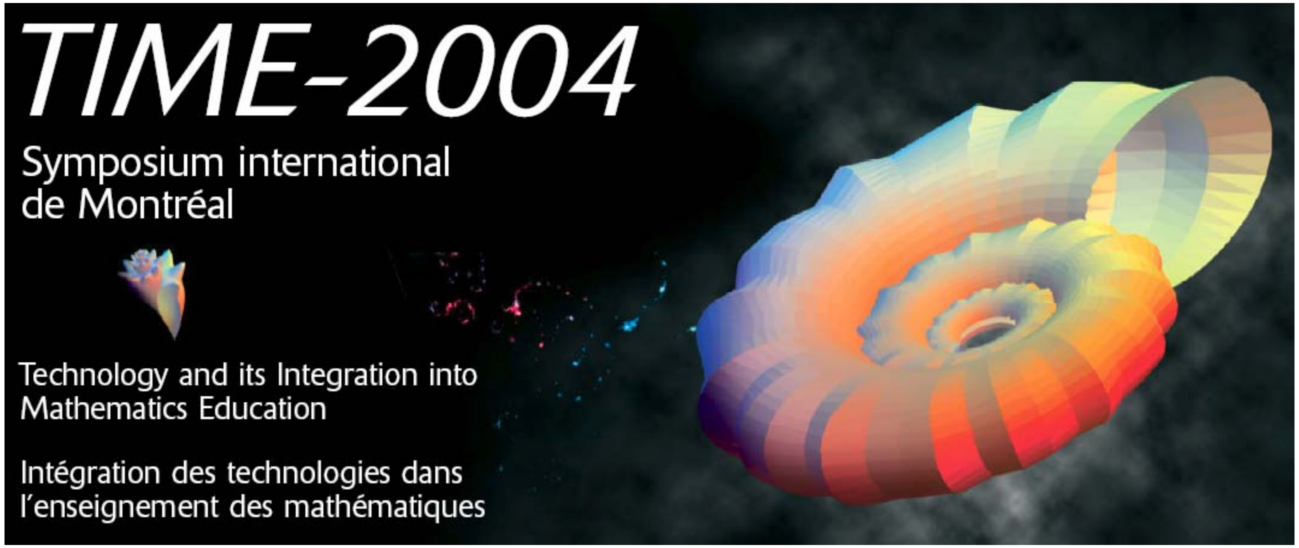
CONTS



Using the TI voyage 200 In Structural Analysis

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École
de technologie
supérieure

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Structural Analysis
TI voyage 200

CONTS

- Examples in Structural Analysis will be presented that illustrate the use of the TI voyage 200, a symbolic and graphic calculator.

- The examples are taken from a first course in Structural Analysis at the undergraduate level. They deal with classical methods of analysis for both statically determinate and statically indeterminate structures.

Structural Analysis
TI voyage 200

- The use of the TI voyage 200 greatly reduces the mathematical difficulties in problem solving, and thus allows the students to spend more time in developing a good understanding of the behaviour of structures.

- The TI voyage 200 is a complex tool and the examples illustrate how to use the calculator in such a way that the students will want to take advantage of its computational power throughout their careers in structural engineering.

CONTS

Structural Analysis
TI voyage 200

- The examples are taken from a textbook used in Structural Analysis at the École de technologie supérieure (Samikian,1994).

- The examples are an attempt to update the contents of the textbook by including the use of a symbolic calculator. The have been developed over the last four years by the speaker. Presently, they are part of the class notes and eventually will be included in a new edition of the textbook.

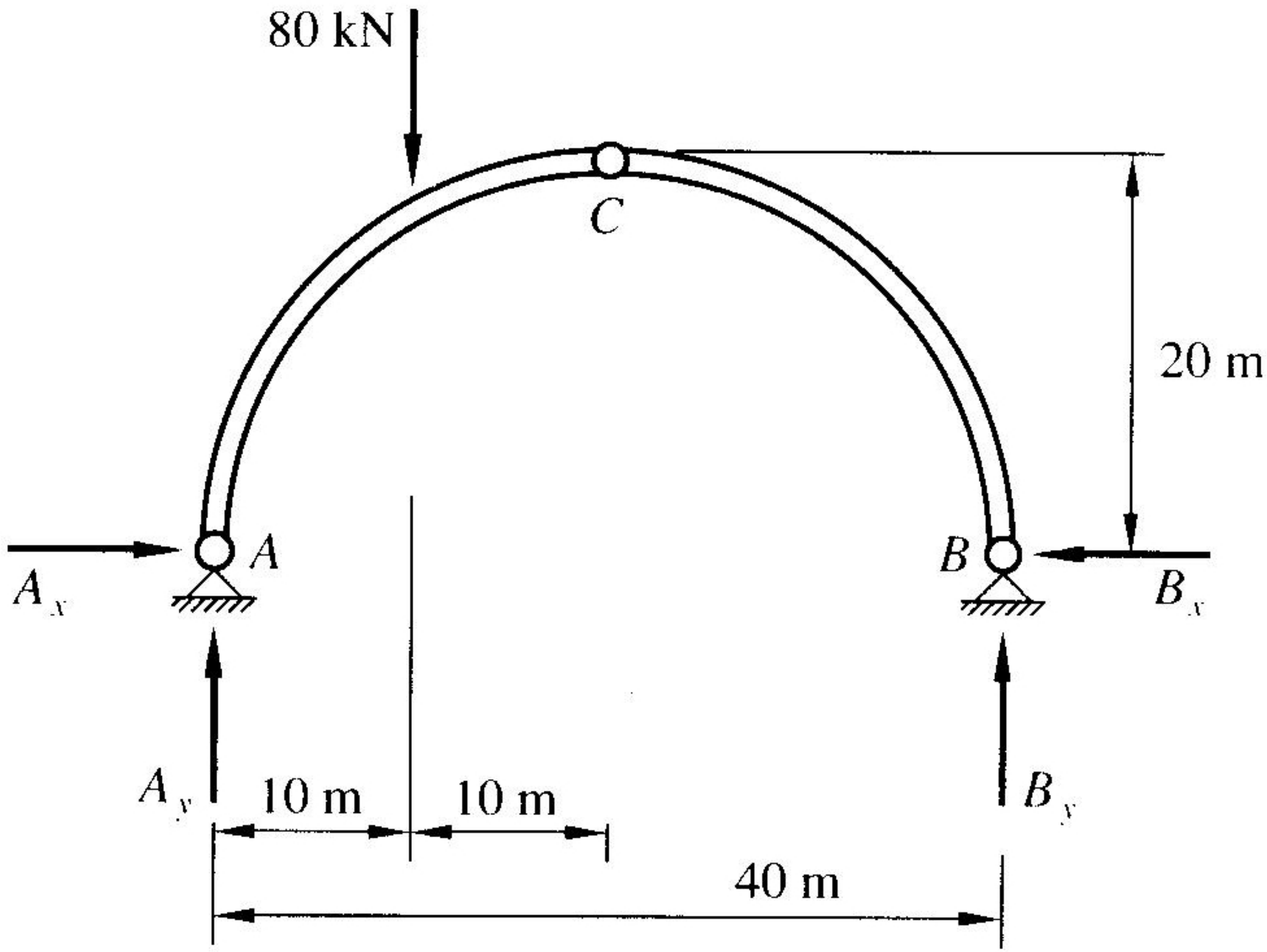
Structural Analysis
TI voyage 200

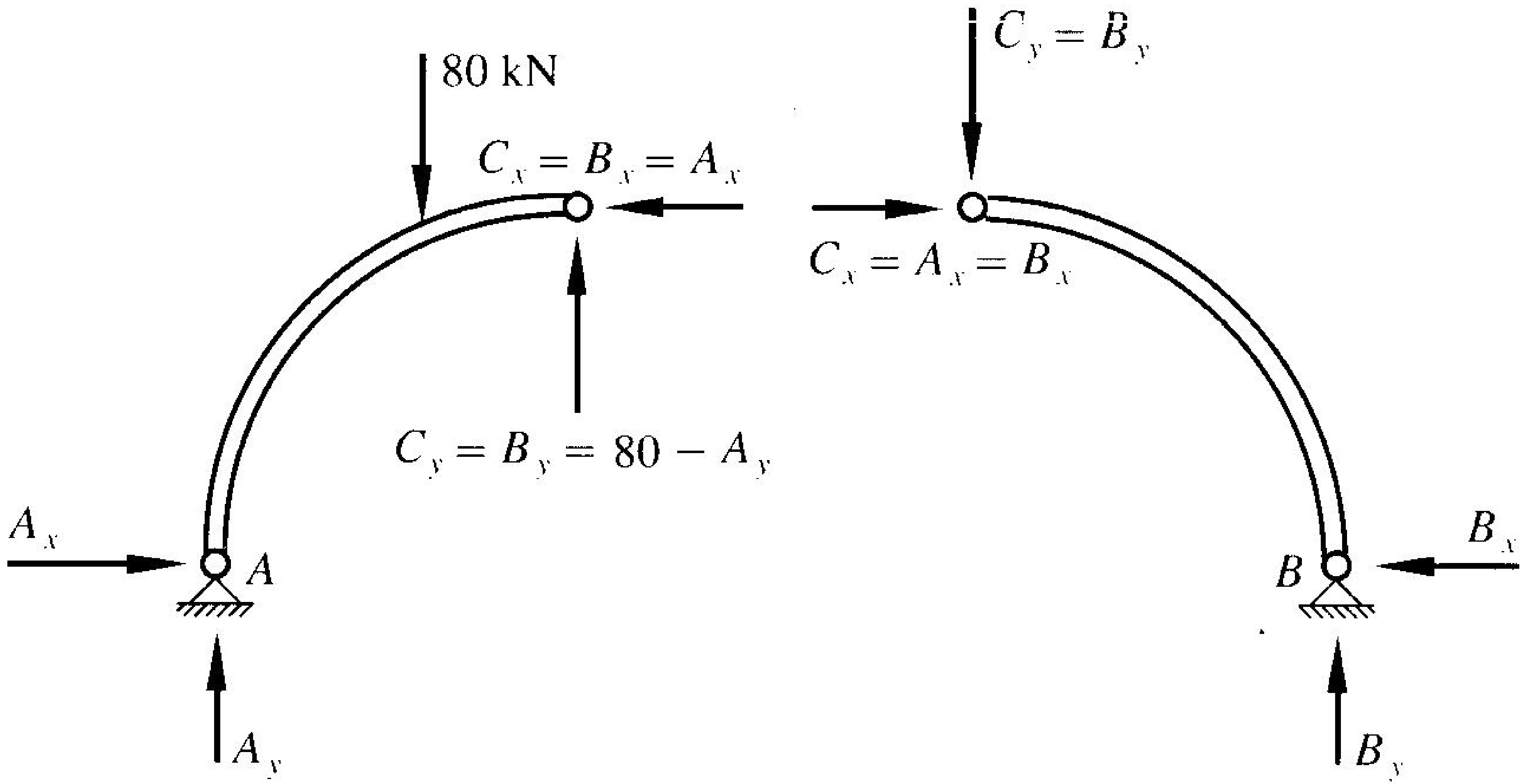
- Three-Hinged Arch (Samikian, example 2.4)
  - *System of linear simultaneous equations*

CONTS

Example 2.4

Determine the support reactions for the three-hinged arch shown in the Figure below.




80 kN
$C_x = B_x = A_x$
$C_y = B_y$
$C_x = A_x = B_x$
$C_y = B_y = 80 - A_y$
A_x
A
B
B_x
A_y
B_y

CONTS

$$\sum M_C = 20A_y - 20A_x - (80 \times 10) = 0$$

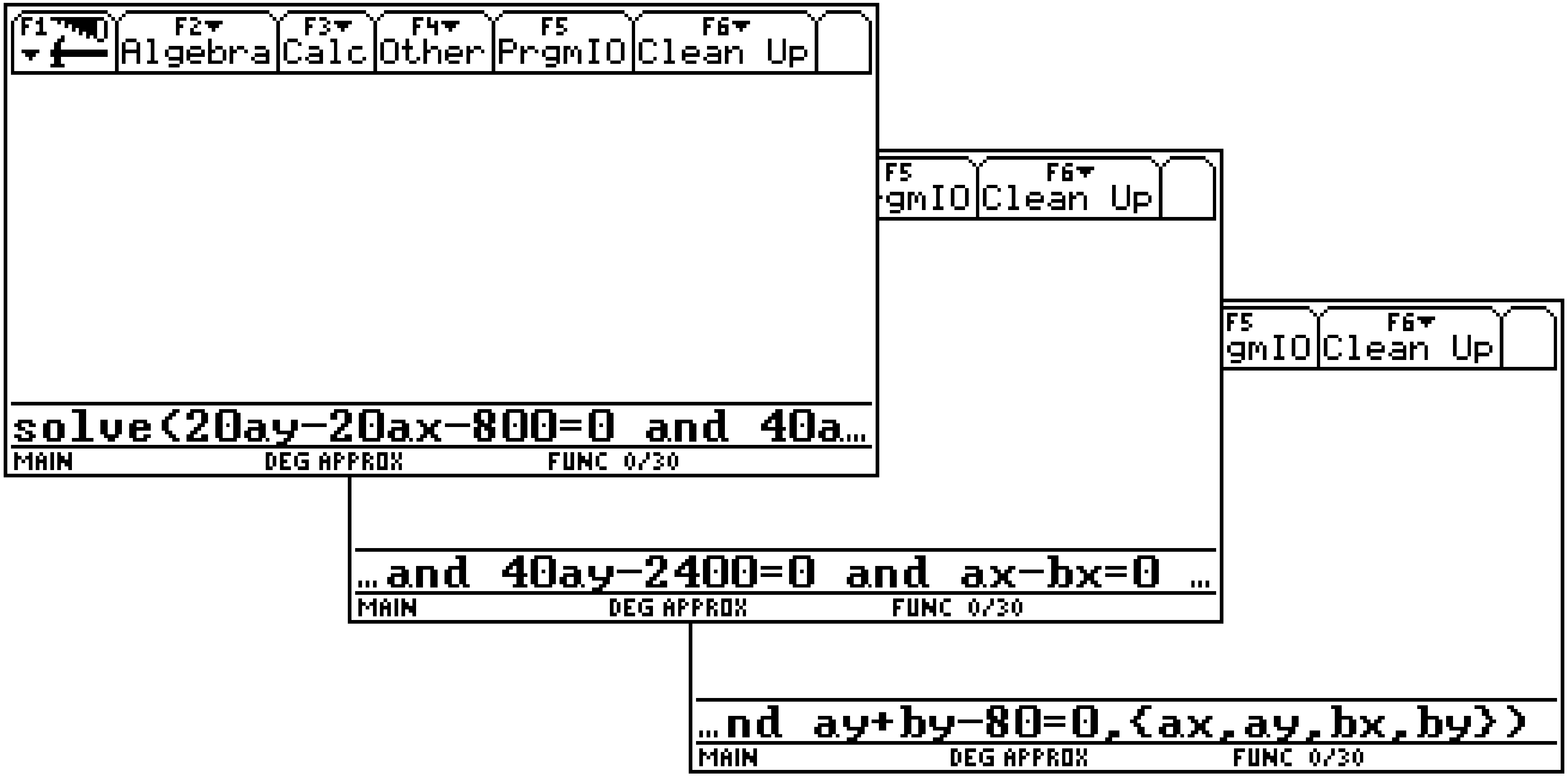
$$\sum M_B = 40A_y - (80 \times 30) = 0$$

