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Developing control over the use of a CAS

The teacher's perspective

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The context

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- École de technologie supérieure (ÉTS)
  - Technical engineering school in Montreal.
  - Use of CAS integrated in the first year common core mathematics and science courses.
  - Since 1999, TI CAS mandatory for all full-time students.
  - Students expected to use it in class, at home, and during exams.
- Emergence of a common culture among math teachers for computer-assisted calculus
  - Use of the same textbook for a given course.
  - Design of a common final exam for all groups.
  - Exchange of "good problems".
  - Ad hoc discussions.
  - Traditional calculus maintained with some problems where use of CAS is forbidden.

Still…

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- Questions and concerns remain on:
  - Virtues and potential risks of this integration
    - **The issue of control**
  - Impact on students' competencies
  - Identification of best practices
- Individual teacher variations can be observed in:
  - Goals pursued
  - Nature of tasks given to students
  - Assessment

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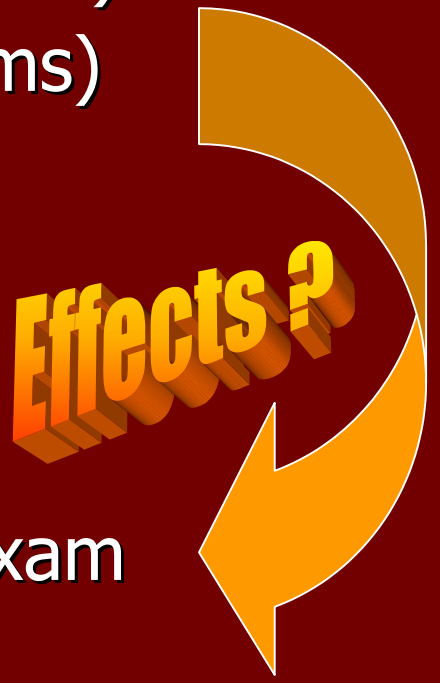
The study

- 5 teachers of Calculus I
  - Interviews (goals, representations, …)
  - Task analysis (homework and exams)
  - Comparison of assessment scales for the common final exam
- 200 students
  - Productions in the final common exam (competencies displayed)



Zooming in : the teacher component

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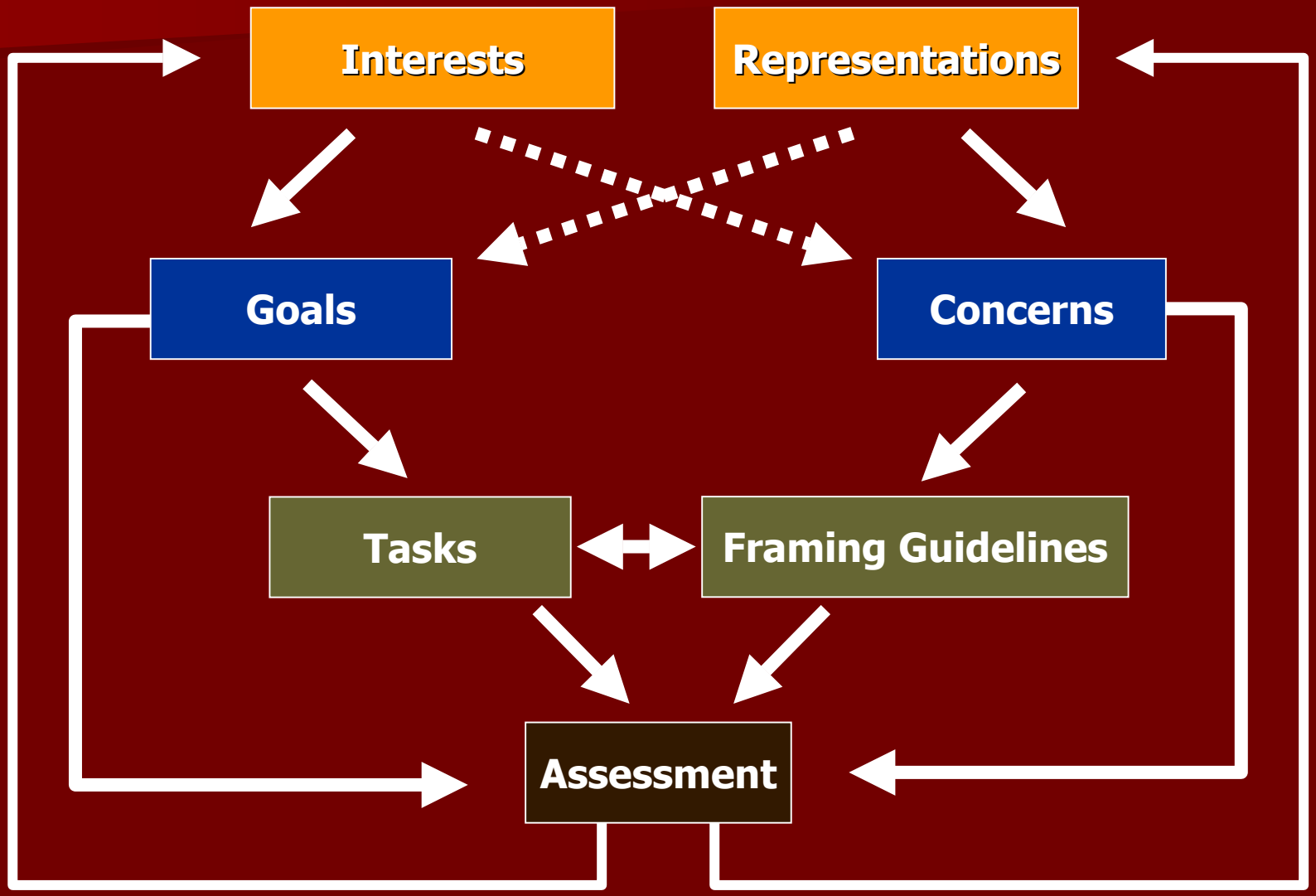

