CONTS

The great revelation !

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Objective

- Demonstrate how the use of a computer algebra system (Maple) in a physics course can ease the introduction of difficult math subjects by putting forth their use as a tool for solving real situations.

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Content :

- The author’s background
- The students involved
- Using a computer algebra system (Maple) in a physics course, not a mathematics course…
- Using Maple for new discoveries
- Conclusions

The author’s background

- Physics teacher for more than 20 years.
- Discovered Maple a few years ago.
- Organised conferences to stimulate the integration of CAS in math and science teaching at the collegial level (pre-university) in Quebec.
- Uses Maple in the regular physics courses (mechanics, electricity and optics).
- Created an advanced physics course to better prepare students for engineering studies and in which Maple is used more extensively.

Subject examples from this advanced physics course

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- 3D equilibrium situations
- Integrating the drag force in dynamic situations
- Rotation dynamics
- Damped and forced simple vibrating systems
- Exploring the Maxwell equations
- Electromagnetic waves

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The students involved

- Science students out of high school.
- Introduced to calculus in two 75 hour courses.
- Students may choose to take a third calculus course.
- Preparing for first year in engineering degree, some of them take the advanced physics course.
- Their expertise of Maple is..

Calculus introduction subjects

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- Limits and continuity
- Derivation and techniques of derivation
- Introduction to extremum problems
- Introduction to infinite series
- Integration and techniques of integration
- Assortment of theorems related to calculus

- *Most textbooks, either in math or physics, are constructed to avoid situations where complex numbers would appear or numerical analysis would be necessary….*

Advanced calculus subject examples :

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- Two variables functions
- Conics
- Assortment of theorems related to calculus
- Introduction to differential equations
- Introduction to double integration
- Complex variable only taught superficially

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Using a computer algebra system (Maple) in a physics course, not a mathematics course…

- To produce plots
- For fast computation
- For mathematical treatment
- Examples given here are out of the usual path and taken from all of the physics course, regular or advanced.

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To produce plots :

- 3D plot of a potential function from an exercise in the textbook where no visual representation is proposed

```
> V:=2*x^3*y-3*x*y^2*z+5*y*z^3;
```

$$V := 2\,x^3\,y - 3\,x\,y^2\,z + 5\,y\,z^3$$

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```
> eq:=V=1000;
```

$$eq := 2\,x^3\,y - 3\,x\,y^2\,z + 5\,y\,z^3 = 1000$$

```
> plots[implicitplot3d](eq,x=-10..10,y=-10..10,z=-10..10,numpoints=2000,scaling=constrained,axes=normal);
```

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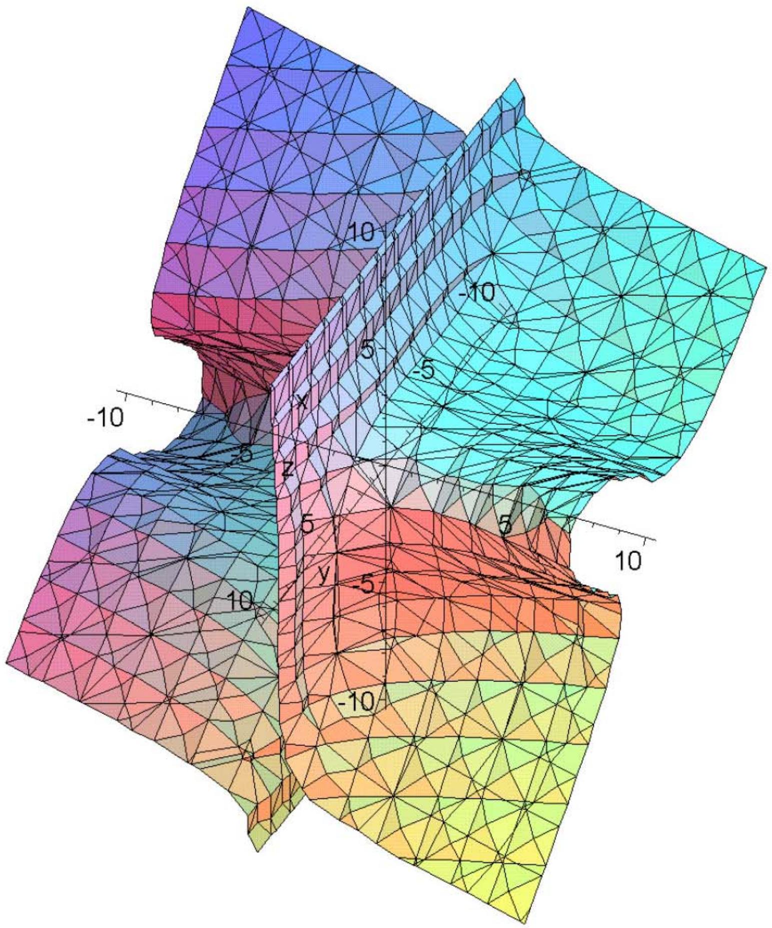

>

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To produce plots :

- Space trajectory of an electric charge in a magnetic field. The figure helps understanding the effect of perpendicular components in the velocity of the particul.

```
> plots[display]({plan,v_perp1,v_perp2,v_para1,v_para2,v_complet1,v_complet2,charge,chemin,BA1,BA2,BB1,BB2,BC1,BC2,BD1,BD2,
nom1,nom2},scaling=constrained,projection=0.9);
```

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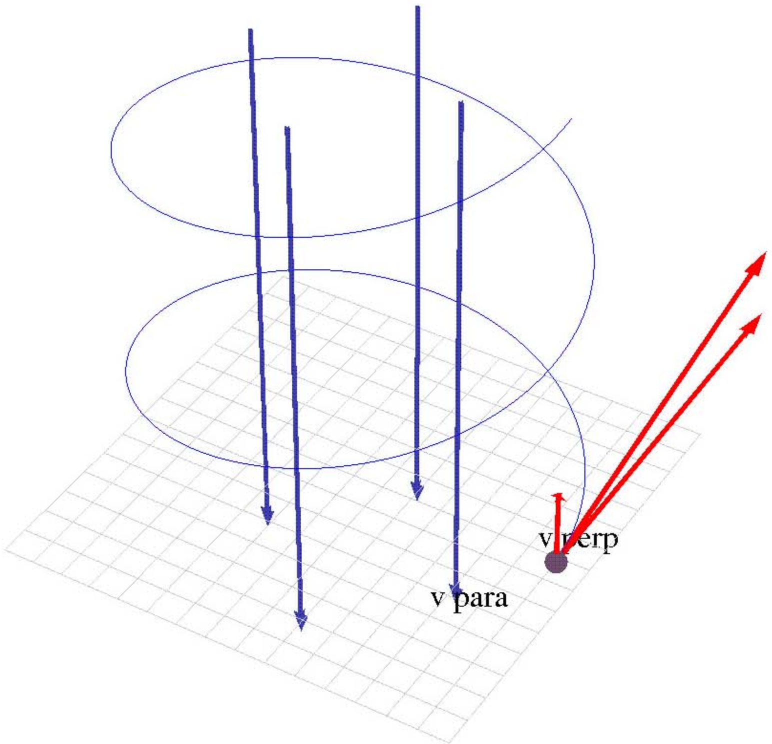

For fast computation :

- Error calculation in a laboratory exercise on lens, focal distances and refraction indexes. The calculation of a refraction index and its estimated error is needed. From a scientist stand point the calculation of the error needs to be obtained from derivation.