$$\sum F_x = A_x - B_x = 0$$

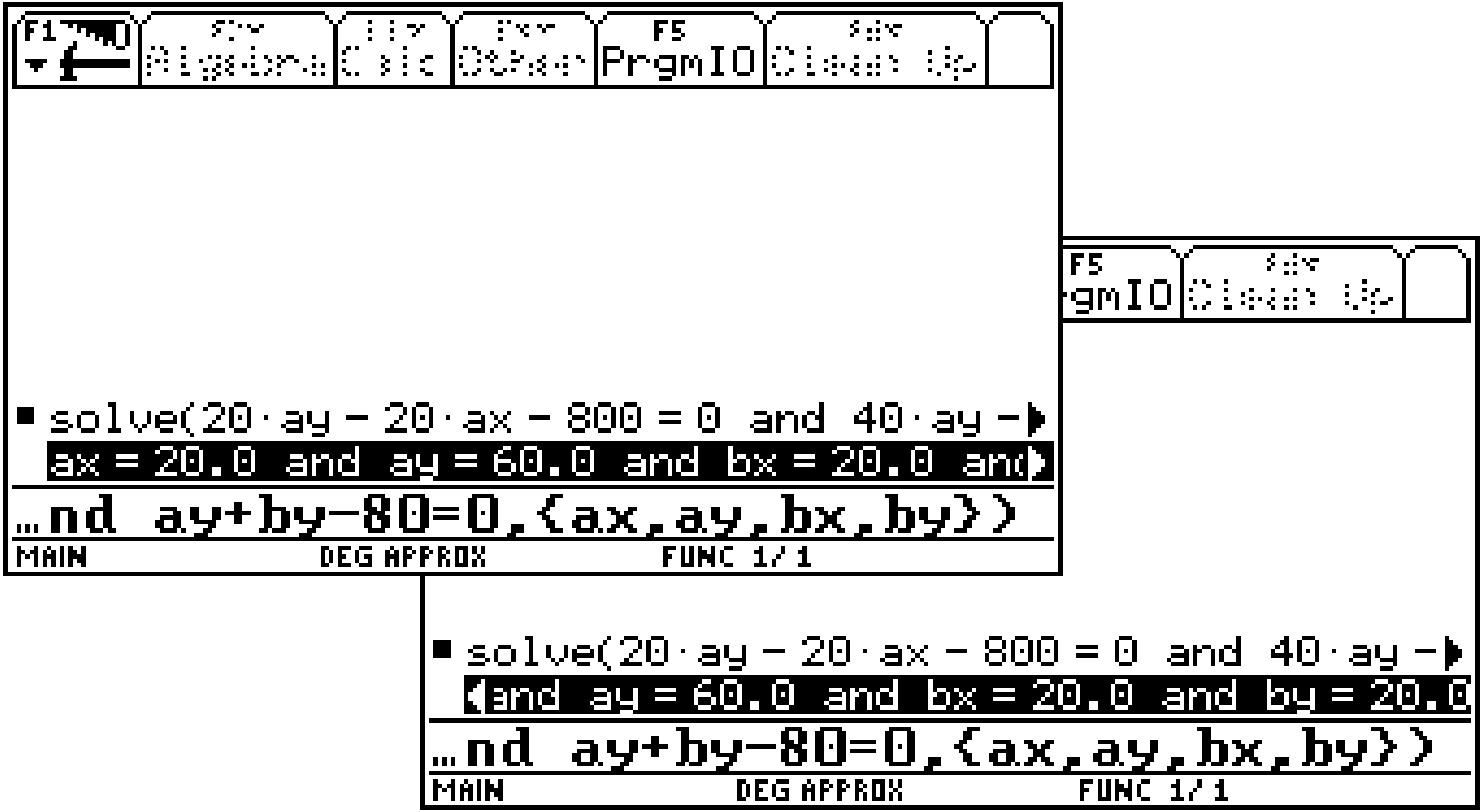
$$\sum F_y = A_y + B_y - 80 = 0$$

CONTS

```
solve(20ay-20ax-800=0 and 40ay-2400=0 and ax-bx=0 and ay+by-80=0,{ax,ay,bx,by})
```


F1 F2 Algebra F3 Calc F4 Other F5 PrgmIO F6 Clean Up
solve(20ay-20ax-800=0 and 40a…
MAIN DEG APPROX FUNC 0/30
…and 40ay-2400=0 and ax-bx=0 …
MAIN DEG APPROX FUNC 0/30
…nd ay+by-80=0,{ax,ay,bx,by})
MAIN DEG APPROX FUNC 0/30

CONTS


F1
Algebra
Calc
Other
F5
PrgmIO
Clean Up
■ solve(20·ay − 20·ax − 800 = 0 and 40·ay −
ax = 20.0 and ay = 60.0 and bx = 20.0 and
…nd ay+by−80=0,{ax,ay,bx,by})
MAIN DEG APPROX FUNC 1/1
F5
gmIO
Clean Up
■ solve(20·ay − 20·ax − 800 = 0 and 40·ay −
and ay = 60.0 and bx = 20.0 and by = 20.0
…nd ay+by−80=0,{ax,ay,bx,by})
MAIN DEG APPROX FUNC 1/1

CONTS

```
ax = 2Ø.Ø and ay = 6Ø.Ø and bx =
                    2Ø.Ø and by = 2Ø.Ø
```

$$A_x = 20\ \mathrm{kN}$$

$$A_y = 60\ \mathrm{kN}$$

$$B_x = 20\ \mathrm{kN}$$

$$B_y = 20\ \mathrm{kN}$$

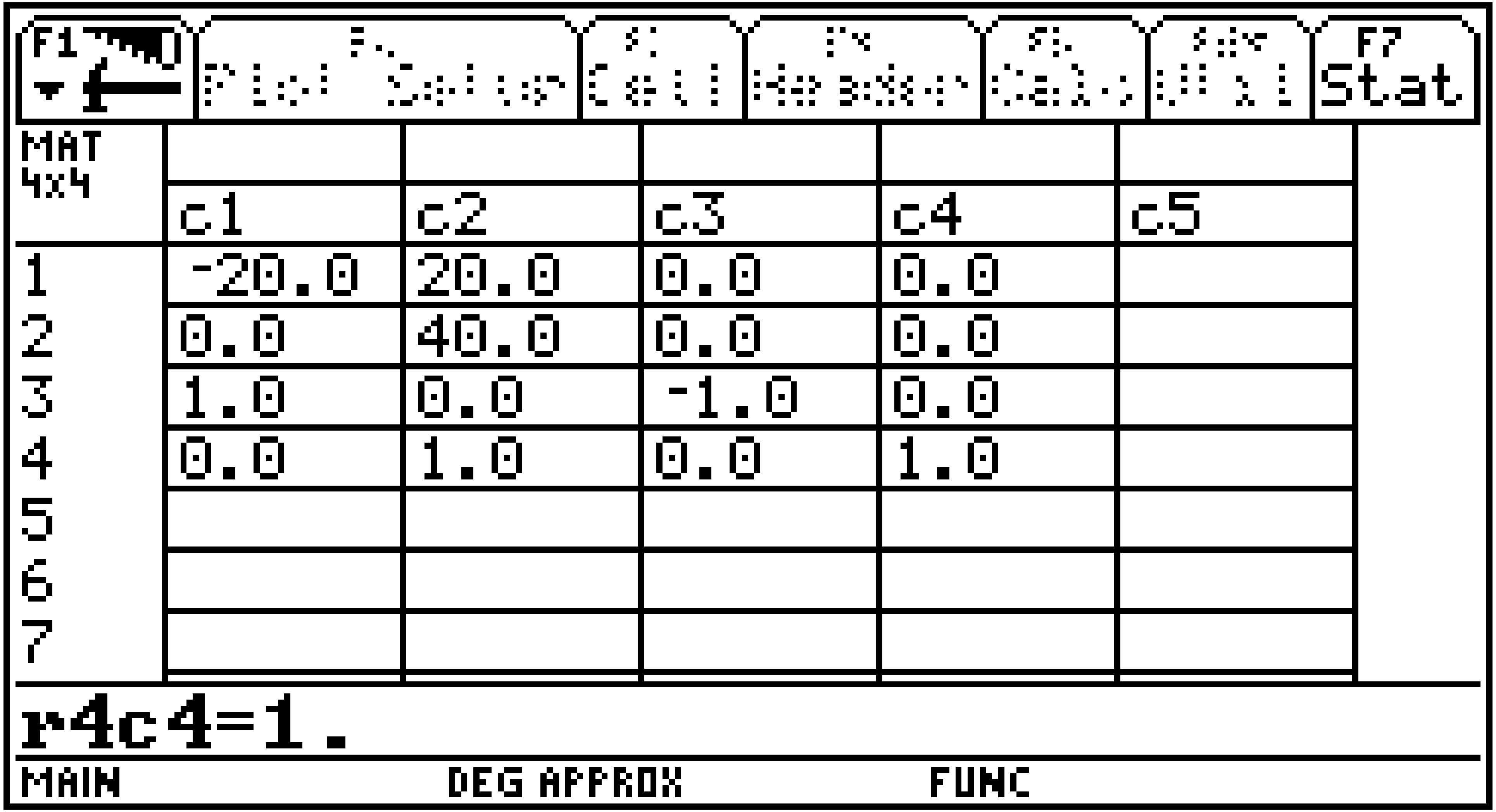
CONTS

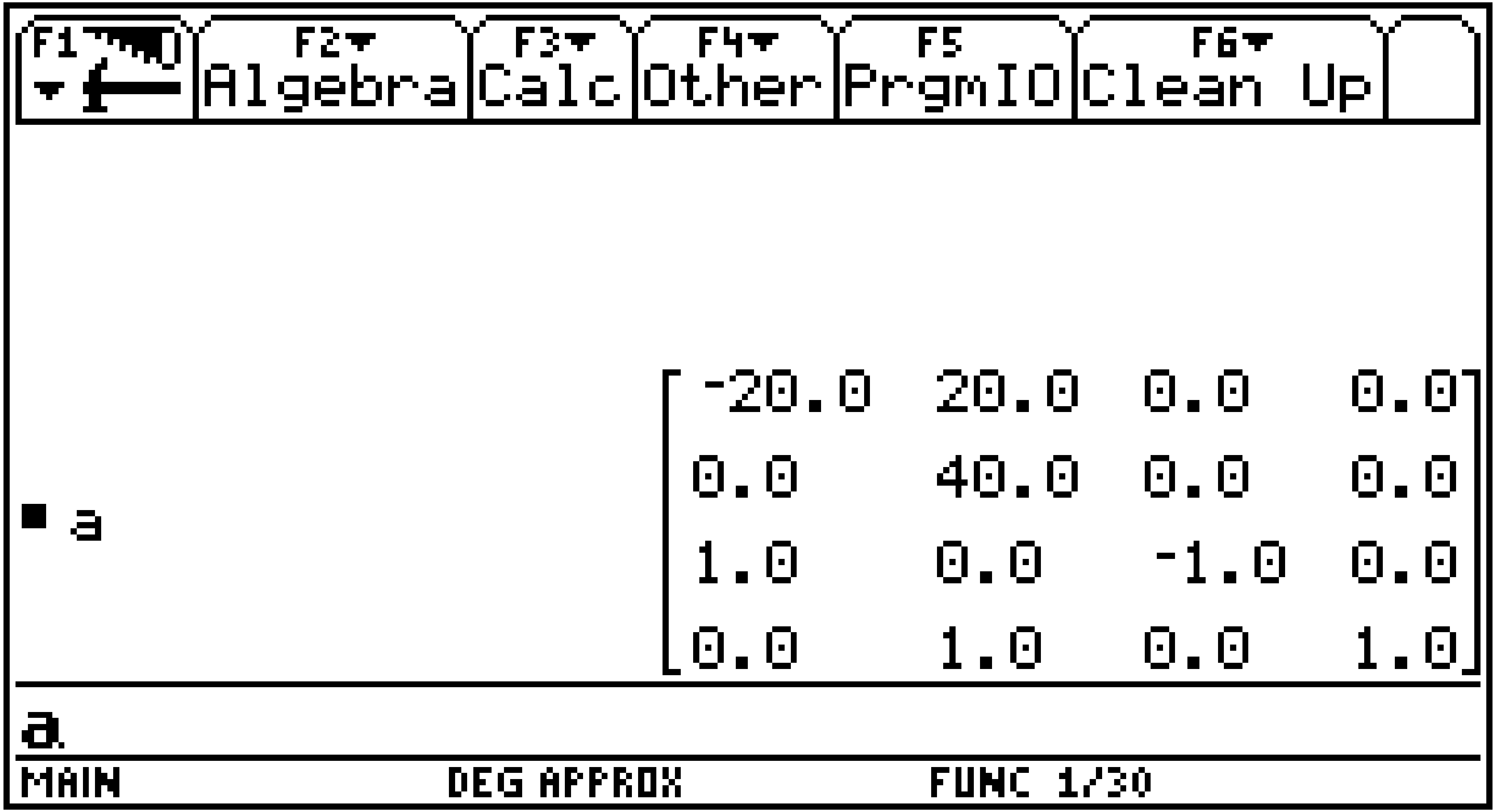
$$
\begin{array}{rrrrcr}
-20A_x & +20A_y & & & = & 800 \\
 & +40A_y & & & = & 2400 \\
A_x & & -B_x & & = & 0 \\
 & A_y & & +B_y & = & 80
\end{array}
$$