Meet the teachers

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- Alfred
- Bob
- Charlotte
- Diane
- Eric

What they like in math…

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Alfred	Precision, deduction Calculus and analysis
Bob	Method of thinking, reasoning, logic Possibility to retrieve old knowledge
Charlotte	Absolute, purity, richness Complementarity of the representations
Diane	Abstraction, rigor and beauty
Eric	Freedom, ability to deduct properties Visualizing with technology

Their perception of their students…

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Alfred	Weak in math; no particular strength; situation is getting worse over time
Bob	Lack a sound basis in math; hard working; facility with technology
Charlotte	Limited scientific culture; see math as a series of algorithms; hands-on
Diane	Heterogeneity in math background; respectful and pleasant
Eric	Technically oriented and resourceful; will appreciate theory if explained

Their teaching objectives: Students should…

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Alfred	Master the basics; grasp the meaning of the concepts
Bob	Develop problem solving methodology; become independent and critical of technology
Charlotte	Perceive and use math as a language and a tool for reflection
Diane	Consolidate their knowledge; develop abstraction, rigor and imagination in solving problems; see the role of math in applications
Eric	See that they can « build » mathematics & understand the necessity of conditions

Perceived value of CAS in learning math

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Alfred	Relieve students from tidious calculations
Bob	Allow use of numerical and graphical; use more complex functions in problems
Charlotte	Allow visualization, build new links and remove focus from algebra
Diane	Allow more complex problems ; shift focus towards modeling, solving, and validating
Eric	Do more in less time; test hypotheses; adapt to students' needs (from a crutch to an exploratory tool); new classroom dynamic

Perceived risks in using CAS while learning math

Alfred	Dependence
Bob	Forgetting algebra; ignoring what the CAS does; believing in anything produced by the CAS; getting lost in the process
Charlotte	Time required for learning how to use it; blind acceptance of answers
Diane	Limited reuse & consolidation of knowledge; reduced incentive for learning calculus
Eric	Dependence; disadvantaging teachers that are not technically-oriented

Task Analysis: Complexity and Competencies

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Levels of complexity	Communication competencies	Evaluation competencies	Intervention competencies
Level 4: **Reformulation**	**Combine complementary models**	**Adapt a solving method**	
Level 3: **Structuration**	**Establish and justify a property**		
Level 2: **Comprehension**	**Interpret data or results**	**Identify distinct cases**	
Level 1: **Association**	**Recognize object**	**Match object-ppty**	**Apply method**

De Terssac (1996); Caron (2001)