```
> eq2:=1/fA=(n-1)*(1/R1-1/R2);
```

$$eq2 := \frac{1}{fA} = (n-1)\left(\frac{1}{R1} - \frac{1}{R2}\right)$$

```
> n:=solve(eq2,n);
```

$$n := \frac{-R1\,R2 - fA\,R2 + fA\,R1}{fA\,(-R2+R1)}$$

```
> dn:='abs(diff(n,R1))*dR1+abs(diff(n,R2))*dR2+abs(diff(n,fA))*dfA';
```

$$dn := \left|\frac{\partial}{\partial R1} n\right| dR1 + \left|\frac{\partial}{\partial R2} n\right| dR2 + \left|\frac{\partial}{\partial fA} n\right| dfA$$

```
> dn:=eval(dn);
```

$$dn := \left|\frac{-R2+fA}{fA\,(-R2+R1)} - \frac{-R1\,R2 - fA\,R2 + fA\,R1}{fA\,(-R2+R1)^2}\right| dR1 + \left|\frac{-R1-fA}{fA\,(-R2+R1)} + \frac{-R1\,R2 - fA\,R2 + fA\,R1}{fA\,(-R2+R1)^2}\right| dR2 + \left|\frac{1}{fA} - \frac{-R1\,R2 - fA\,R2 + fA\,R1}{fA^2\,(-R2+R1)}\right| dfA$$

```
> R1:=0.2;
  dR1:=.01;
  R2:=-0.23;
  dR2:=.01;
  fA:=0.23;
  dfA:=0.02;
```

$$R1 := 0.2$$

$$dR1 := 0.01$$

$$R2 := -0.23$$

$$dR2 := 0.01$$

$$fA := 0.23$$

$$dfA := 0.02$$

```
> 'n'=n; Delta*'n'=dn;
```

$$n = 1.465116279$$

$$\Delta\, n = 0.06228983939$$

```
>
```

For fast computation :

- Calculating the mean speed during a certain time interval. The vanishing time interval translates into instantaneous speed.

Commençons par aller voir le graphique de position en fonction du temps :

```
> plot(x(t),t=0..5,0..16,color=blue,labels=[`t`,`x(t)`],labelfont=[TIMES,ROMAN,12]);
```

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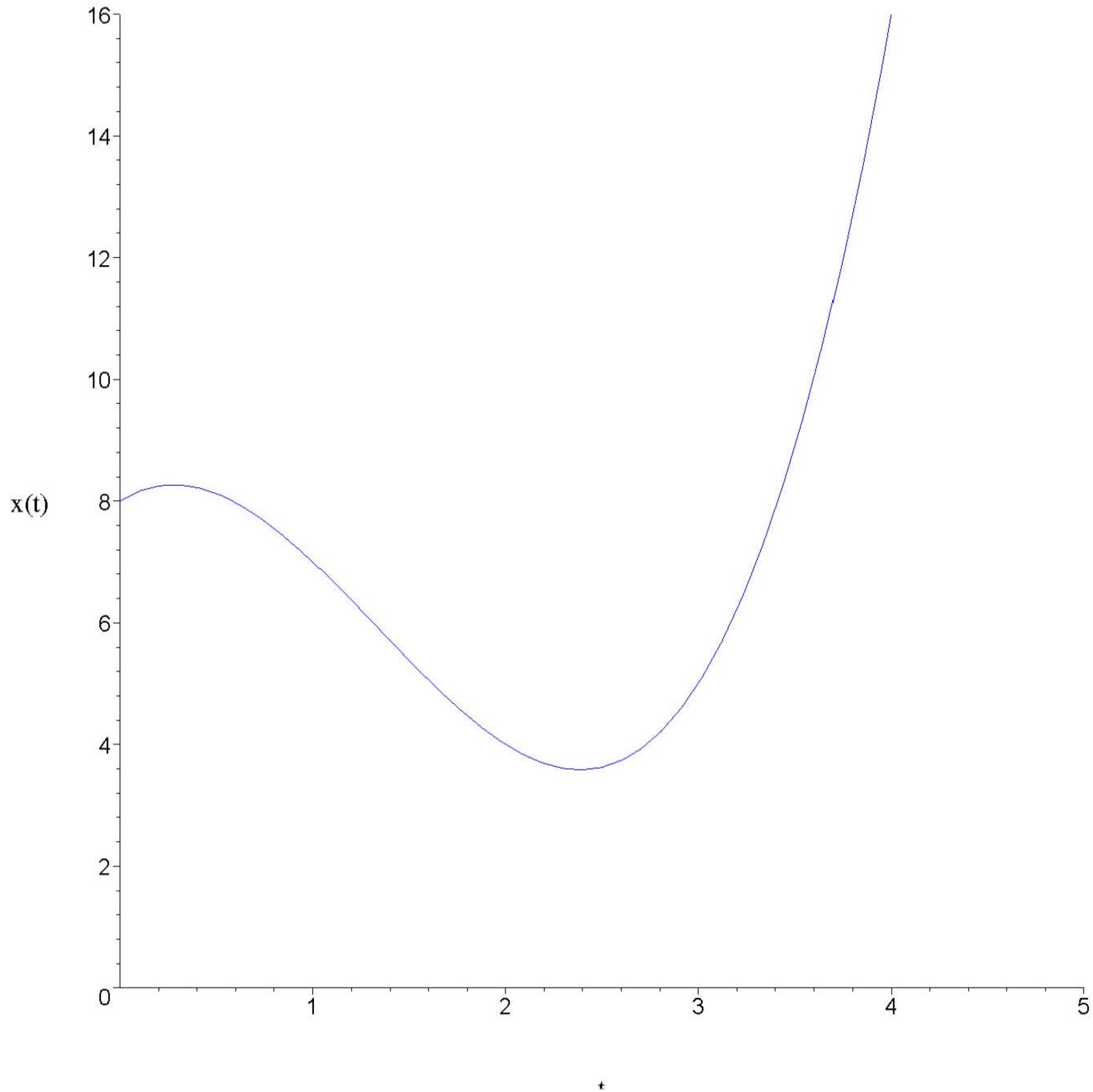

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Calculons une vitesse moyenne :