$$[A]\{x\}=\{B\}$$

```
simult(a,b)
```

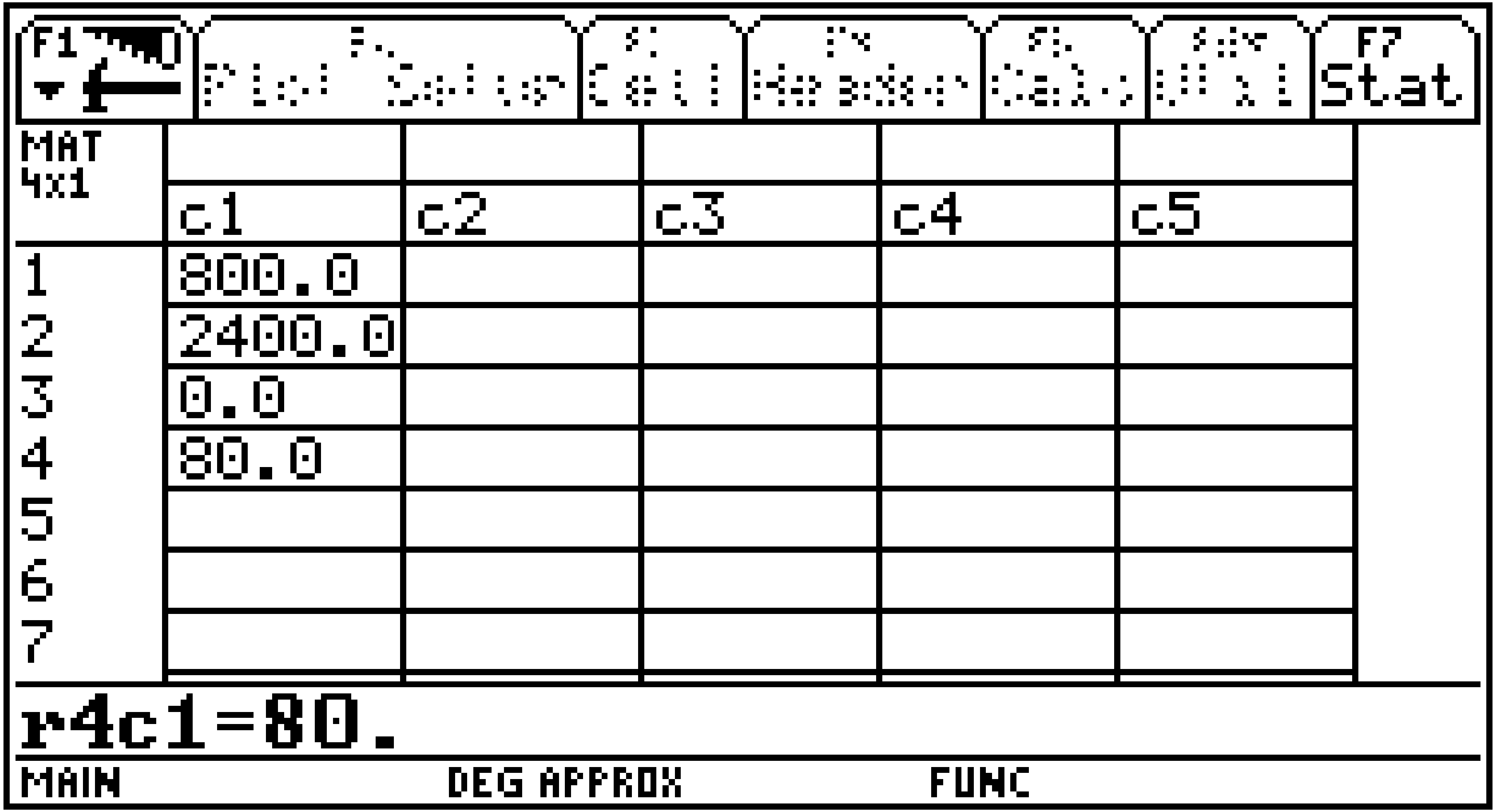
$$[A]=\begin{bmatrix} -20 & 20 & 0 & 0 \\ 0 & 40 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

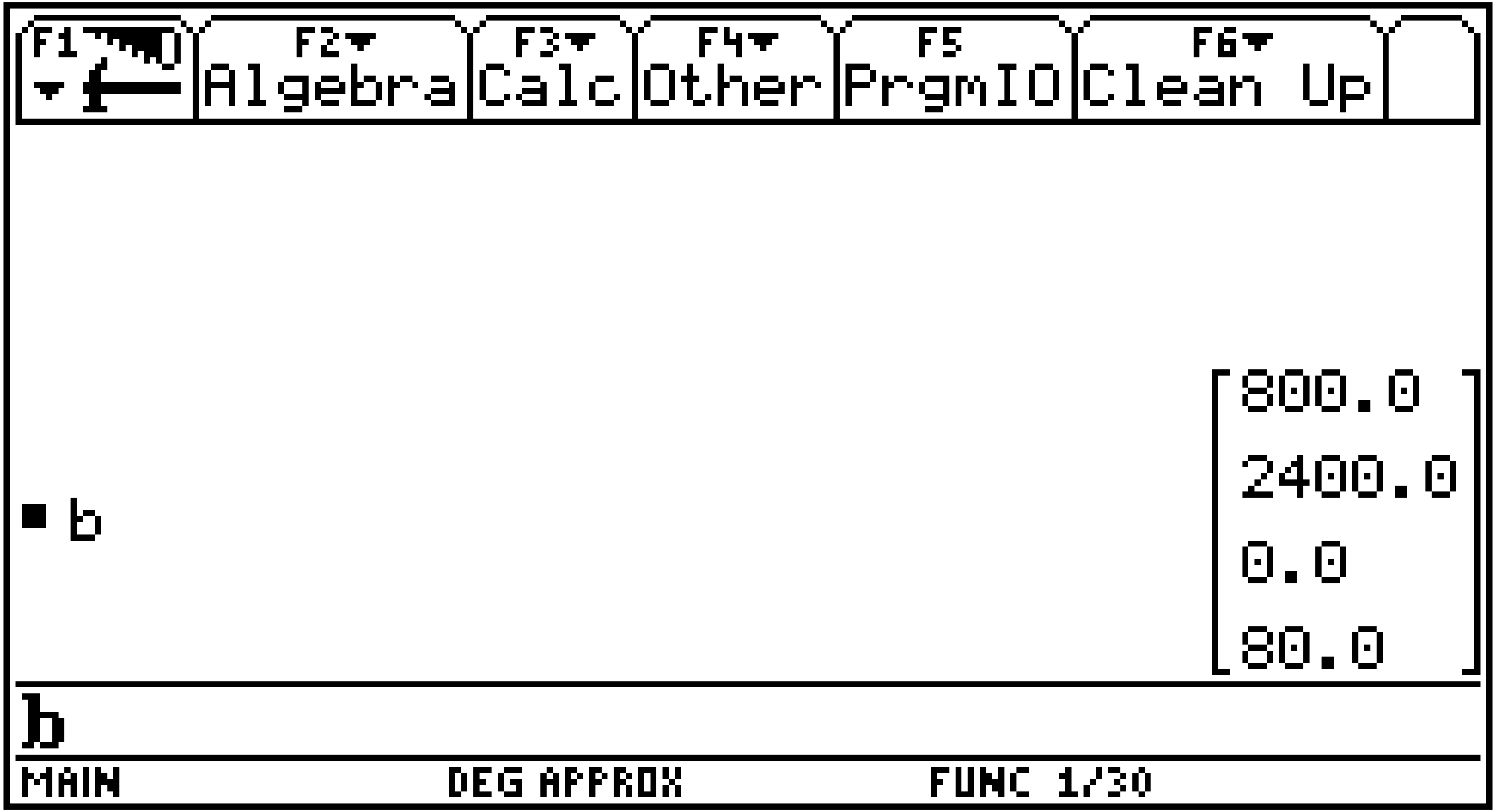

F1
F7
Stat
MAT
4x4
c1
c2
c3
c4
c5
1 ⁻20.0 20.0 0.0 0.0
2 0.0 40.0 0.0 0.0
3 1.0 0.0 ⁻1.0 0.0
4 0.0 1.0 0.0 1.0
5
6
7
r4c4=1.
MAIN
DEG APPROX
FUNC


F1
F2 Algebra
F3 Calc
F4 Other
F5 PrgmIO
F6 Clean Up
■ a
⁻20.0 20.0 0.0 0.0
0.0 40.0 0.0 0.0
1.0 0.0 ⁻1.0 0.0
0.0 1.0 0.0 1.0
a
MAIN
DEG APPROX
FUNC 1/30

CONTS

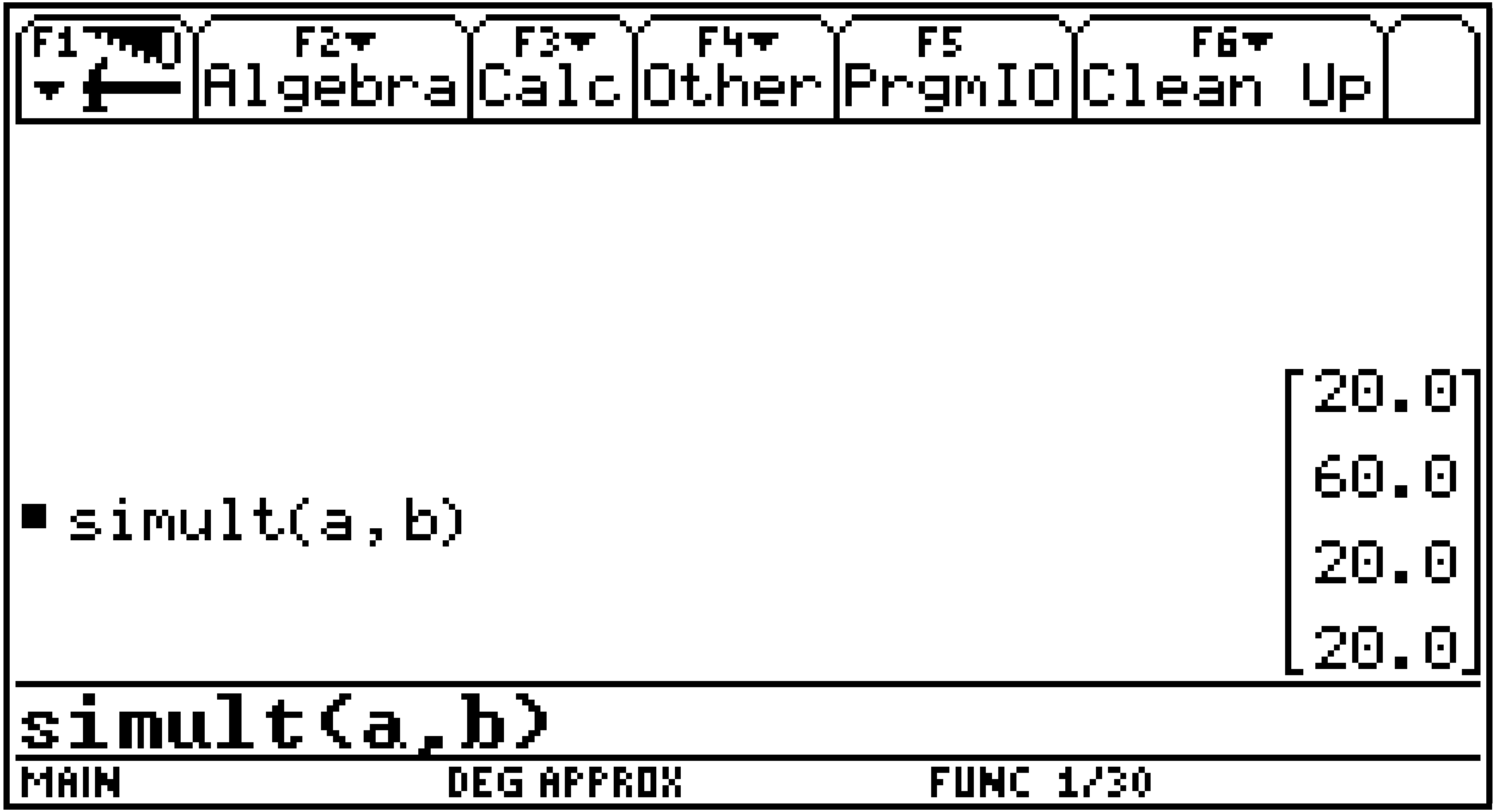
$$\{B\} = \begin{Bmatrix} 800 \\ 2400 \\ 0 \\ 80 \end{Bmatrix}$$


F1
F7
Stat
MAT
4x1
c1
c2
c3
c4
c5
1
2
3
4
5
6
7
800.0
2400.0
0.0
80.0
r4c1=80.
MAIN
DEG APPROX
FUNC



F2 Algebra
F3 Calc
F4 Other
F5 PrgmIO
F6 Clean Up
■ b
800.0
2400.0
0.0
80.0
b
MAIN
DEG APPROX
FUNC 1/30

CONTS



F1
F2
Algebra
F3
Calc
F4
Other
F5
PrgmIO
F6
Clean Up
simult(a, b)
[20.0
60.0
20.0
20.0]
simult(a,b)
MAIN
DEG APPROX
FUNC 1/30

Structural Analysis
TI voyage 200

Parabolic Cables (Samikian, example 4.4)

- *System of non linear simultaneous equations*

Example 4.4

Determine the maximum tension in a cable under a uniform load of 12 kN/m, for the geometry shown in Figure 4.12.