Distribution of Tasks across Complexity Levels

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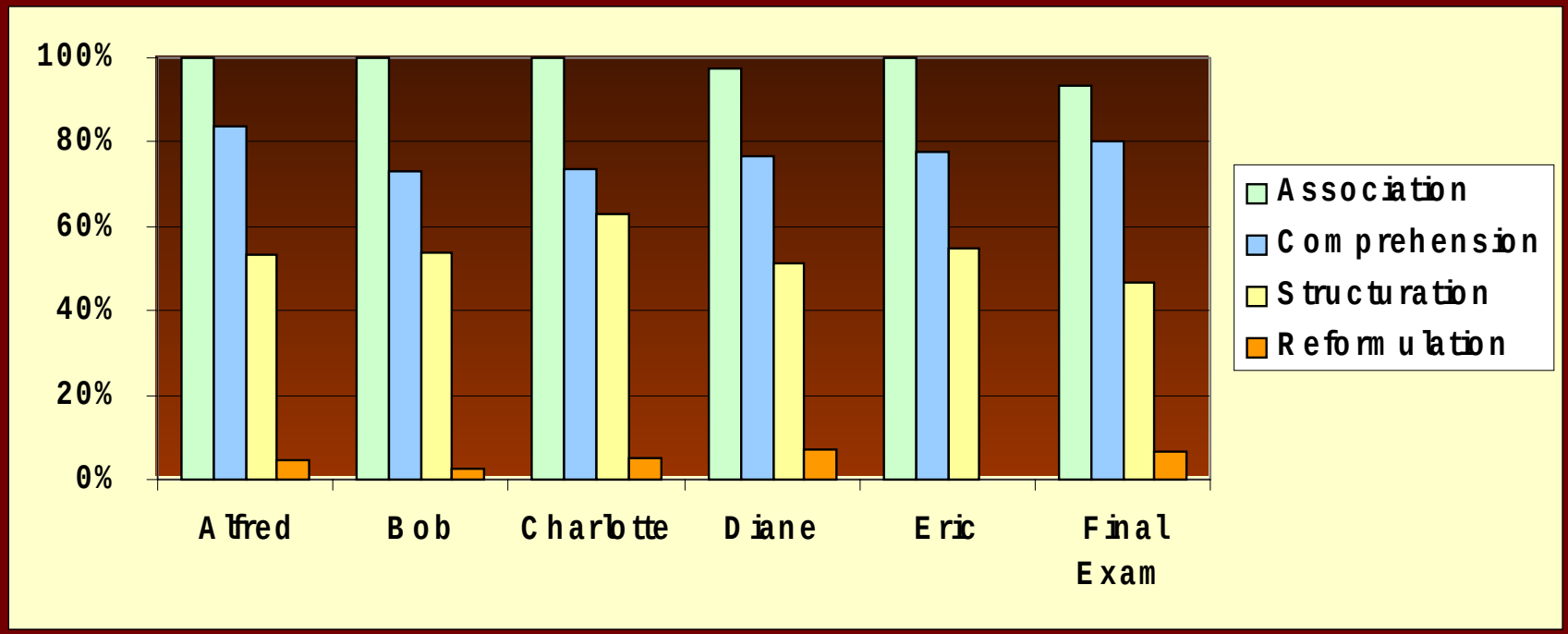

Some typical tasks -1

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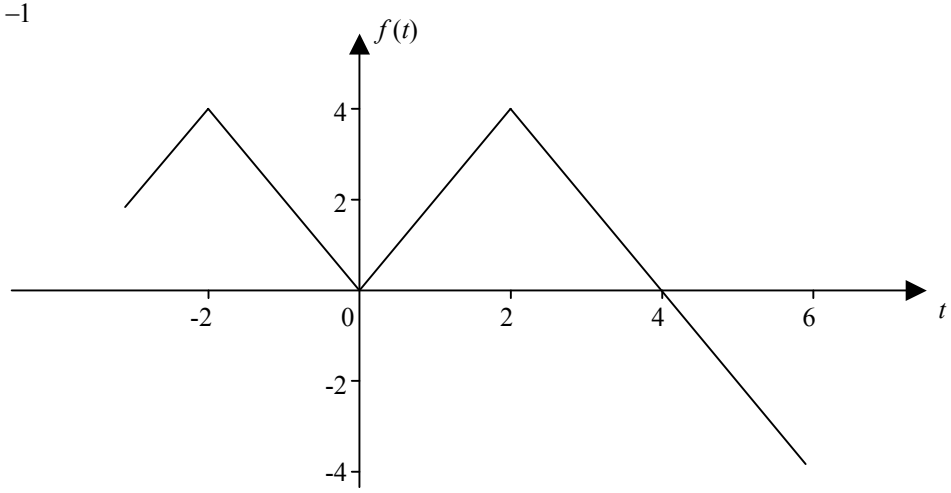
Soit $G(x) = \int_{-1}^{x} f(t)dt$ où le graphe de $f(t)$ est donné à la figure suivante :