```
> t1:=1;
> x(t1);
> dt:=2;
```

$$t1 := 1$$

$$7$$

$$dt := 2$$

```
> vmoy:='(x(t1+dt)-x(t1))/dt'; 'vmoy'=vmoy;
```

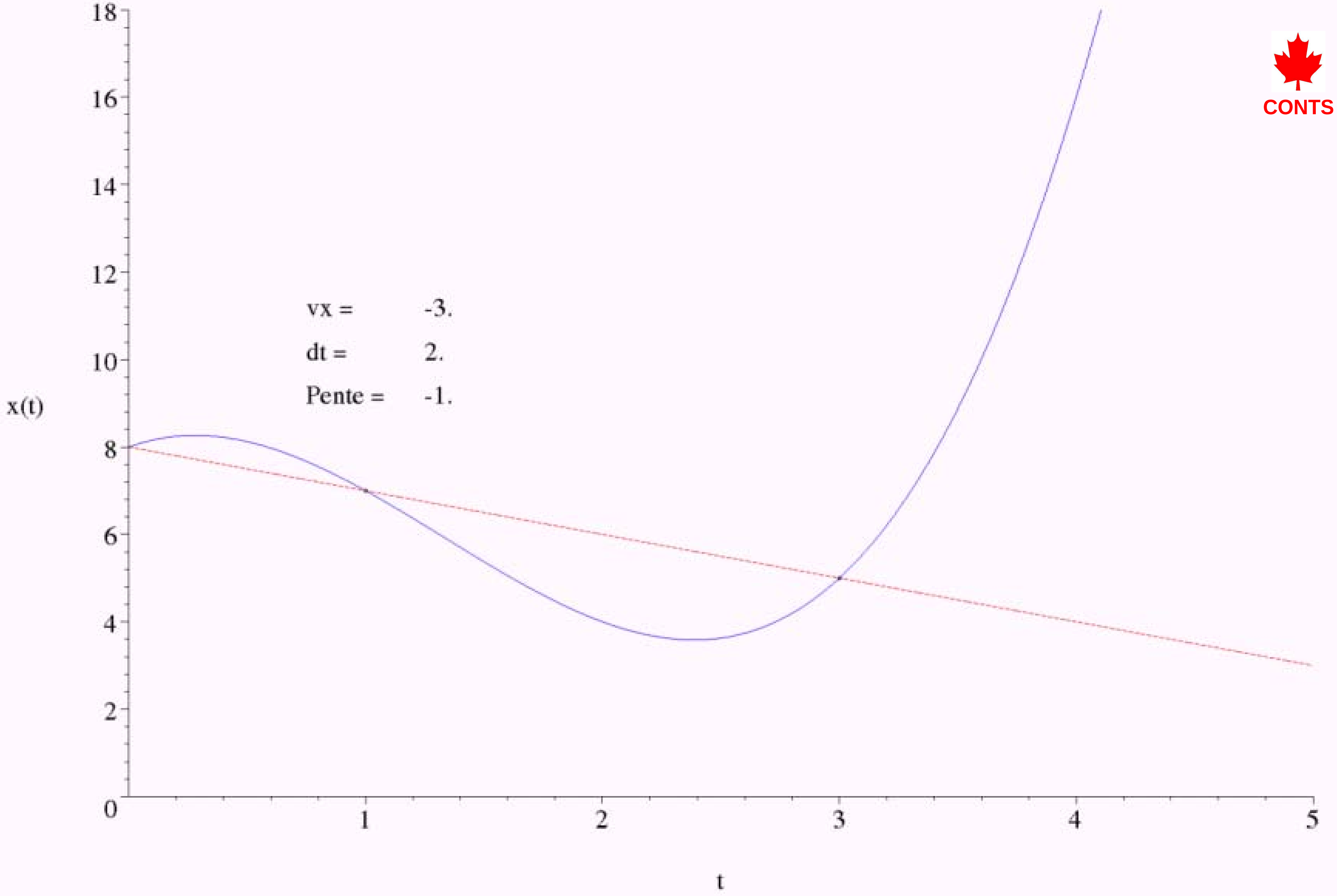
$$vmoy := \frac{x(t1 + dt) - x(t1)}{dt}$$

$$vmoy = -1$$

Retour sur la courbe du haut pour comprendre ce que nous venons de faire...

Et maintenant, amusons-nous un peu :

```
> voir(x,t1,dt);
```



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For mathematical treatment :

- Computation of forces in 3D equilibrium problem. Six linear equations of forces and momentum have to be resolved. Calculation can be done by hand but involves many steps through which an error can appear.

On charge une librairie et on crée une commande manquante :

```
> with(linalg):
  norme:=proc(V::vector)
  sqrt(dotprod(V,V));
  end:
Warning, the protected names norm and trace have been redefined and unprotected
```

On fixe deux constantes :

```
> g:=9.8; m:=100;
```

$$g := 9.8$$

$$m := 100$$

Le vecteur unitaire de T1 et le vecteur T1

```
> L1V:=vector(3,[-4,-1.5,2.5]);
  u1V:=evalm(L1V/norme(L1V));
  T1V:=evalm(T1*u1V);
```

$$L1V := [-4, -1.5, 2.5]$$

$$u1V := [-0.8081220356, -0.3030457634, 0.5050762722]$$

$$T1V := [-0.8081220356\, T1, -0.3030457634\, T1, 0.5050762722\, T1]$$

Le vecteur unitaire de T2 et le vecteur T2

```
> L2V:=vector(3,[-2.5,+1.5,2.5]);
  u2V:=evalm(L2V/norme(L2V));
  T2V:=evalm(T2*u2V);
```

$$L2V := [-2.5, 1.5, 2.5]$$

$$u2V := [-0.6509445550, 0.3905667330, 0.6509445550]$$

$$T2V := [-0.6509445550\ T2, 0.3905667330\ T2, 0.6509445550\ T2]$$

Les autres vecteurs et le vecteur r pour tous les vecteurs

```
> FgV:=vector(3,[0,0,-m*g]);
  DV:=vector(3,[0,D,0]);
  CV:=vector(3,[Cx,Cy,Cz]);
```

$$FgV := [0, 0, -980.0]$$

$$DV := [0, D, 0]$$

$$CV := [Cx, Cy, Cz]$$

Les équations par rapport au point D

```
> restart:
```

```
> m:=100;
  g:=9.81;
```

$$m := 100$$

$$g := 9.81$$

```
> eq1:=-0.808*T1-0.651*T2+Cx=0;

> eq2:=-0.303*T1+0.391*T2+Cy+Dy=0;

> eq3:=+0.505*T1+0.651*T2+Cz-m*g=0;

> eq4:=Cy=0;
```

```
> eq5:=-Cx+2.5*m*g-2.5*0.651*T2-4*0.505*T1;
> eq6:=2.5*0.391*T2+4*(-0.303)*T1;
```

$$eq1 := -0.808\ T1 - 0.651\ T2 + Cx = 0$$

$$eq2 := -0.303\ T1 + 0.391\ T2 + Cy + Dy = 0$$

$$eq3 := 0.505\ T1 + 0.651\ T2 + Cz - 981.00 = 0$$

$$eq4 := Cy = 0$$

$$eq5 := -Cx + 2452.500 - 1.6275\ T2 - 2.020\ T1$$

$$eq6 := 0.9775\ T2 - 1.212\ T1$$

```
> solD:=solve({seq(eq||i,i=1..6)},{T1,T2,Dy,Cx,Cy,Cz});
```

$$solD := \{Cy = 0., Cz = 411.7368813, Dy = -78.87070021, Cx = 700.7142857, T1 = 433.8322344, T2 = 537.9075888\}$$

Les équations par rapport au point C

```
> eq4:=-Dy+0.303*T1-0.391*T2=0;
> eq5:=2*m*g-0.808*T1-4*0.505*T1-3.5*0.651*T2=0;
> eq6:=-4*0.303*T1+2.5*0.391*T2=0;
```

$$eq4 := -Dy + 0.303\ T1 - 0.391\ T2 = 0$$

$$eq5 := 1962.00 - 2.828\ T1 - 2.2785\ T2 = 0$$

$$eq6 := 0.9775\ T2 - 1.212\ T1 = 0$$

```
> solC:=solve({seq(eq||i,i=1..6)},{T1,T2,Dy,Cx,Cy,Cz});
```

$$solC := \{Cz = 525.5895050, Dy = -63.09656017, Cx = 560.5714286, T1 = 347.0657875, T2 = 430.3260711, Cy = 0.\}$$

```
> solD;
  solC;
```

$$\{Cy = 0., Cz = 411.7368813, Dy = -78.87070021, Cx = 700.7142857, T1 = 433.8322344, T2 = 537.9075888\}$$

$$\{Cz = 525.5895050, Dy = -63.09656017, Cx = 560.5714286, T1 = 347.0657875, T2 = 430.3260711, Cy = 0.\}$$

>

Using Maple for new discoveries

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- In certain situations treated with Maple through the advanced physics course, the output from the software involves subjects which have never been known to the students.
- Instead of avoiding them and creating less interesting problems, the author has chosen to address these situation and use them to discover powerful new mathematical tools.
- In most cases, these subjects will be treated in first year university math courses.
- Let’s review some examples…

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Complex numbers :

- The value of the force between two unknown electric charges is given. The value of these charges has to be found.