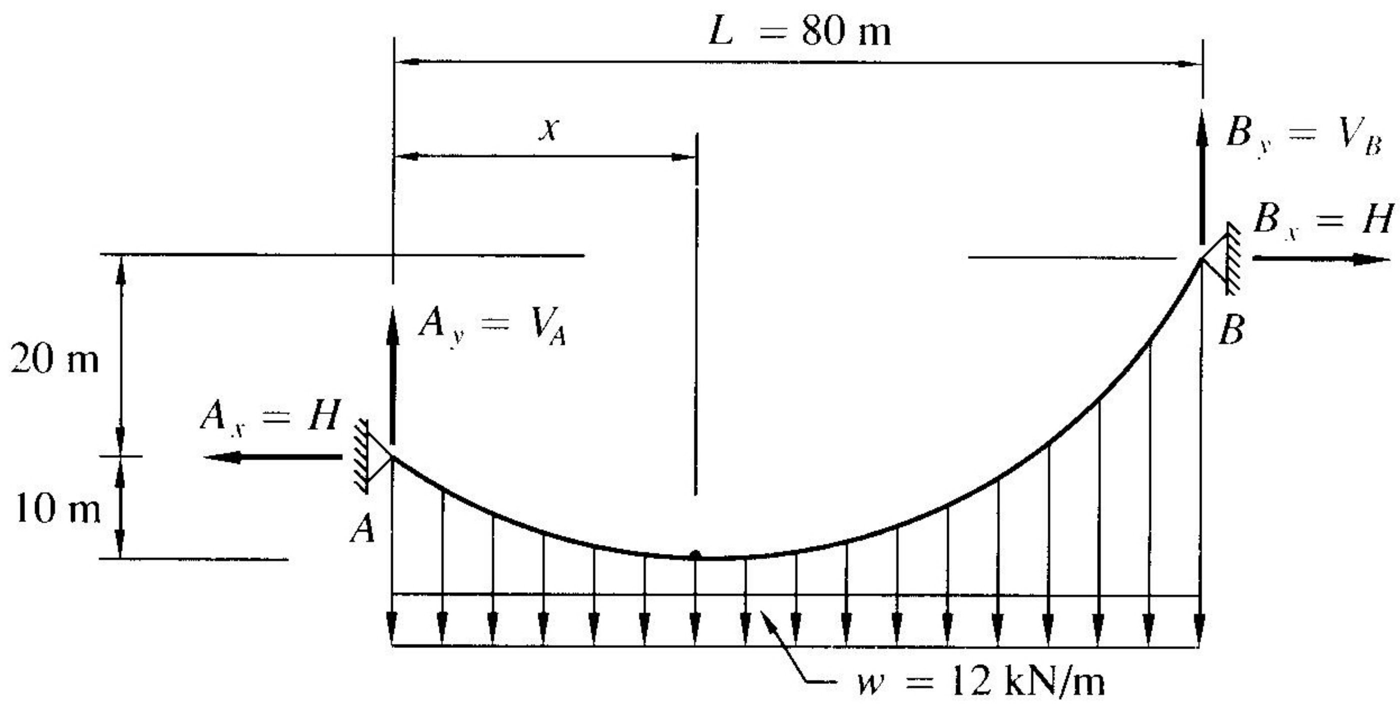


Figure 4.12

CONTS

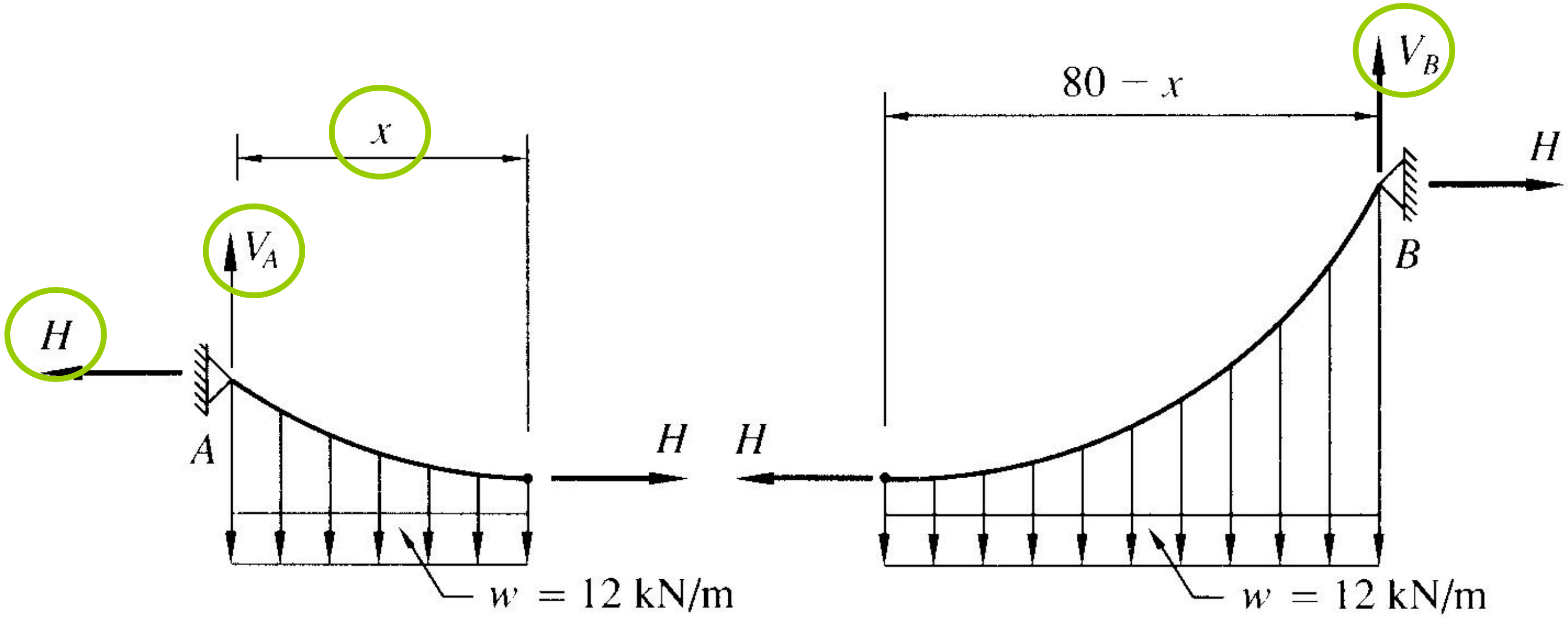


Figure 4.13

From Equation 2.8, we have for the left-hand part

$$\sum M_A = -10H + \frac{wx^2}{2} = 0 \tag{1}$$

For the right-hand part, we have

$$\sum M_B = 30H - \frac{w(L-x)^2}{2} = 0 \tag{2}$$

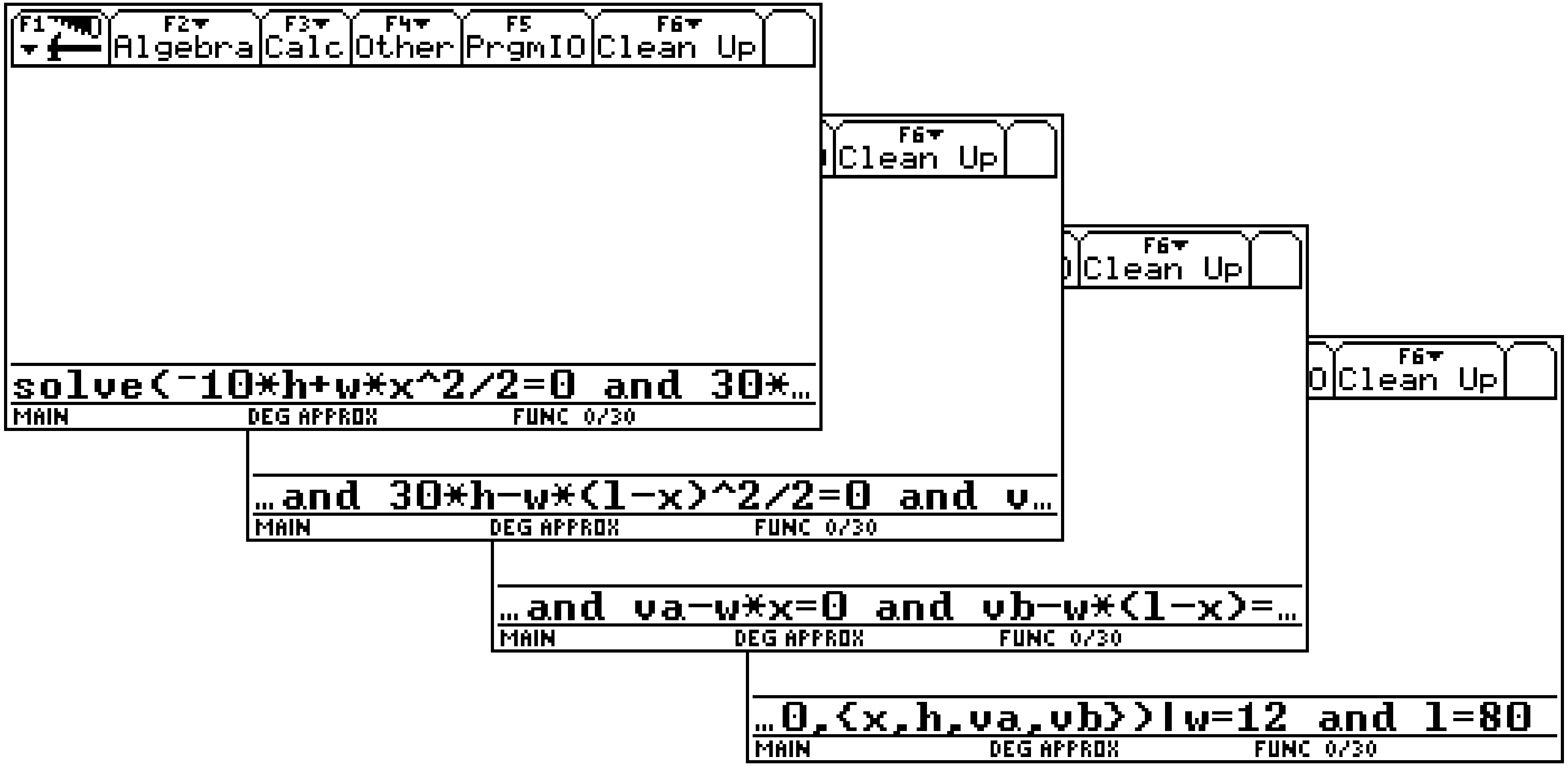
From Equation 2.7, we have for the left-hand part

$$\sum F_y = V_A - wx = 0 \tag{3}$$

For the right-hand part, we have

$$\sum F_y = V_B - w(L-x) = 0 \tag{4}$$

```
solve(-10h+w*x^2/2=0 and
 30h-w*(1-x)^2/2=0 and va-w*x=0 and
 vb-w*(l-x)=0,{x,h,va,vb})|w=12 and l=80
```

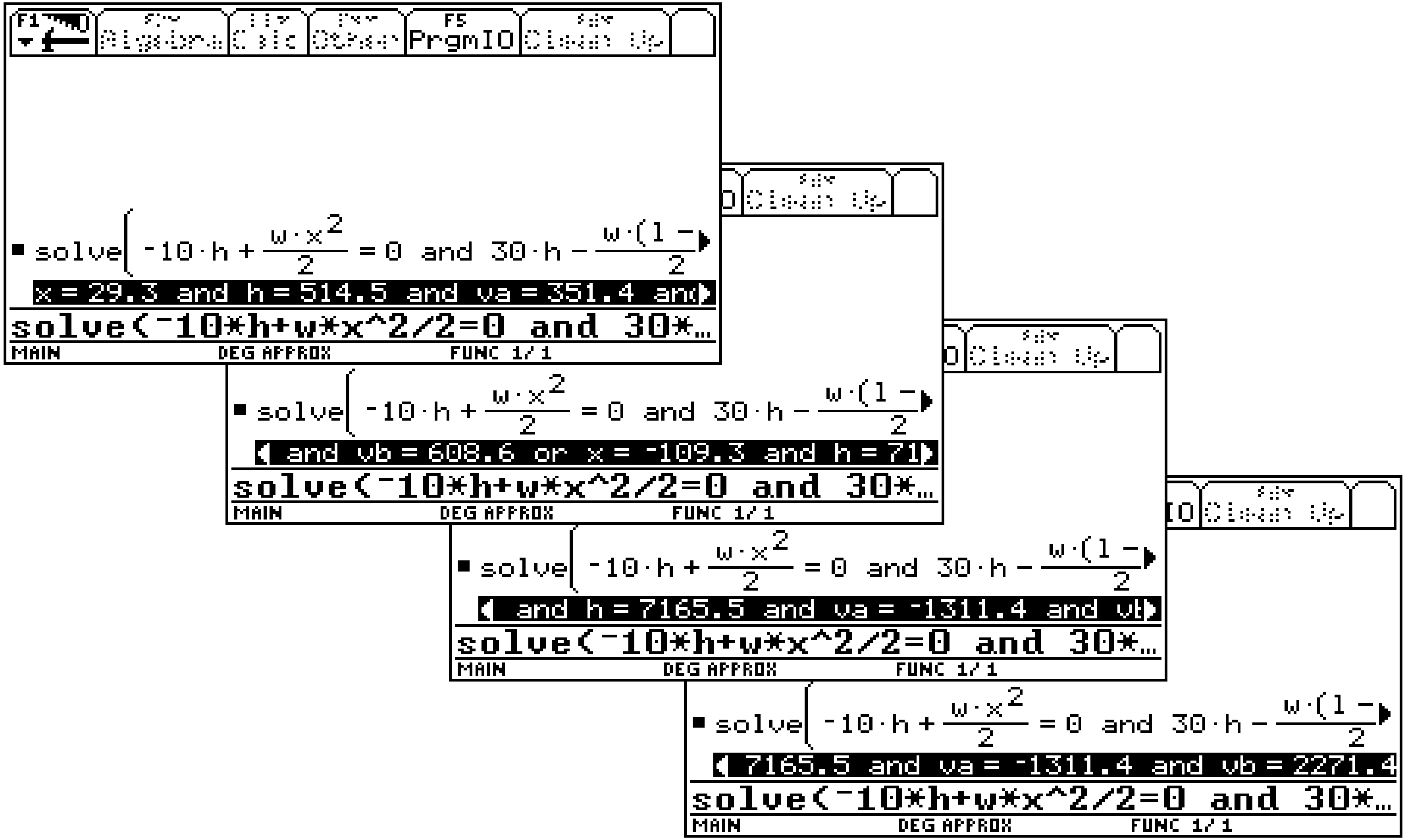


CONTS

x = 29.3 and h = 514.5 and va = 351.4 and vb = 608.6

or

x = -109.3 and h = 7165.5 and va = -1311.4 and vb = 2271.4



CONTS

Therefore, we find

$$x = 29.3,\ \ H = 514.5,\ \ V_A = 351.4,\ \ V_B = 608.6$$

or

$$x = -109.3,\ \ H = 7165.5,\ \ V_A = 1311.4,\ \ V_B = 2271.4$$

where only the first solution has physical meaning.