a) Estimez $G(2)$ et $G(-2)$. *Justifiez.*

b) Estimez $G'(1)$. *Justifiez.*

c) La fonction $G(x)$ est croissante sur quel intervalle entre -1 et 6? *Justifiez.*

d) La fonction $G(x)$ est concave vers le haut sur quel intervalle entre -1 et 6? *Justifiez.*

Some typical tasks -2

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Trouvez la valeur positive k telle que l'aire de la région comprise entre le graphe $y = k \cos x$ et $y = k x^2$ est 2.

Indiquez clairement l'intégrale définie utilisée.

On veut fabriquer une boîte de base carrée ouverte ayant un volume de 32 m^3. Quelles doivent être les dimensions de la boîte pour minimiser le coût ?

Tasks Where Use of CAS is...

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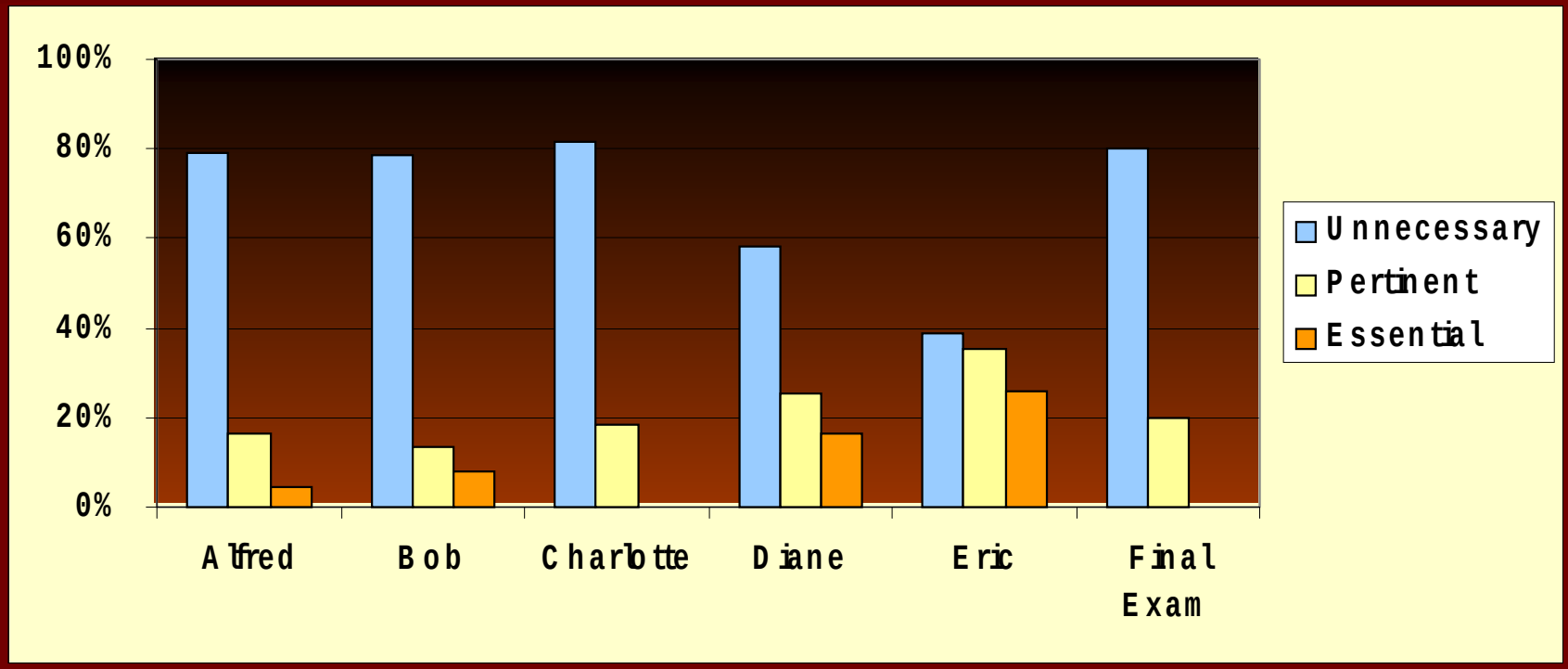