```
> restart:
> eq1:=q1=2e-6-q2;
```

$$eq1 := q1 = 0.2\ 10^{-5} - q2$$

```
> eq2:=q1=15e-12/q2;
```

$$eq2 := q1 = \frac{0.15\ 10^{-10}}{q2}$$

```
> solve({eq1,eq2},{q1,q2});
```

$$\{q2 = 0.1000000000\ 10^{-5} - 0.3741657387\ 10^{-5}\ I,\ q1 = 0.1000000000\ 10^{-5} + 0.3741657387\ 10^{-5}\ I\},$$
$$\{q1 = 0.1000000000\ 10^{-5} - 0.3741657387\ 10^{-5}\ I,\ q2 = 0.1000000000\ 10^{-5} + 0.3741657387\ 10^{-5}\ I\}$$

```
> plot({2e-6-q2,15e-12/q2},q2=-1e-5..1e-5,q1=-3e-5..3e-5,tickmarks=[0,0]);
```

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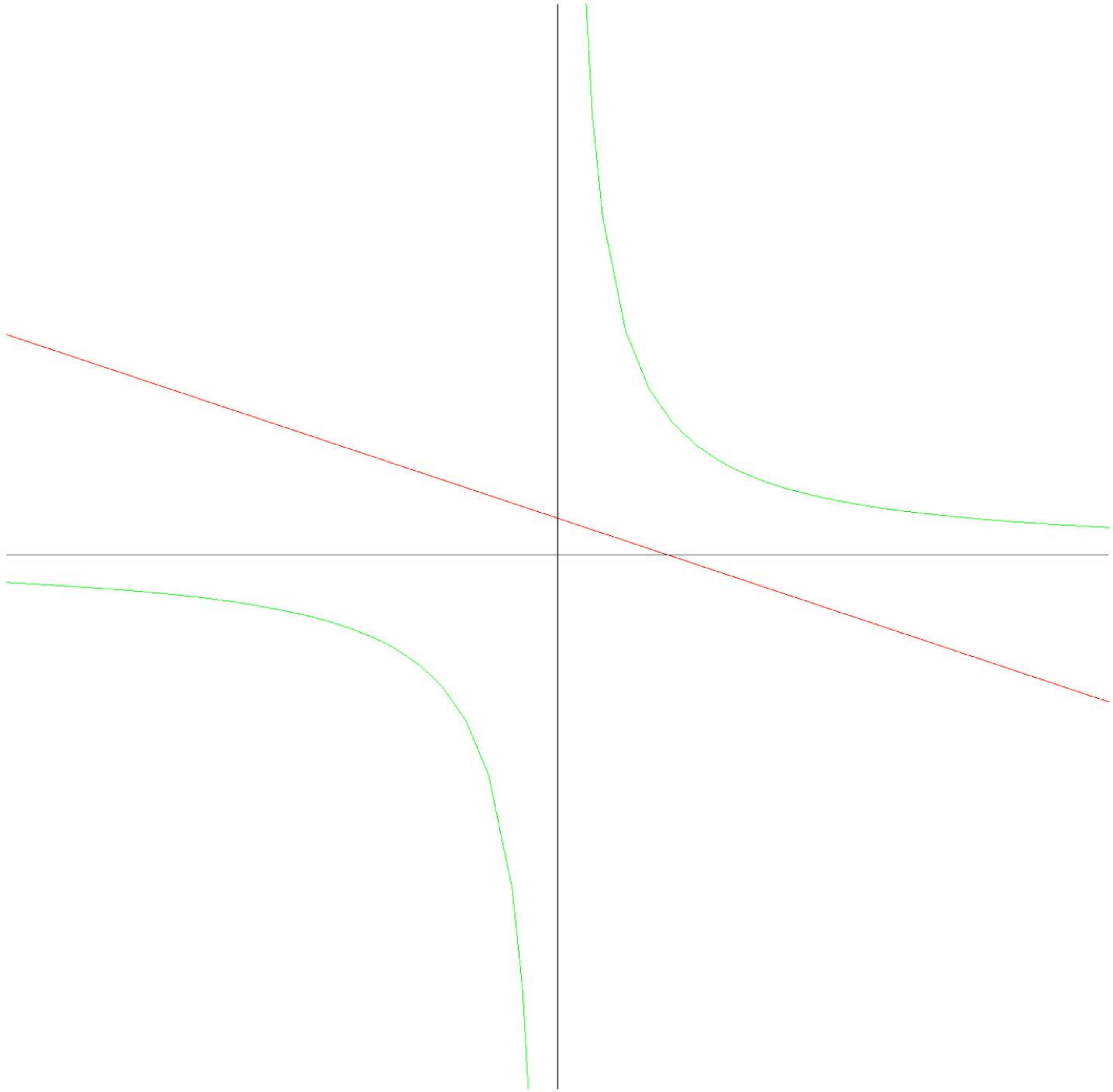

Complex numbers and numerical analysis :

- Calculating the electric field beside a uniformly charged ring.

```
> restart:
```

Cherchons à exprimer l'intégrale qui donne le champ total. Je dois d'abord m'assurer qu'aucun problème ne surgira avec les fonctions trigonométriques avec les relations suivantes:

```
> cos(theta)=cosT;
  cos(alpha)=cosA;
```

$$\cos(\theta) = cosT$$

$$\cos(\alpha) = cosA$$

Le champ électrique total au point considéré n'aura qu'une composante horizontale, i.e. selon z. Chaque parcelle infinitésimale de champ selon z est donnée par :

```
> eq1:=dE[z]=k*dq*cosA/r^2;
```

$$eq1 := dE_z = \frac{k\, dq\, cosA}{r^2}$$

Mais :

```
> dq:=lambda*dl;
```

$$dq := \lambda\, dl$$

On se sert de l'arc de cercle pour changer une première fois la variable d'intégration :

```
> dl:=a*dT;
```

$$dl := a\, dT$$

On écrase les segments a et r sur l'axe z :

```
> eq2:=a*cosT+r*cosA=z;
```

$$eq2 := a\ cosT + r\ cosA = z$$

De cette équation on tire une expression pour cos(α)

```
> cosA:=solve(eq2,cosA);
```

$$cosA := \frac{-a\ cosT + z}{r}$$

On applique la loi des cosinus au côté *r* du triangle formé par *a*, *r* et *z* :

```
> eq3:=r^2=a^2+z^2-2*a*z*cosT;
```

$$eq3 := r^2 = a^2 + z^2 - 2\ a\ z\ cosT$$

On en tire une relation pour cos(θ)

```
> cosT:=solve(eq3,cosT);
```

$$cosT := \frac{-r^2 + a^2 + z^2}{2\ a\ z}$$

Ce qui change la valeur de cos(α)

```
> cosA:=simplify(cosA);
```

$$cosA := \frac{r^2 - a^2 + z^2}{2\ z\ r}$$

On dérive l'expression de cos(θ) pour changer la variable d'intégration une autre fois :