Finally,

$$T_A = \sqrt{(V_A)^2 + (H)^2} = \sqrt{(351,4)^2 + (514,4)^2} = 623 \text{ kN}$$

et

$$T_B = \sqrt{(V_B)^2 + (H)^2} = \sqrt{(608.6)^2 + (514.4)^2} = 796.9 \text{ kN}$$

The maximum tension in the cable is at support *B*, with the highest elevation.

Structural Analysis
TI voyage 200

- Method of Virtual Work (Samikian, chapter 9)
  - *Symbolic calculations, Mohr's integrals*

CONTS

The work done by the virtual load undergoing the real displacement

is equal to

the work done by the virtual internal forces undergoing the real internal deformations

---

Real Displacement **Real Internal Deformation**

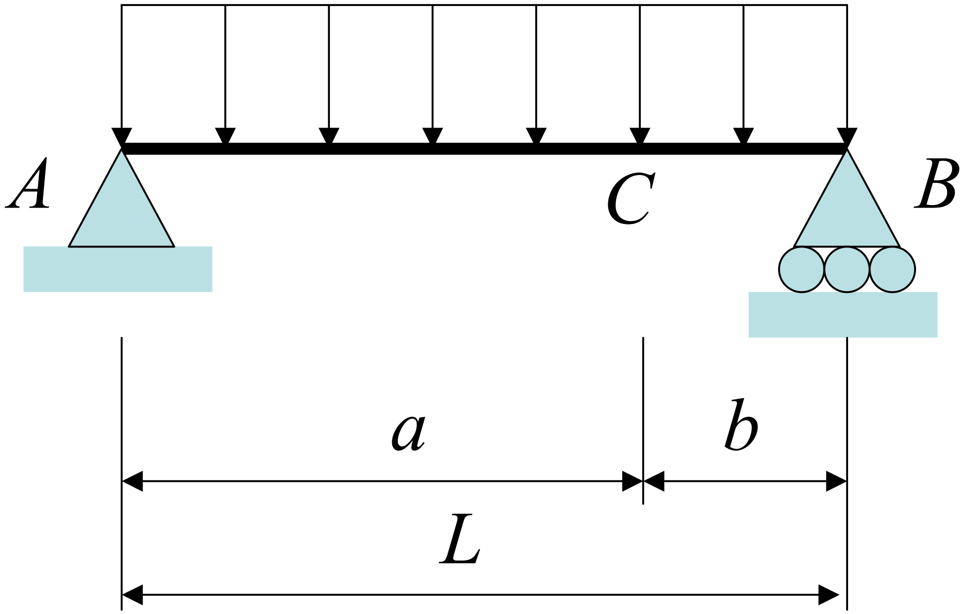
$$\underbrace{1}_{\text{Unit Virtual Load}} \cdot \overbrace{\Delta}^{\text{Real Displacement}} = \int_0^L \underbrace{m}_{\text{Virtual Internal Moment}} \overbrace{\frac{M}{EI}}^{\text{Real Internal Deformation}} dx$$

Unit Virtual Load **Virtual Internal Moment**

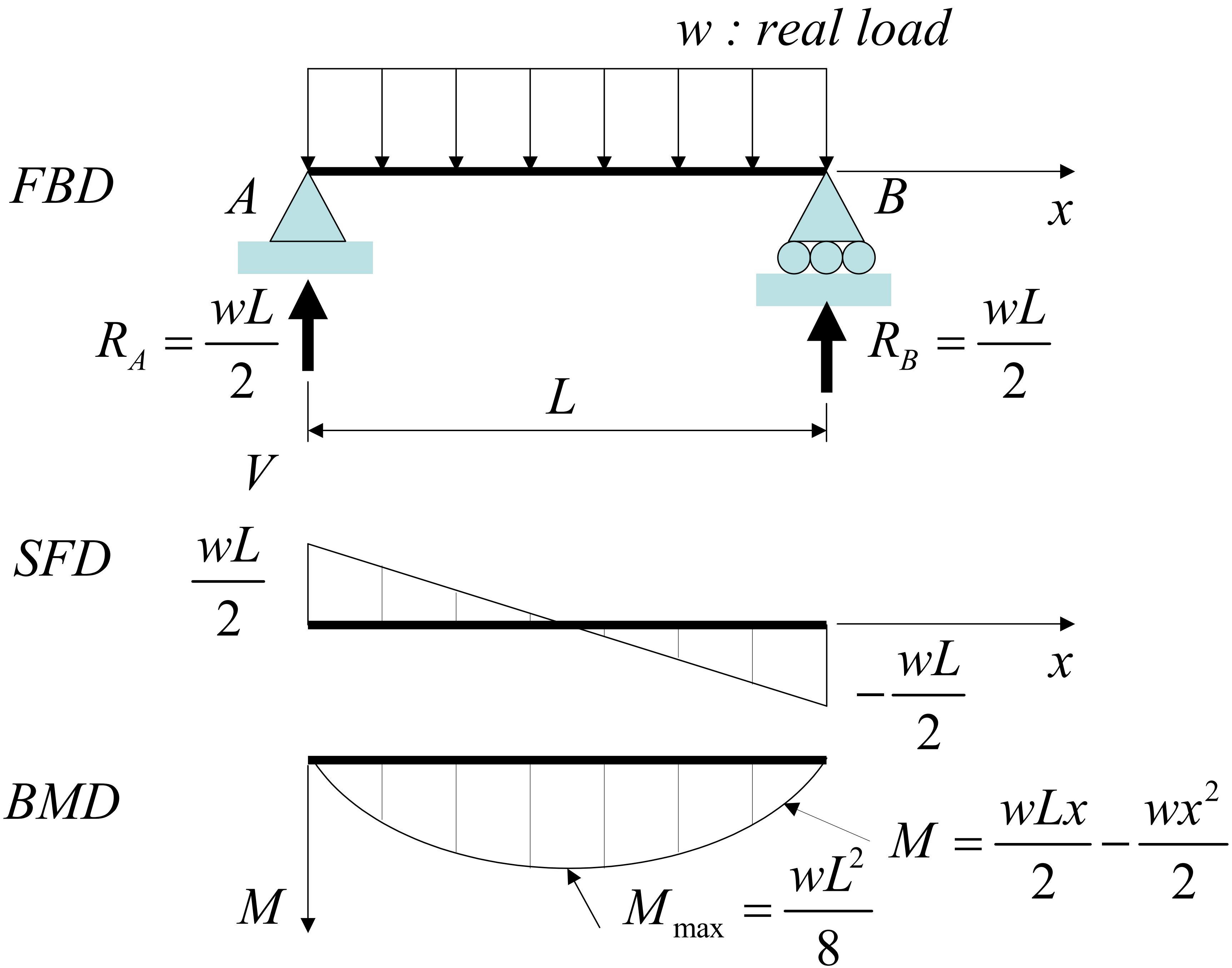
CONTS

Example

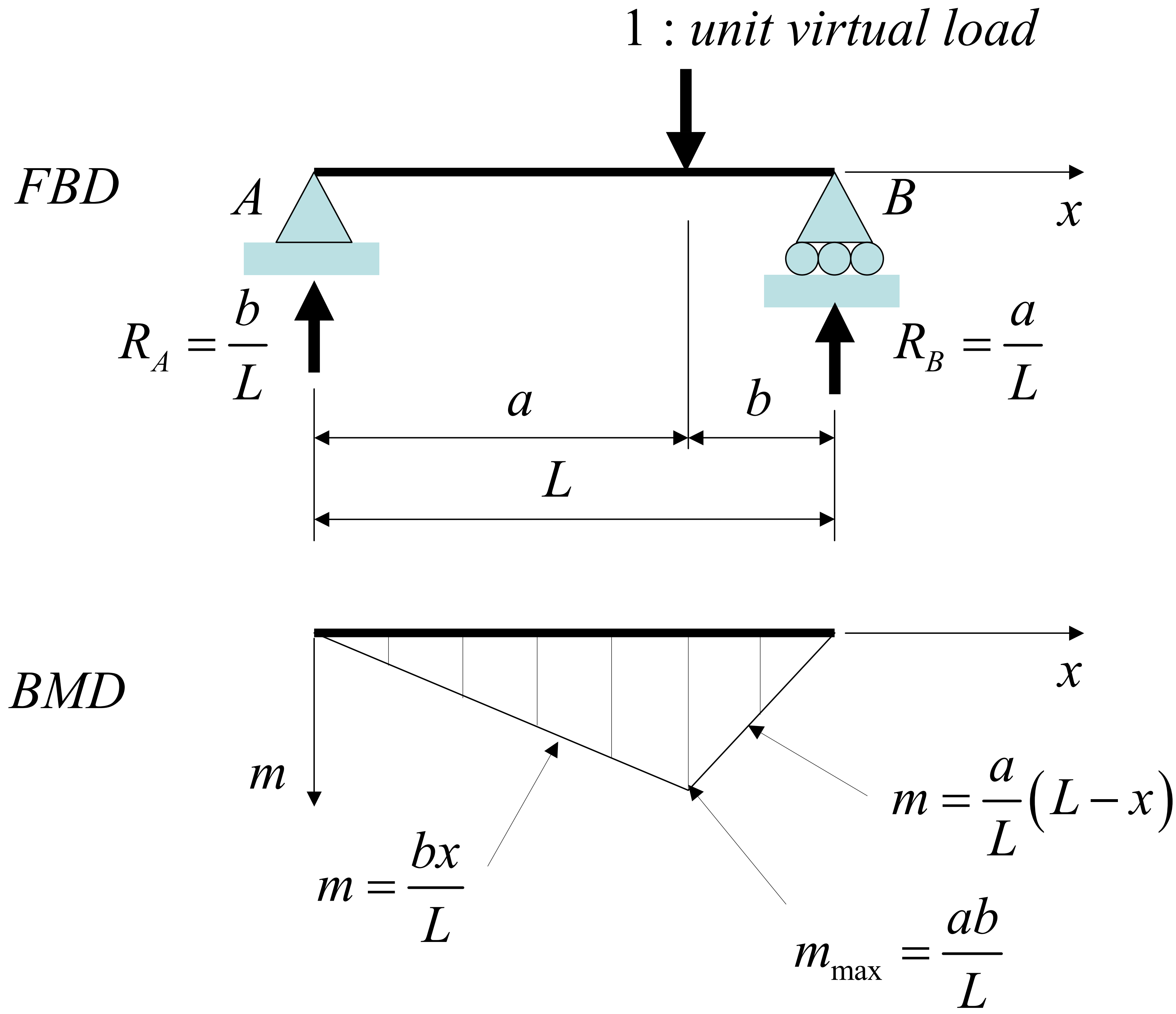
Find the deflection at point C



CONTS



w : real load
FBD
A
B
x
$R_A = \frac{wL}{2}$
$R_B = \frac{wL}{2}$
L
V
SFD
$\frac{wL}{2}$
$-\frac{wL}{2}$
x
BMD
$M = \frac{wLx}{2} - \frac{wx^2}{2}$
M
$M_{\max} = \frac{wL^2}{8}$



1 : unit virtual load
FBD
A
B
x
$R_A = \frac{b}{L}$
$R_B = \frac{a}{L}$
a
b
L
BMD
x
m
$m = \frac{bx}{L}$
$m = \frac{a}{L}(L - x)$
$m_{\max} = \frac{ab}{L}$

