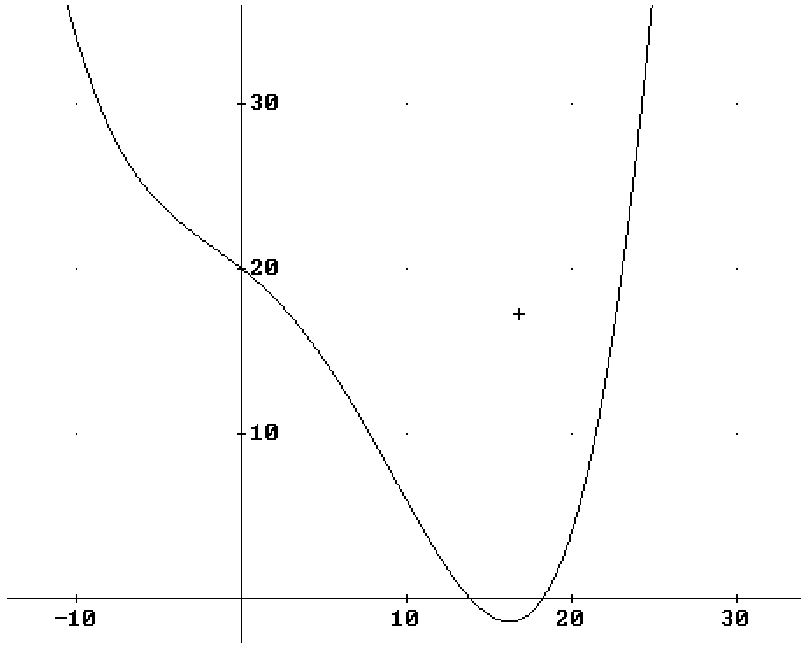
e) Verify the results of (c) and (d) for $a = 10$.

f) Suppose there is another general function $y_b(x)$ with $b > 0$ and $a \neq b$. Give reasons why both graphs can never have real intersection points.

Example 3

Original version is finding a polynomial function of degree ??, which fulfills some conditions (zeros, turning points, inflection points, slopes, ...).

We do it the other way round:



Find a function which describes the given function graph in the best possible way. Explain what you are doing.

How do you evaluate the quality of your approximation?

Possible variations of this task:

- Have different scales on both axes
- Ask the students that only two or three points are allowed to be used (considering the slopes or other conditions)
- Omit the axes in the sketch
- Provide only a hand sketch

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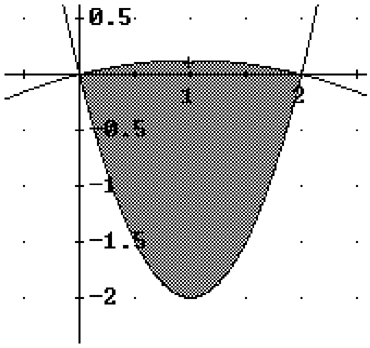
Working with Derive 6 we have another opportunity – loading the graph in the background of the 2D-plot window.

References: Josef Böhm, *Neue Aufgaben für das Unterrichten mit Derive & TI-92/V 200*, bk-teachware

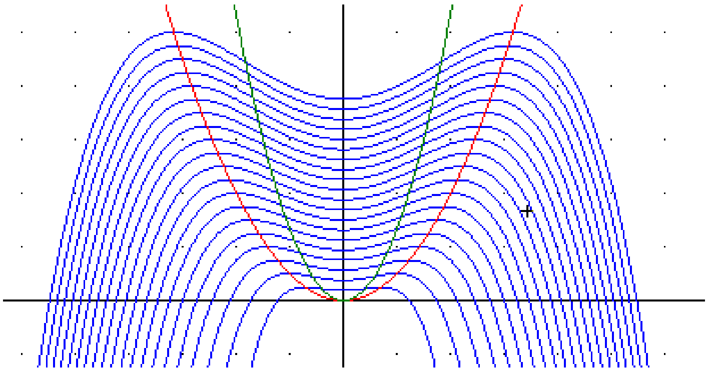
Example 1



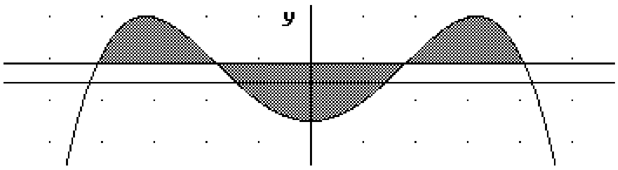
$$\#13:\quad \int_0^2 (g(x) - f(x))\, dx = \frac{256\, k^2 + 1}{24\, k}$$

$$\#14:\quad 2\, x^2 - 4\, x \le y \le \frac{x}{4} - \frac{x^2}{8} \wedge 0 \le x \le 2$$

Example 2



$$\left(f(x, 10) \ge y \ge \frac{80}{9} \wedge -\frac{\sqrt{150}}{3} \le x \le -\frac{\sqrt{30}}{3}\right) \vee \left(f(x, 10) \ge y \ge \frac{80}{9} \wedge \frac{\sqrt{30}}{3} \le x \le \frac{\sqrt{150}}{3}\right) \vee \left(f(x, 10) \le y \le \frac{80}{9} \wedge -\frac{\sqrt{30}}{3} \le x \le \frac{\sqrt{30}}{3}\right)$$



Example 3