```
> eq4:=-sinT*dT=diff(cosT,r)*dr;
```

$$eq4 := -sinT\ dT = -\frac{r\ dr}{a\ z}$$

```
> dT:=r*dr/(a*z*sinT);
```

$$dT := \frac{r\,dr}{a\,z\,sinT}$$

Notre argument est devenu ceci :

```
> eq1;
```

$$dE_z = \frac{k\,\lambda\,dr\,(r^2 - a^2 + z^2)}{2\,r^2\,z^2\,sinT}$$

Mais :

```
> eq5:=sinT='sqrt(1-cosT^2)';
```

$$eq5 := sinT = \sqrt{1 - cosT^2}$$

```
> sinT:=sqrt(1-cosT^2);
```

$$sinT := \frac{\sqrt{4 - \frac{(-r^2 + a^2 + z^2)^2}{a^2\,z^2}}}{2}$$

Donc

```
> eq1;
```

$$dE_z = \frac{k\,\lambda\,dr\,(r^2 - a^2 + z^2)}{r^2\,z^2\sqrt{4 - \frac{(-r^2 + a^2 + z^2)^2}{a^2\,z^2}}}$$

Et voilà, on est prêt à "calculer" l'intégrale, en se rappelant d'abord que le haut et le bas contribuent de la même manière au champ électrique :

```
> E[z]=2*Int(1,E[z]=sur_le_haut.._);
```

$$E_z = 2\int_{sur\_le\_haut}^{\_} 1\, dE_z$$

```
> E:=Int(2*rhs(eq1)/dr,r=z-a..z+a);
```

$$E := \int_{z-a}^{z+a} \frac{2\,k\,\lambda\,(r^2 - a^2 + z^2)}{r^2\, z^2 \sqrt{4 - \frac{(-r^2 + a^2 + z^2)^2}{a^2\, z^2}}}\, dr$$

```
> E:=int(2*rhs(eq1)/dr,r=z-a..z+a);
```

$$E := \int_{z-a}^{z+a} \frac{2\,k\,\lambda\,(r^2 - a^2 + z^2)}{r^2\, z^2 \sqrt{4 - \frac{(-r^2 + a^2 + z^2)^2}{a^2\, z^2}}}\, dr$$

```
> k:=9e9;lambda:=1e-6;a:=.1;
```

$$k := 0.9\ 10^{10}$$

$$\lambda := 0.1\ 10^{-5}$$

$$a := 0.1$$

```
> eval(E,z=1);
```

$$5697.679786 + 0.09245349960\, I$$

```
> Re(eval(E,z=1));
```

$$5697.679786$$

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```
>
```

Complex numbers :

- Damped harmonic motion

```
> k:=200;
> m:=10;
```

$$k := 200$$

$$m := 10$$

```
> omega:='sqrt(k/m)';
```

$$\omega := \sqrt{\frac{k}{m}}$$

```
> x_0:=5;
> v_x0:=10;
```

$$x\_0 := 5$$

$$v\_x0 := 10$$

```
> x:='exp(-alpha*t)*(x_0*cos(mu*t)+((v_x0+alpha*x_0)/mu)*sin(mu*t))';
```

$$x := e^{(-\alpha\, t)} \left(x\_0 \cos(\mu\, t) + \frac{(v\_x0 + \alpha\, x\_0) \sin(\mu\, t)}{\mu} \right)$$

```
> 'x'=x;
```

$$x = e^{(-\alpha\, t)} \left(5 \cos(\mu\, t) + \frac{(10 + 5\, \alpha) \sin(\mu\, t)}{\mu} \right)$$

```
> mu:='sqrt(k/m-alpha^2)';
```

$$\mu := \sqrt{\frac{k}{m} - \alpha^2}$$

```
> alpha:='c/(2*m)';
```

$$\alpha := \frac{c}{2\,m}$$

CONTS

Le cas sous-amorti

```
> c_sa:='(1/10)*2*m*omega';
```

$$c\_sa := \frac{m\,\omega}{5}$$

```
> x_sa:=subs({c=c_sa},x);
```

$$x\_sa := \mathbf{e}^{\left(-\frac{\sqrt{5}\,t}{5}\right)}\left(5\cos\left(\frac{\sqrt{7920}\,t}{20}\right)+\frac{1}{396}\,(10+\sqrt{5}\,)\sqrt{7920}\,\sin\left(\frac{\sqrt{7920}\,t}{20}\right)\right)$$

Le cas critique

```
> c_cr:='2*m*omega';
```

$$c\_cr := 2\,m\,\omega$$

```
> x_cr:=limit(x,c=c_cr);
```

$$x\_cr := \frac{5+10\sqrt{5}\,t+10\,t}{(\mathbf{e}^{(\sqrt{5}\,t)})^2}$$

Le cas surcritique

```
> c_sc:='2*2*m*omega';
```

$$c\_sc := 4\,m\,\omega$$

```
> x_sc:=subs({c=c_sc},x);
```

$$x\_sc := \mathbf{e}^{(-4\sqrt{5}\,t)}\left(5\cos\left(\frac{\sqrt{-24000}\;t}{20}\right) - \frac{1}{1200}\,(10 + 20\sqrt{5}\,)\sqrt{-24000}\,\sin\left(\frac{\sqrt{-24000}\;t}{20}\right)\right)$$

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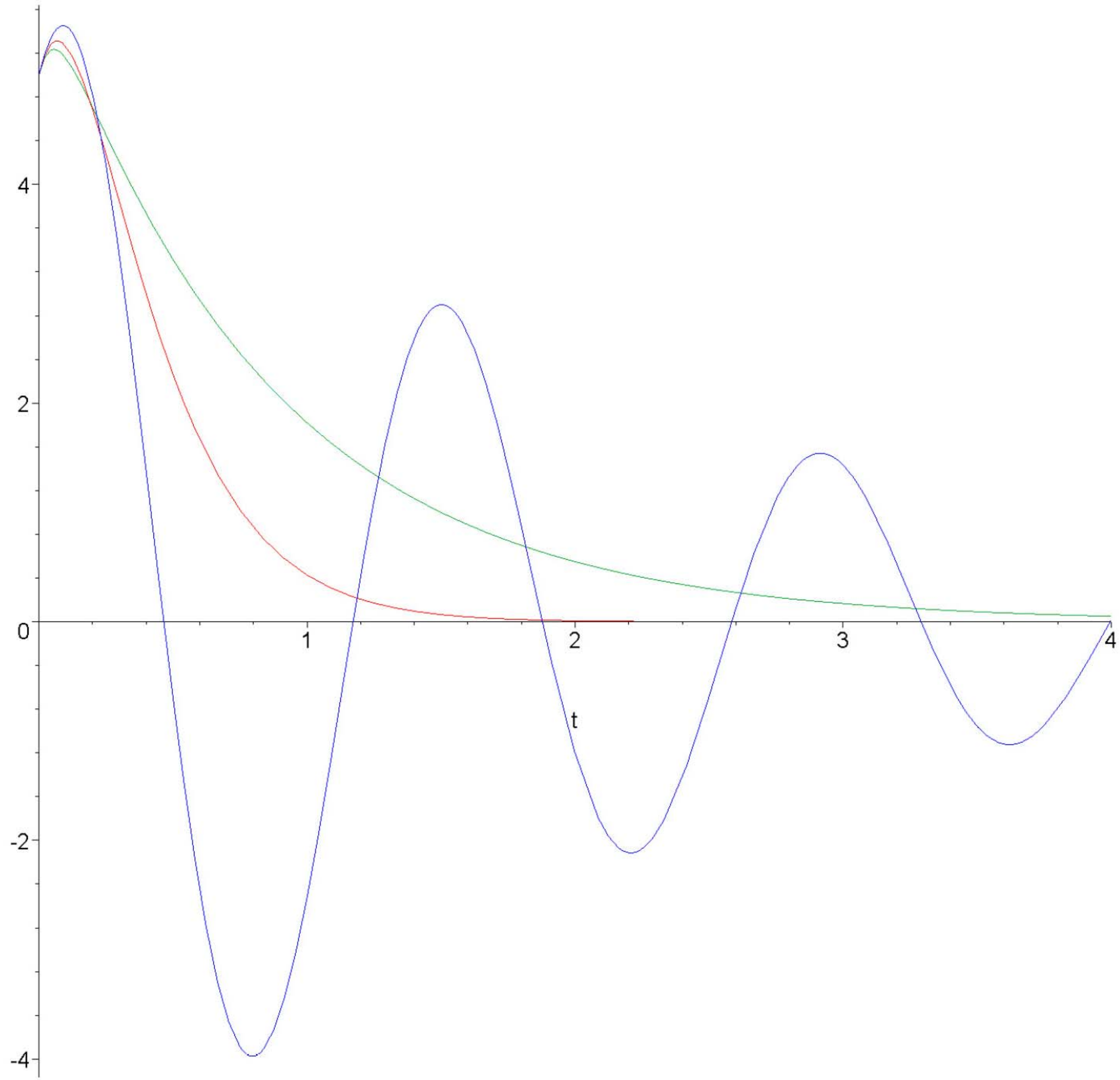
```
> plot([x_sa,x_cr,x_sc],t=0..4,color=[blue, red,COLOR(RGB, 0, .55, .13)]);
```