CONTS

$$M = \frac{wLx}{2} - \frac{wx^2}{2}$$

$$m = \frac{bx}{L},\ 0 \le x \le a$$

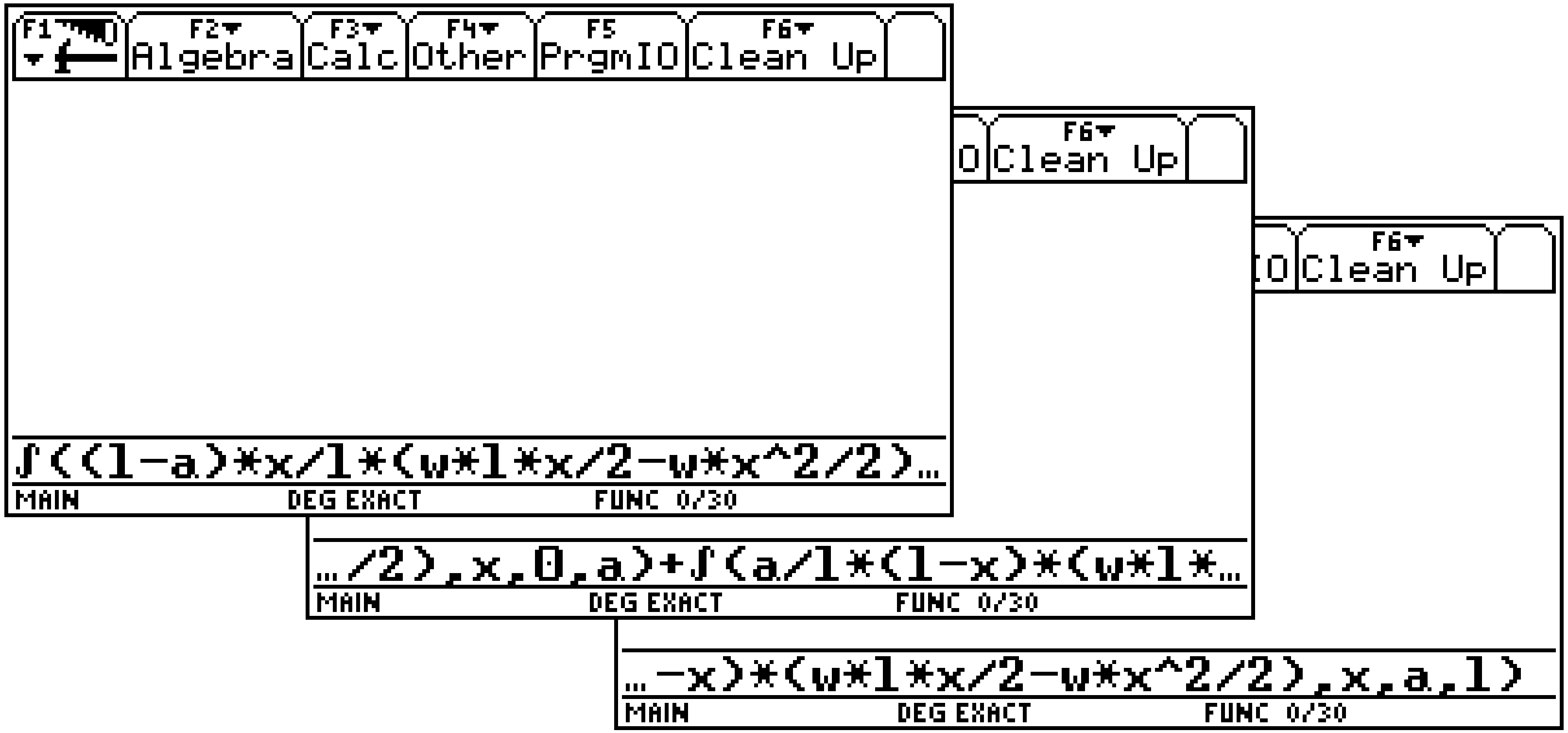
$$m = \frac{a}{L}\left(L - x\right),\ a \le x \le L$$

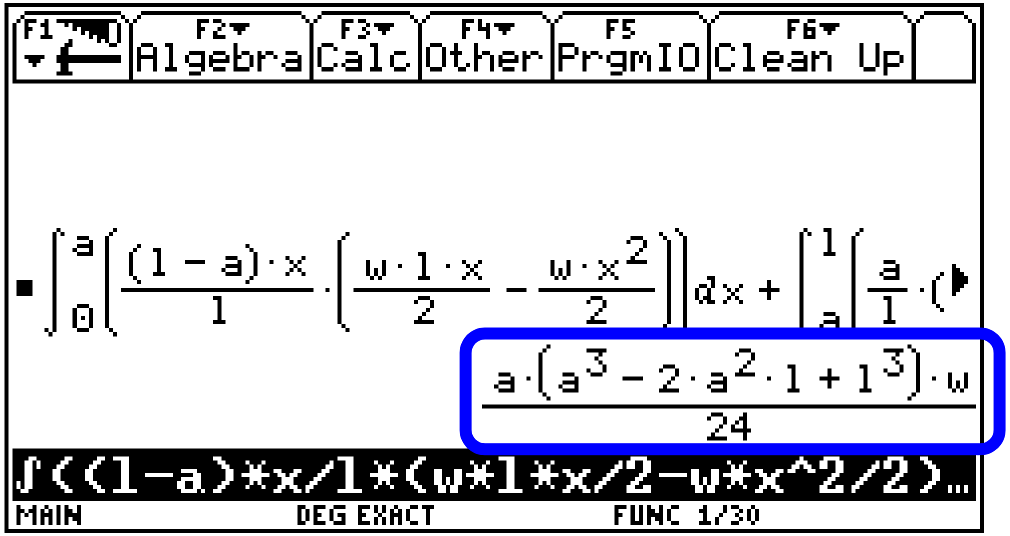
$$1 \cdot \Delta_C = \int_0^L m \frac{M}{EI} dx$$

$$\Delta_C = \int_0^a \frac{(L-a)x}{L}\left(\frac{wLx}{2} - \frac{wx^2}{2}\right) dx + \int_a^L \frac{a}{L}\left(L - x\right)\left(\frac{wLx}{2} - \frac{wx^2}{2}\right) dx$$

CONTS

$$\int_0^a \frac{(L-a)x}{L}\left(\frac{wLx}{2}-\frac{wx^2}{2}\right)dx + \int_a^L \frac{a}{L}(L-x)\left(\frac{wLx}{2}-\frac{wx^2}{2}\right)dx$$





Simple Beam — uniformly distributed load

$R = V$ .... $= \frac{w\ell}{2}$

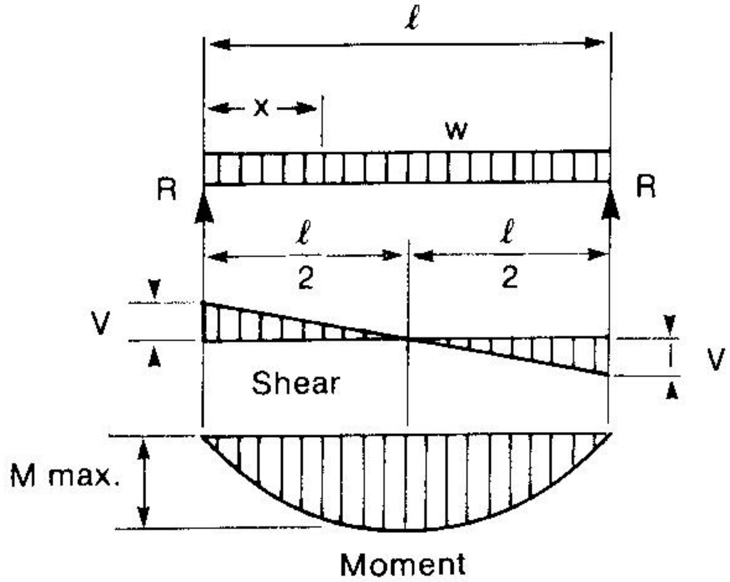
V_x .... $= w\left(\frac{\ell}{2} - x\right)$

M max. (at centre) .... $= \frac{w\ell^2}{8}$

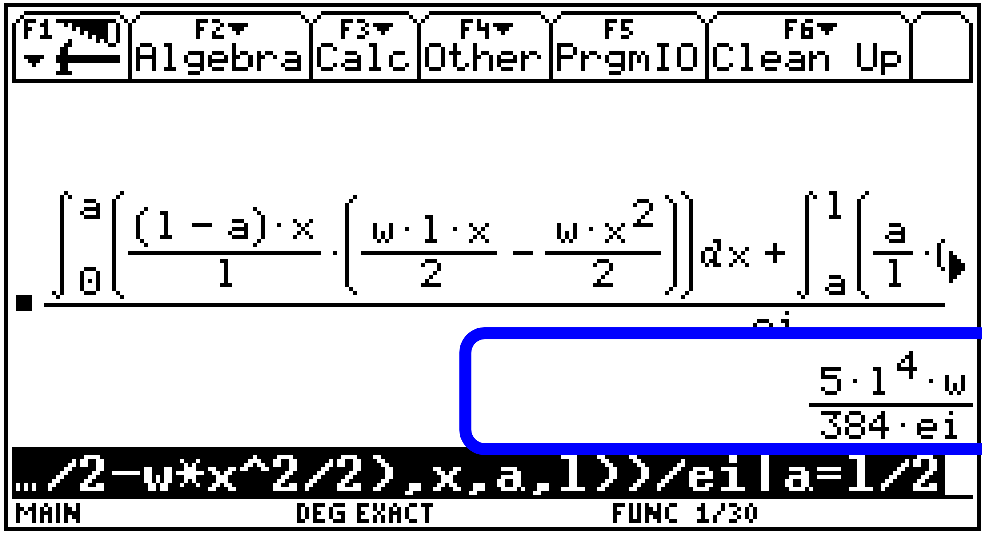
M_x .... $= \frac{wx}{2}(\ell - x)$

Δmax. (at centre) .... $= \frac{5\,w\ell^4}{384\,EI}$

Δ_x .... $= \frac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$



CONTS



When $a = \dfrac{L}{2}$

Simple Beam — uniformly distributed load

$R = V$ .... $= \dfrac{w\ell}{2}$

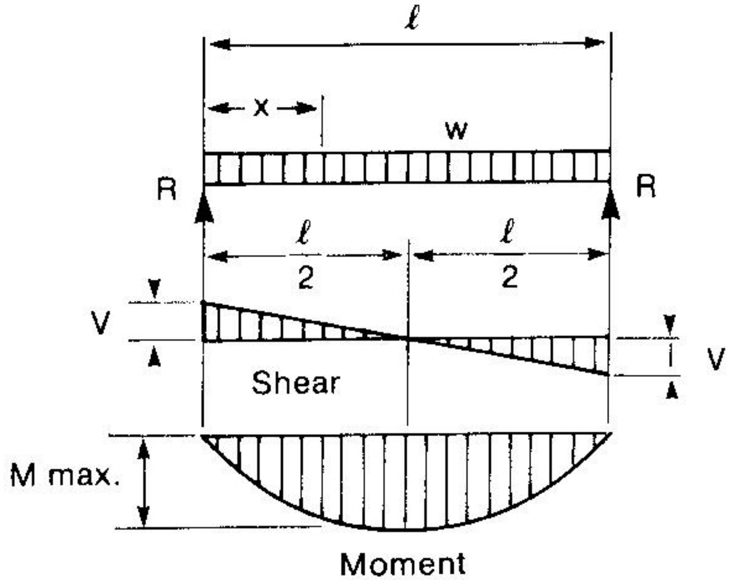
V_x .... $= w\left(\dfrac{\ell}{2} - x\right)$

M max. (at centre) .... $= \dfrac{w\ell^2}{8}$

M_x .... $= \dfrac{wx}{2}(\ell - x)$

Δmax. (at centre) .... $= \boxed{\dfrac{5\,w\ell^4}{384\,EI}}$

Δ_x .... $= \dfrac{wx}{24\,EI}(\ell^3 - 2\ell x^2 + x^3)$

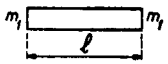
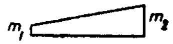
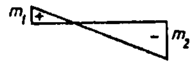
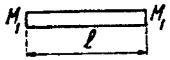
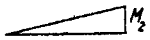
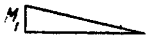
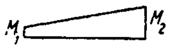
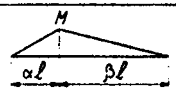


Mohr's Integrals

$$\frac{1}{\ell}\int_0^{\ell} Mm\,dx$$

Reference : « Techniques de l'ingénieur »,
Construction Series, Volume C5, Chapter C2555

Tableau I. — Valeur des intégrales $(1/\ell)\int_0^\ell M\, m\, dx$.

	m			
M				
 $M(x) = M_1$	$M m_1$	$\frac{1}{2} M_1 m_1$	$\frac{1}{2} M_1 (m_1 + m_2)$	$\frac{1}{2} M_1 (m_1 - m_2)$
 $M(x) = M_2 \frac{x}{\ell}$	$\frac{1}{2} M_2 m_1$	$\frac{1}{3} M_2 m_1$	$\frac{1}{6} M_2 (m_1 + 2 m_2)$	$\frac{1}{6} M_2 (m_1 - 2 m_2)$
 $M(x) = M_1 \left(1 - \frac{x}{\ell}\right)$	$\frac{1}{2} M_1 m_1$	$\frac{1}{6} M_1 m_1$	$\frac{1}{6} M_1 (2 m_1 + m_2)$	$\frac{1}{6} M_1 (2 m_1 - m_2)$
 $M(x) = M_1 + \frac{x}{\ell}(M_2 - M_1)$	$\frac{1}{2}(M_1 + M_2) m_1$	$\frac{1}{6}(M_1 + 2 M_2) m_1$	$\frac{1}{6}(2 M_1 m_1 + M_1 m_2 + M_2 m_1 + 2 M_2 m_2)$	$\frac{1}{6}(2 M_1 m_1 + M_2 m_1 - M_1 m_2 - 2 M_2 m_2)$
 $M(x) = M_1 - \frac{x}{\ell}(M_2 + M_1)$	$\frac{1}{2}(M_1 - M_2) m_1$	$\frac{1}{6}(M_1 - 2 M_2) m_1$	$\frac{1}{6}(2M_1 m_1 + M_1 m_2 - M_2 m_1 - 2 M_2 m_2)$	$\frac{1}{6}(2 M_1 m_1 - M_1 m_2 - M_2 m_1 + 2 M_2 m_2)$
 $0 < x < \alpha\ell \quad M(x) = \frac{M}{\alpha}\frac{x}{\ell}$ $\alpha\ell < x < \ell \quad M(x) = \frac{M}{\beta}\left(1 - \frac{x}{\ell}\right)$	$\frac{1}{2} M m_1$	$\frac{1}{6} M m_1 (1 + \alpha)$	$\frac{1}{6} M [m_1 (1 + \beta) + m_2 (1 + \alpha)]$	$\frac{1}{6} M [m_1 (1 + \beta) - m_2 (1 + \alpha)]$
 $M(x) = M_2 \frac{x^2}{\ell^2}$	$\frac{1}{3} M_2 m_1$	$\frac{1}{4} M_2 m_1$	$\frac{1}{12} M_2 (m_1 + 3 m_2)$	$\frac{1}{12} M_2 (m_1 - 3 m_2)$
 $M(x) = M_2 \left(\frac{2x}{\ell} - \frac{x^2}{\ell^2}\right)$	$\frac{2}{3} M_2 m_1$	$\frac{5}{12} M_2 m_1$	$\frac{1}{12} M_2 (3 m_1 + 5 m_2)$	$\frac{1}{12} M_2 (3 m_1 - 5 m_2)$