Some specificities in tasks

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Alfred	Working out and validating algebra Numerical analysis
Bob	Comprehension through modeling Structuration through justifying
Charlotte	Comprehension through translation Structuration through justifying Exercising critical judgment over CAS result
Diane	Applications and modeling Graphing and estimating to orient and validate Comparison of TI calculator with Maple
Eric	Exploring objects and discovering properties Extensive use of symbolic and graphical functionalities

Numerical Analysis with Alfred

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a) Montrez analytiquement que $\int_0^1 \frac{4}{1+x^2}\,dx=\pi$

b) Complétez le tableau suivant (en gardant 6 décimales)

	$n = 10$	$n = 20$
GAUCHE (n)		
DROITE (n)		
MI (n)		
TRAP (n)		
SIMPSON (n)		

c) Déterminez le nombre de subdivisions nécessaires pour obtenir une précision de 8 décimales (i.e. Erreur $\leq 5 \times 10^{-9}$) avec la méthode du trapèze.

d) Déterminez le nombre de subdivisions nécessaires pour obtenir une précision de 8 décimales (i.e. Erreur $\leq 5 \times 10^{-9}$) avec la méthode de Simpson.

[Aide: pour les deux dernières sous-questions, servez-vous de la feuille distribuée en classe et n'évaluez pas les sommes]

Exploring with Eric

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Montrez que $\frac{d^n}{dx^n}(e^x \cos x) = 2^{n/2} e^x \cos(x + \frac{n\pi}{4})$ $(n \in N^*)$

Applying with Diane

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On cherche à calibrer un bâton vertical déposé au fond d'un réservoir de façon à savoir quand le réservoir est rempli au quart, à la demie ou aux trois quarts.

Le réservoir est obtenu par la rotation autour de l'axe des y de la courbe

$$y = 10 - \sqrt{100 - x^2}$$

où $-10 \le x \le 10$. Le bâton correspond au segment de l'axe des y où $0 \le y \le 10$. Ici, x et y sont mesurés en centimètres. Déterminez à quelle hauteur on doit inscrire les marques de calibration sur le bâton.

Expliquez bien chaque étape du raisonnement, posez clairement vos intégrales. Tracez une figure illustrant le réservoir et le bâton calibré (à la main ou avec un logiciel). Arrondissez à la quatrième décimale.

Framing Guidelines

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Teacher	What to include in the solution	What to do with the CAS	Mathematical Hints	Technical Advices
Alfred	✓ ✓	✓ ✓	✓	
Bob	✓	✓		
Charlotte	✓ ✓ ✓	✓ ✓	✓ ✓	✓
Diane *	✓ ✓ ✓	✓ ✓ ✓	✓ ✓	✓ ✓ ✓
Eric	✓	✓ ✓	✓	

* Includes modeling considerations

Assessment Scales for the common exam

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Teacher	More weight to more complex problems	Mapping of solution components with their positive contribution	Mapping of typical errors with their negative contribution
Alfred	✓ ✓	✓ ✓	
Bob	✓ ✓	✓	✓ ✓
Charlotte	✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓
Diane	✓ ✓ ✓	✓ ✓ ✓	✓ ✓ ✓
Eric	✓	✓ ✓	

Some preliminary conclusions

- Teachers' representations do influence, to some extent, their choice of tasks and their assessment.
- Main differences in choice of tasks reside in :
  - Necessity of using the CAS (technical complexity of problems)
  - Functionalities of the CAS used (numerical, graphical, symbolic)
  - Emphasis on modeling and/or applications
  - Validation schemes
- More detailed framing guidelines seem to go with more rigorous assessment scales.
- Impact on students' control and competencies ?
- Role of classroom dynamics ?

To be continued …