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Differential equations :

- Forced harmonic motion.

```
> equa_diff:=k*x(t)+c*diff(x(t),t)+m*diff(x(t),t$2)=Fe*cos(omega[e]*t);
```

$$equa\_diff := k\,x(t) + c\left(\frac{d}{dt}x(t)\right) + m\left(\frac{d^2}{dt^2}x(t)\right) = Fe\cos(\omega_e\,t)$$

```
> with(DEtools):
```

```
> condition:=x(0)=x0,D(x)(0)=v_x0;
```

$$condition := x(0) = x0,\ D(x)(0) = v\_x0$$

```
> x:=rhs(dsolve( {equa_diff,condition} , {x(t)} ));
```

$$\begin{aligned}
x := {} & \frac{1}{2}\,e^{\left(-\frac{(c-\sqrt{c^2-4\,k\,m})\,t}{2\,m}\right)}\big(-Fe\,\omega_e^{\,2}\,m\,c + 2\,v_x0\,m\,k^2 + 2\,v_x0\,m^3\,\omega_e^{\,4} + 2\,v_x0\,m\,c^2\,\omega_e^{\,2} - 4\,v_x0\,m^2\,k\,\omega_e^{\,2} + x0\,c^3\,\omega_e^{\,2} - c\,Fe\,k + c\,x0\,k^2 \\
& - \sqrt{c^2-4\,k\,m}\,Fe\,k + \sqrt{c^2-4\,k\,m}\,x0\,k^2 + c\,x0\,\omega_e^{\,4}\,m^2 + \sqrt{c^2-4\,k\,m}\,Fe\,\omega_e^{\,2}\,m + \sqrt{c^2-4\,k\,m}\,x0\,c^2\,\omega_e^{\,2} + \sqrt{c^2-4\,k\,m}\,x0\,\omega_e^{\,4}\,m^2 \\
& - 2\,c\,x0\,k\,m\,\omega_e^{\,2} - 2\sqrt{c^2-4\,k\,m}\,x0\,k\,m\,\omega_e^{\,2}\big) \Big/ \left(\sqrt{c^2-4\,k\,m}\,(c^2\,\omega_e^{\,2} + k^2 - 2\,k\,m\,\omega_e^{\,2} + \omega_e^{\,4}\,m^2)\right) + \frac{1}{2}\,e^{\left(-\frac{(c+\sqrt{c^2-4\,k\,m})\,t}{2\,m}\right)}\big(\\
& -2\,v_x0\,m^3\,\omega_e^{\,4} - c\,x0\,\omega_e^{\,4}\,m^2 + \sqrt{c^2-4\,k\,m}\,x0\,\omega_e^{\,4}\,m^2 + 4\,v_x0\,m^2\,k\,\omega_e^{\,2} + \sqrt{c^2-4\,k\,m}\,Fe\,\omega_e^{\,2}\,m + Fe\,\omega_e^{\,2}\,m\,c - 2\,v_x0\,m\,c^2\,\omega_e^{\,2} \\
& - 2\sqrt{c^2-4\,k\,m}\,x0\,k\,m\,\omega_e^{\,2} + 2\,c\,x0\,k\,m\,\omega_e^{\,2} + \sqrt{c^2-4\,k\,m}\,x0\,c^2\,\omega_e^{\,2} - x0\,c^3\,\omega_e^{\,2} - 2\,v_x0\,m\,k^2 + c\,Fe\,k - \sqrt{c^2-4\,k\,m}\,Fe\,k - c\,x0\,k^2 \\
& + \sqrt{c^2-4\,k\,m}\,x0\,k^2\big) \Big/ \left(\sqrt{c^2-4\,k\,m}\,(c^2\,\omega_e^{\,2} + k^2 - 2\,k\,m\,\omega_e^{\,2} + \omega_e^{\,4}\,m^2)\right) + \frac{((k-\omega_e^{\,2}\,m)\cos(\omega_e\,t) + \omega_e\sin(\omega_e\,t)\,c)\,Fe}{\omega_e^{\,4}\,m^2 + (c^2-2\,k\,m)\,\omega_e^{\,2} + k^2}
\end{aligned}$$

On se donne les conditions de mouvement du cas sous-amorti :

```
> k:=20;

> m:=10;
```

CONTS

```
> omega[n]:=sqrt(k/m);
> c:=(1/10)*2*m*omega[n];
> Fe:=20;
> omega[e]:=0.6*omega[n];
> x0:=5;
> v_x0:=1;
```