CONTS

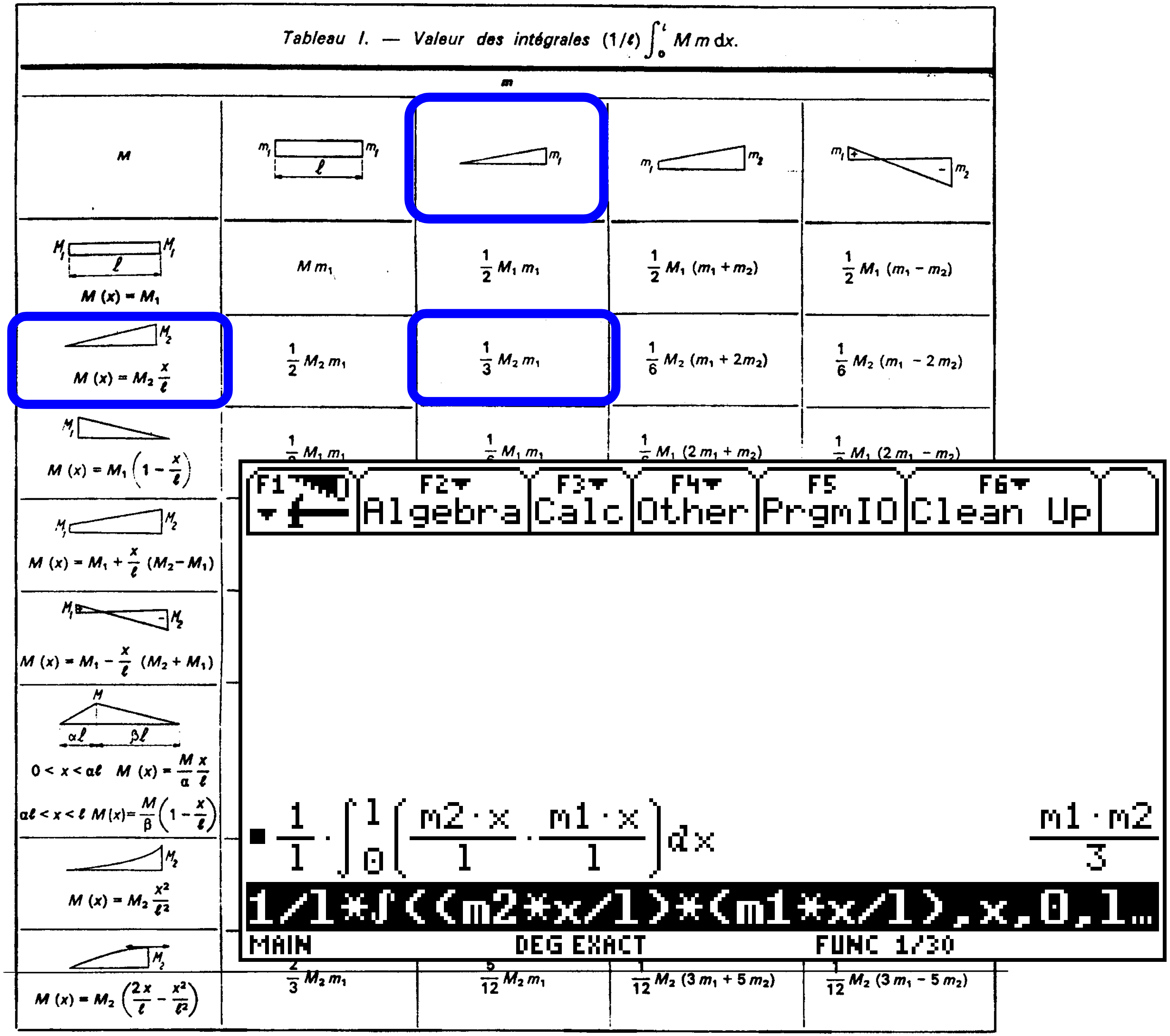


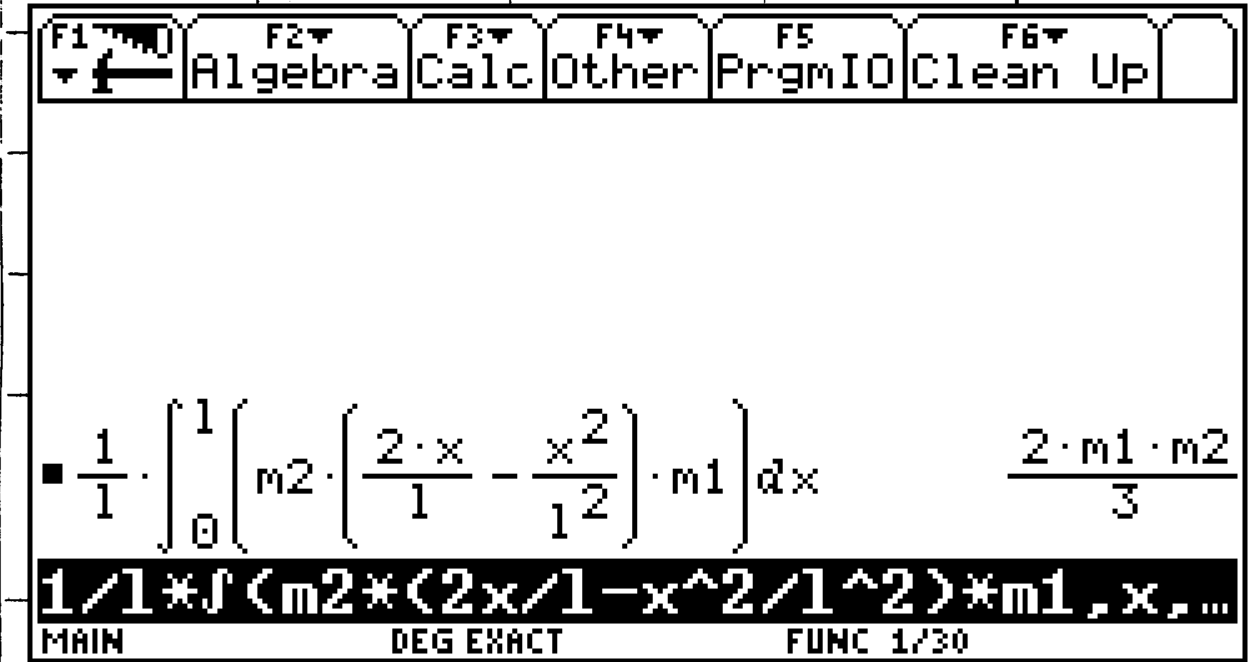
Tableau I. — Valeur des intégrales $(1/\ell)\int_0^\ell M\,m\,dx$.

M	m (rectangle, m_1)	m (triangle, m_1)	m (trapèze, m_1, m_2)	m (m_1 +, − m_2)
$M(x) = M_1$	$M\,m_1$	$\frac{1}{2} M_1 m_1$	$\frac{1}{2} M_1 (m_1 + m_2)$	$\frac{1}{2} M_1 (m_1 - m_2)$
$M(x) = M_2 \frac{x}{\ell}$	$\frac{1}{2} M_2 m_1$	$\frac{1}{3} M_2 m_1$	$\frac{1}{6} M_2 (m_1 + 2m_2)$	$\frac{1}{6} M_2 (m_1 - 2m_2)$
$M(x) = M_1\left(1 - \frac{x}{\ell}\right)$	$\frac{1}{[illegible]} M_1 m_1$	$\frac{1}{[illegible]} M_1 m_1$	$\frac{1}{[illegible]} M_1 (2m_1 + m_2)$	$\frac{1}{[illegible]} M_1 (2m_1 - m_2)$
$M(x) = M_1 + \frac{x}{\ell}(M_2 - M_1)$				
$M(x) = M_1 - \frac{x}{\ell}(M_2 + M_1)$				
$0 < x < \alpha\ell \quad M(x) = \frac{M}{\alpha}\frac{x}{\ell}$ $\alpha\ell < x < \ell \quad M(x) = \frac{M}{\beta}\left(1 - \frac{x}{\ell}\right)$				
$M(x) = M_2 \frac{x^2}{\ell^2}$				
$M(x) = M_2\left(\frac{2x}{\ell} - \frac{x^2}{\ell^2}\right)$	$\frac{2}{3} M_2 m_1$	$\frac{5}{12} M_2 m_1$	$\frac{1}{12} M_2 (3m_1 + 5m_2)$	$\frac{1}{12} M_2 (3m_1 - 5m_2)$

CONTS

Tableau I. — Valeur des intégrales $(1/\ell)\int_0^\ell M\,m\,dx$.

M \ m	m_1 ▭ m_1 (ℓ)	◺ m_1	m_1 ⏢ m_2	m_1 (+) (−) m_2
$M(x) = M_1$	$M\,m_1$	$\frac{1}{2} M_1 m_1$	$\frac{1}{2} M_1 (m_1 + m_2)$	$\frac{1}{2} M_1 (m_1 - m_2)$
$M(x) = M_2 \frac{x}{\ell}$	$\frac{1}{2} M_2 m_1$	$\frac{1}{3} M_2 m_1$	$\frac{1}{6} M_2 (m_1 + 2m_2)$	$\frac{1}{6} M_2 (m_1 - 2m_2)$
$M(x) = M_1 \left(1 - \frac{x}{\ell}\right)$				
$M(x) = M_1 + \frac{x}{\ell}(M_2 - M_1)$				
$M(x) = M_1 - \frac{x}{\ell}(M_2 + M_1)$				
$\alpha\ell$, $\beta\ell$; $0 < x < \alpha\ell$ $M(x) = \frac{M}{\alpha}\frac{x}{\ell}$; $\alpha\ell < x < \ell$ $M(x) = \frac{M}{\beta}\left(1 - \frac{x}{\ell}\right)$				
$M(x) = M_2 \frac{x^2}{\ell^2}$				
$M(x) = M_2 \left(\frac{2x}{\ell} - \frac{x^2}{\ell^2}\right)$	$\frac{2}{3} M_2 m_1$	$\frac{5}{12} M_2 m_1$	$\frac{1}{12} M_2 (3m_1 + 5m_2)$	$\frac{1}{12} M_2 (3m_1 - 5m_2)$



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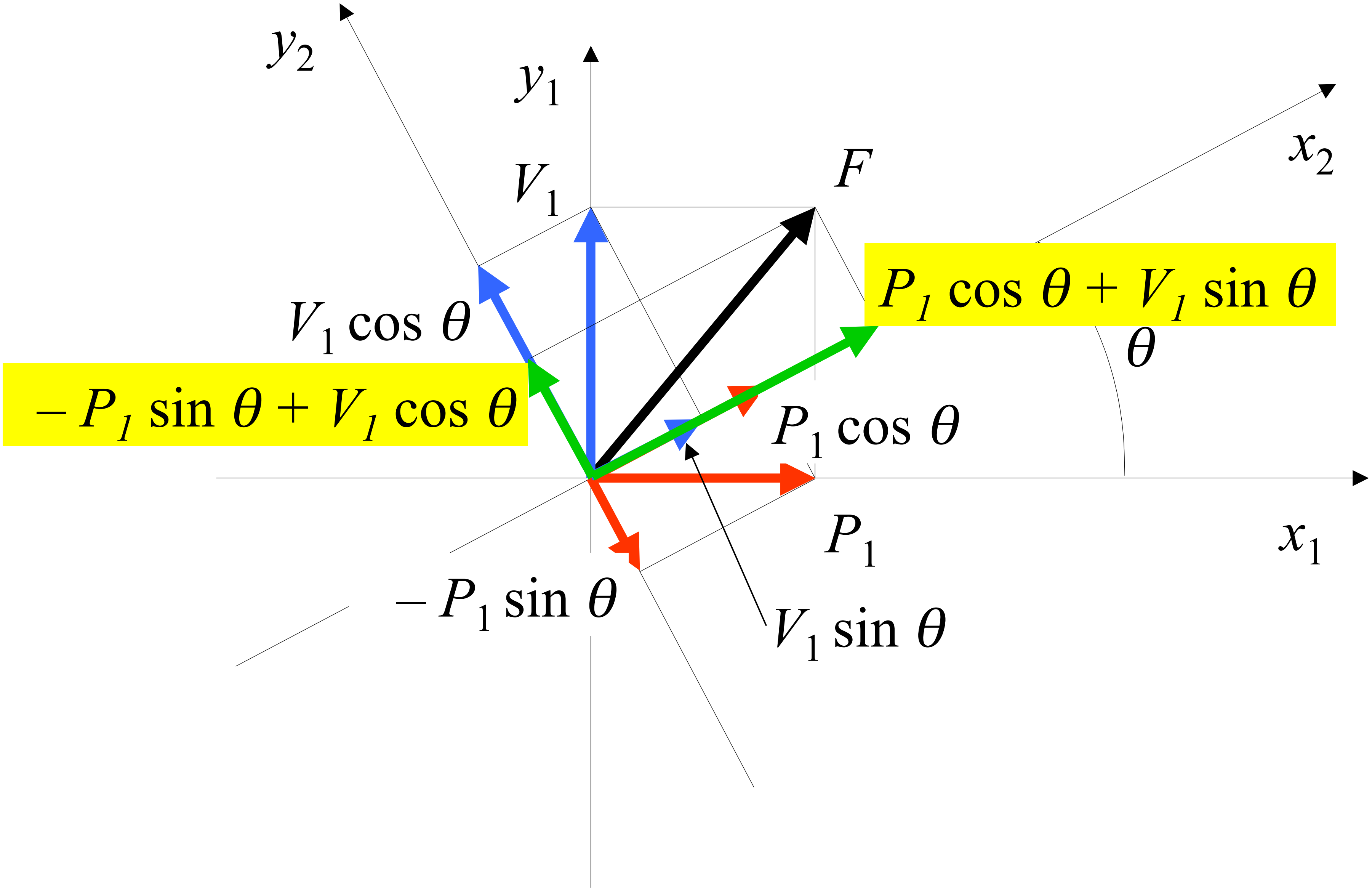
- Coordinate Transformations