$$k := 20$$

$$m := 10$$

$$\omega_n := \sqrt{2}$$

$$c := 2\sqrt{2}$$

$$Fe := 20$$

$$\omega_e := 0.6\sqrt{2}$$

$$x0 := 5$$

$$v\_x0 := 1$$

```
> x;
```

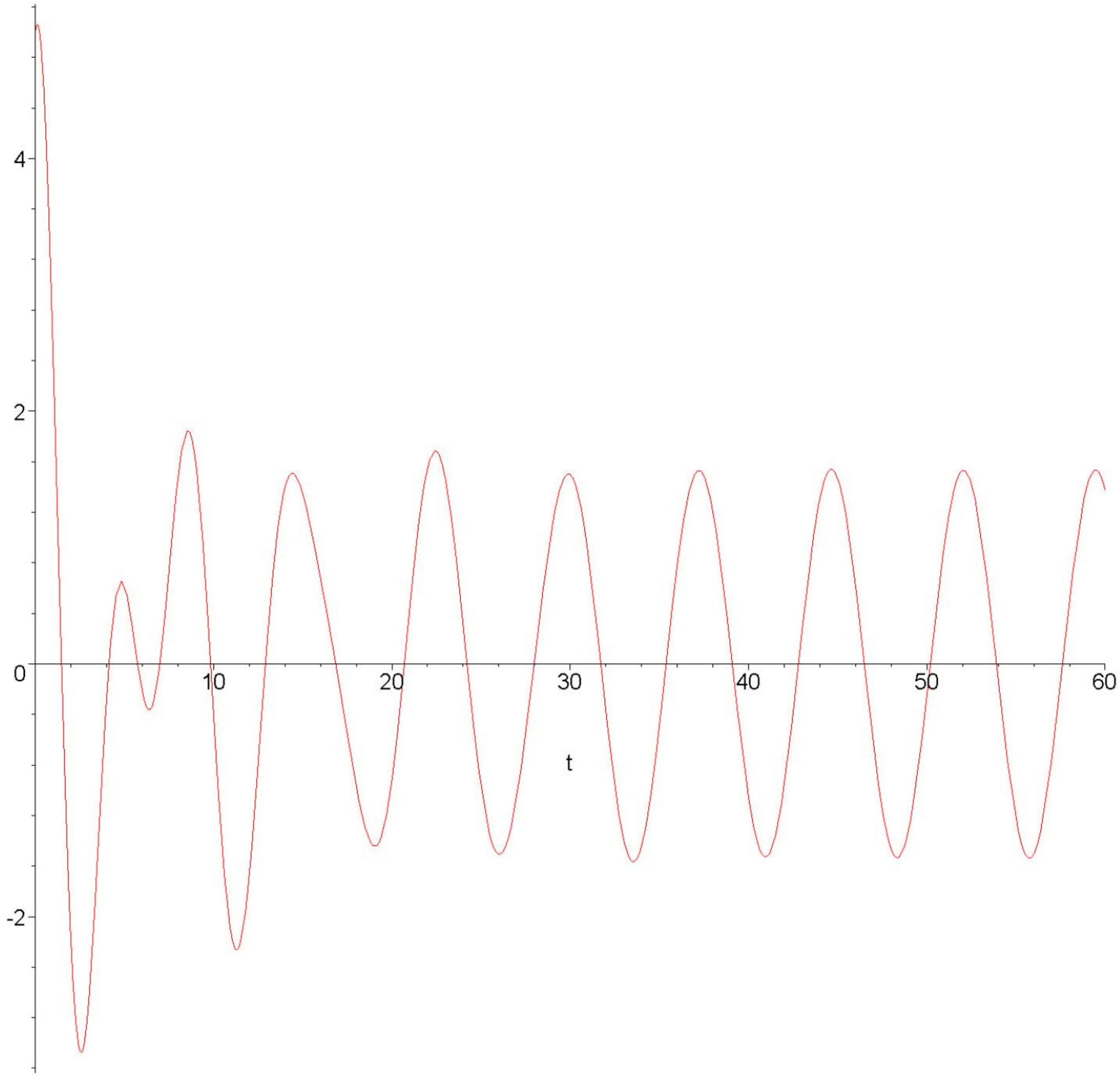
$$-0.3722365162\ 10^{-5}\ \mathbf{e}^{\left(-\frac{(2\sqrt{2}-\sqrt{-792})\,t}{20}\right)}(608.0000\sqrt{2}+3392.0000+592.0000\sqrt{-792})\sqrt{-792}$$
$$-0.3722365162\ 10^{-5}\ \mathbf{e}^{\left(-\frac{(2\sqrt{2}+\sqrt{-792})\,t}{20}\right)}(-3392.0000-608.0000\sqrt{2}+592.0000\sqrt{-792})\sqrt{-792}+1.509433962\cos(0.6\sqrt{2}\ t)$$
$$+0.2830188679\sin(0.6\sqrt{2}\ t)$$

```
> simplify(convert(simplify(expand(x)),trig));
```

$$0.8908173424 \cosh(0.1414213562\, t) \sin(1.407124728\, t) - 0.8908173424 \sinh(0.1414213562\, t) \sin(1.407124728\, t)$$
$$+ 3.490566038 \cosh(0.1414213562\, t) \cos(1.407124728\, t) - 3.490566038 \sinh(0.1414213562\, t) \cos(1.407124728\, t)$$
$$+ 1.509433962 \cos(0.8485281372\, t) + 0.2830188679 \sin(0.8485281372\, t)$$

CONTS

```
> plot(x,t=0..60,numpoints=100);
```



CONTS

Numerical analysis :

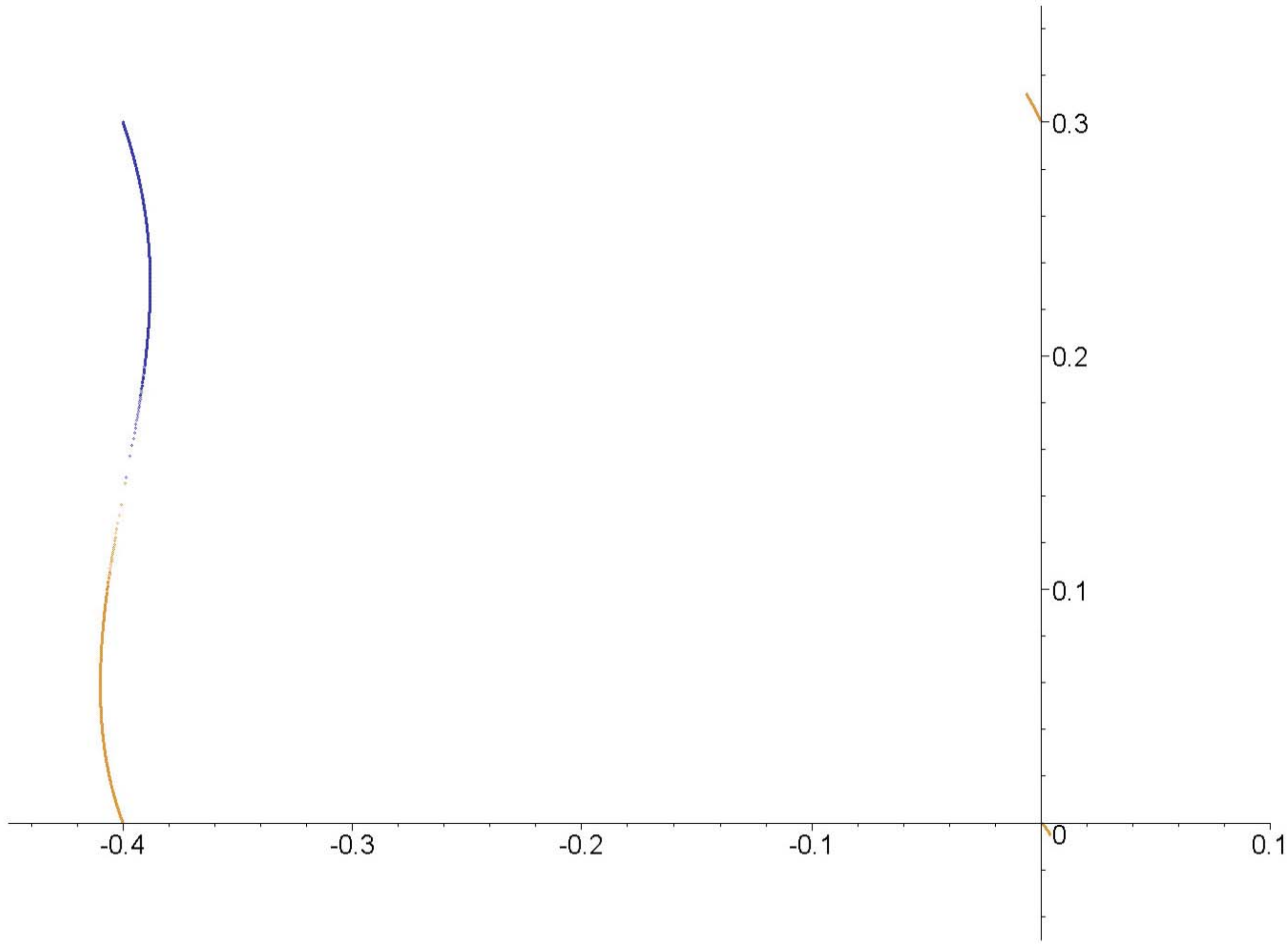
- Movement of four electric charges under their own influence