Coordinate Transformations



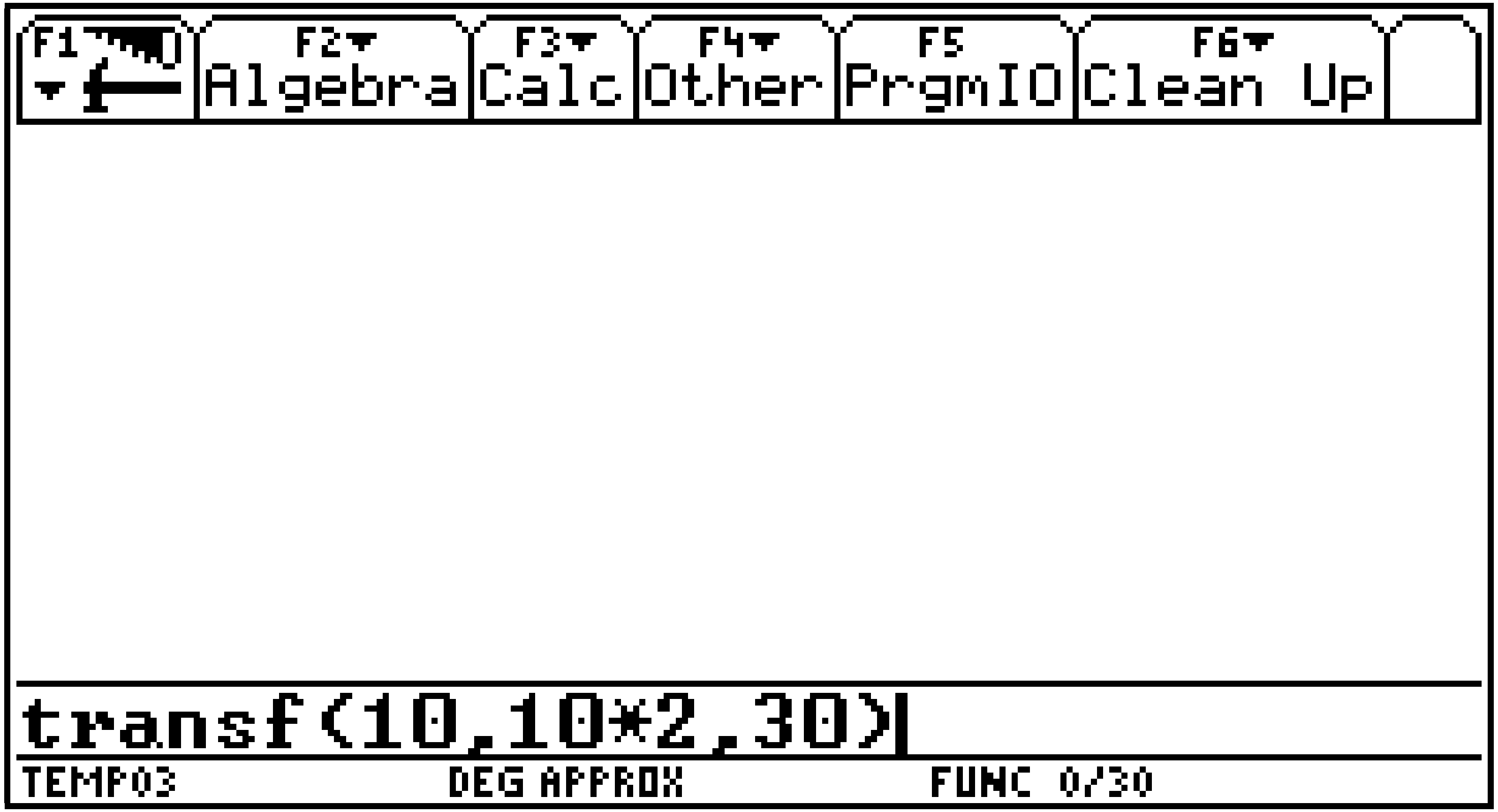
The `transf(p1,v1,θ)` program transforms the components (p1,v1) of a vector defined in the $x_1 y_1$ Cartesian coordinate system into components (p2,v2) in the $x_2 y_2$ Cartesian coordinate system.

The $x_2 y_2$ coordinate system makes an angle θ with the $x_1 y_1$ system, where θ is positive when measured anticlockwise.

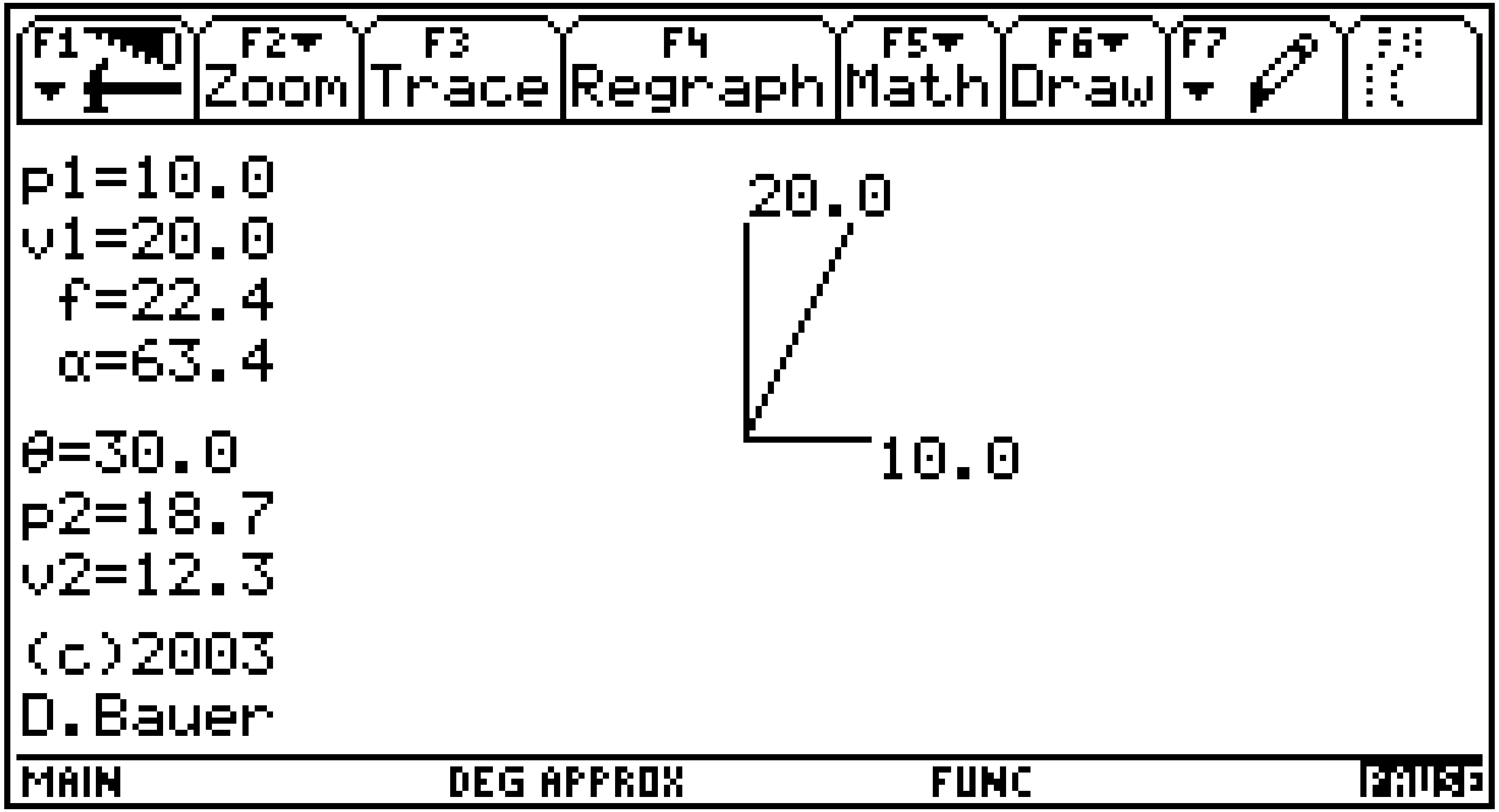
1. Download the `transf` program (file : transf.9xp) from the Web site onto your computer

2. Transfer the `transf` program from your computer to the calculator using TI Connect (USB cable) or TI-Graph Link (serial cable)

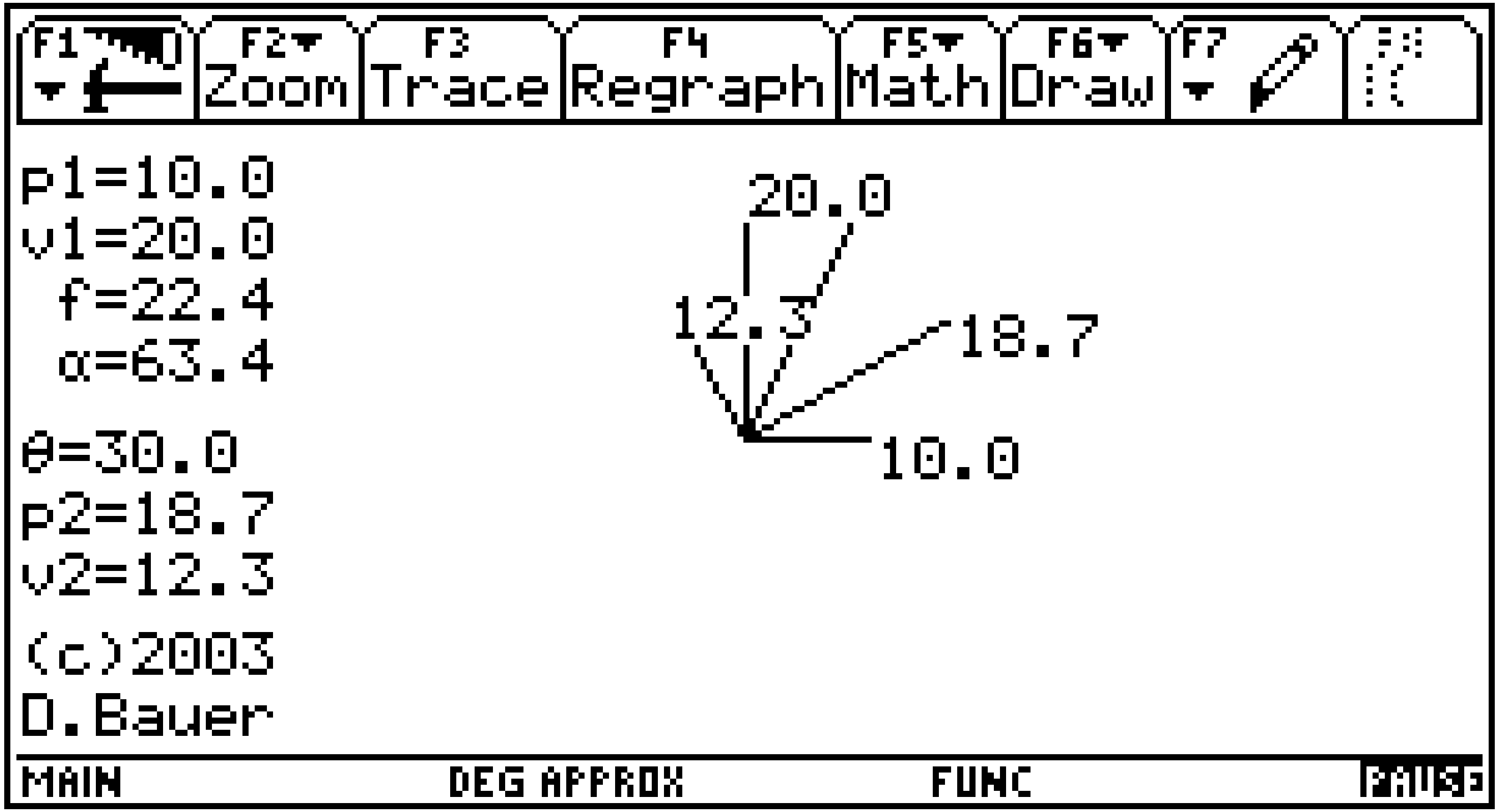
3. Once the program is loaded in the calculator, give the command `transf(p1,v1,θ)`. In the following example, p1=10, v1=10*2=20 and $\theta = 30°$.



The program displays the values of p1, v1, θ and p2, v2. Also, a drawing is shown for checking purposes.



CONTS



F1
F2
Zoom
F3
Trace
F4
Regraph
F5
Math
F6
Draw
F7
p1=10.0
v1=20.0
f=22.4
α=63.4
θ=30.0
p2=18.7
v2=12.3
(c)2003
D.Bauer
20.0
12.3
18.7
10.0
MAIN
DEG APPROX
FUNC
PAUSE

CONTS

```
(p1,v1,θ)
Prgm

√(p1^2+v1^2)→f

tan⁻¹(v1/p1)→α

p1*cos(θ)+v1*sin(θ)→p2

⁻p1*sin(θ)+v1*cos(θ)→v2
```

```
ClrDraw
setGraph("axes","off")

max({p1,v1,p2*cos(θ),p2*sin(θ),
⁻v2*sin(θ),v2*cos(θ)})→gmax

min({p1,v1,p2*cos(θ),p2*sin(θ),
⁻v2*sin(θ),v2*cos(θ)})→gmin

max(abs(gmax),abs(gmin))→gmax

⁻1.2*gmax→xmin

⁻1.2*gmax→ymin

1.2*gmax→xmax

1.2*gmax→ymax

ZoomSqr
```

```
PxlText "p1="&string(p1),5,Ø
PxlText "v1="&string(v1),15,Ø
PxlText " f="&string(f),25,Ø
PxlText " α="&string(α),35,Ø

PxlText "θ="&string(θ),5Ø,Ø
PxlText "p2="&string(p2),6Ø,Ø
PxlText "v2="&string(v2),7Ø,Ø

PxlText "(c)2ØØ3",83,Ø
PxlText "D.Bauer",93,Ø
```

CONTS

```
Line  Ø,Ø,p1,v1
Line  Ø,Ø,p1,Ø
Line  Ø,Ø,Ø,v1

PtText string(p1),p1,Ø
PtText string(v1),Ø,v1

Pause
```

CONTS

```
Line  Ø,Ø,p2*cos(θ),p2*sin(θ)
Line  Ø,Ø,⁻v2*sin(θ),v2*cos(θ)

PtText string(p2),p2*cos(θ),p2*sin(θ)
PtText string(v2),⁻v2*sin(θ),v2*cos(θ)

Pause

setGraph("axes","on")
DispHome
EndPrgm
```

CONTS

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- Acknowledgments
  – *The present research was made possible by grants from the École de technologie supérieure (PSIRE-ENS 2001,2003).*