Graphique des trajectoires

```
> gr1:=pointplot(position1,color=couleur(1),symbol=CIRCLE,symbolsize=5):
  gr2:=pointplot(position2,color=couleur(2),symbol=CIRCLE,symbolsize=5):
  gr3:=pointplot(position3,color=couleur(3),symbol=CIRCLE,symbolsize=5):
  gr4:=pointplot(position4,color=couleur(4),symbol=CIRCLE,symbolsize=5):
> display({gr1,gr2,gr3,gr4},scaling=constrained,view=[-0.45..0.1,-0.05..0.35]);
```

CONTS

CONTS



Differential equations and numerical analysis :

- More complete differential equation of a flying rocket

```
> restart:
```

Nous allons maintenant utiliser la fonctionnalité de Maple qui résout les équations différentielles. Il suffit d'écrire nos variables dans un langage que reconnaît Maple:

```
> equa_diff:=-(m0-alpha*t)*g+v_exp*alpha-k*v_y(t)^2=(m0-alpha*t)*diff(v_y(t),t);
```

$$equa\_diff := -(m0-\alpha\, t)\, g + v\_exp\, \alpha - k\, \mathrm{v\_y}(t)^2 = (m0 - \alpha\, t)\left(\frac{d}{dt}\mathrm{v\_y}(t)\right)$$

```
> condition:=v_y(0)=0;
```

$$condition := \mathrm{v\_y}(0) = 0$$

Et de résoudre...

```
> v:=rhs(dsolve( {equa_diff, condition} , {v_y(t)} ));
```

$$v := \Big(-k\, g\, (m0-\alpha\, t)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} m0\,\sqrt{\alpha} + 2\, k^{(3/2)}\, g\, (m0-\alpha\, t)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} m0\,\sqrt{v\_exp} + k\, g\, (m0-\alpha\, t)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} \alpha^{(3/2)}\, t$$

$$- 2\, k^{(3/2)}\, g\, (m0-\alpha\, t)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} \alpha\, t\,\sqrt{v\_exp}\Big)\, \mathrm{hypergeom}\left([\ ], \left[\frac{2\,(\sqrt{\alpha}+\sqrt{v\_exp}\,\sqrt{k})}{\sqrt{\alpha}}\right], -\frac{k\, g\,(m0-\alpha\, t)}{\alpha^2}\right) \Big/ \Big(\Big(\Big(-$$

$$\left(m0^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right) \cos\left(\frac{\pi\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right) 16^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} \Gamma\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)^2 \Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{2\sqrt{\alpha}}\right)^2 \Big($$

$$k\, v\_exp\, g\, \mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right) m0$$

$$- \alpha^{(3/2)}\, v\_exp^{(3/2)}\sqrt{k}\, \mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right)\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\Big)$$

CONTS

$$\mathrm{hypergeom}\left([\ ], \left[\frac{\sqrt{\alpha}-2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right], -\frac{k\,g\,(m0-\alpha\,t)}{\alpha^2}\right)(m0-\alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)} \Bigg/ \Bigg(\pi^2\left(\left(-\frac{k\,g\,m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2\Bigg($$

$$\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,\mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\alpha^2\,v\_exp$$

$$+\sqrt{k}\,\sqrt{v\_exp}\,\mathrm{BesselI}\left(-\frac{-\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{\alpha}\,g\,m0\Bigg)\Bigg)$$

$$+\mathrm{hypergeom}\left([\ ], \left[\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right], -\frac{k\,g\,(m0-\alpha\,t)}{\alpha^2}\right)(m0-\alpha\,t)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\Bigg)k\,(-\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k})\,(\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k})$$

$$\sqrt{\alpha}\Bigg)+\Bigg(\Bigg(\left(m0^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2\sin\left(\frac{\pi\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)\cos\left(\frac{\pi\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)16^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\,\Gamma\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)^2\,\Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{2\sqrt{\alpha}}\right)^2\Bigg($$

$$k\,v\_exp\,g\,\mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)m0$$

$$-\alpha^{(3/2)}\,v\_exp^{(3/2)}\sqrt{k}\,\mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,(m0-\alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\alpha^2\sqrt{v\_exp}\,\sqrt{k} \Bigg/ \Bigg(\pi^2$$

$$\left(\left(-\frac{k\,g\,m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2\Bigg(\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,\mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\,\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\alpha^2\,v\_exp$$

$$+\sqrt{k}\sqrt{v\_exp}\,\mathrm{BesselI}\left(-\frac{-\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}},2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{\alpha}\,g\,m0\Bigg)\Bigg)-4\left(m0^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2\sin\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)$$

CONTS

$$\cos\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)16^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\Gamma\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)^2\Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{2\sqrt{\alpha}}\right)^2\Bigg($$

$$k\,v\_exp\,g\,\mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}},2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)m0$$

$$-\alpha^{(3/2)}v\_exp^{(3/2)}\sqrt{k}\,\mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}},2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,(m0-\alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\alpha\,v\_exp^{(3/2)}k^{(3/2)}\Bigg/\Bigg($$

$$\pi^2\left(\left(-\frac{k\,g\,m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2\Bigg(\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,\mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}},2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\alpha^2\,v\_exp$$

$$+\sqrt{k}\sqrt{v\_exp}\,\mathrm{BesselI}\left(-\frac{-\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}},2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{\alpha}\,g\,m0\Bigg)\Bigg)\Bigg)$$

$$\mathrm{hypergeom}\left([\ ],\left[\frac{\sqrt{\alpha}-2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right],-\frac{k\,g\,(m0-\alpha\,t)}{\alpha^2}\right)+$$

$$\left((m0-\alpha\,t)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\alpha^2\sqrt{v\_exp}\sqrt{k}-4\,(m0-\alpha\,t)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\alpha\,v\_exp^{(3/2)}k^{(3/2)}\right)$$

$$\mathrm{hypergeom}\left([\ ], \left[\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right], -\frac{k\,g\,(m0-\alpha\,t)}{\alpha^2}\right)+\left(\left(m0^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)\cos\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right) 16^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right.$$

$$\Gamma\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)^2 \Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{2\sqrt{\alpha}}\right)^2 \left(k\,v\_exp\,g\,\mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right) m0\right.$$

$$-\alpha^{(3/2)}\,v\_exp^{(3/2)}\sqrt{k}\,\mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,(m0-\alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)} k\,g\,m0\sqrt{\alpha}\Bigg/\left(\pi^2\right.$$

$$\left(\left(-\frac{k\,g\,m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \left(\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,\mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\alpha^2\,v\_exp\right.$$

$$\left.\left.+\sqrt{k}\sqrt{v\_exp}\,\mathrm{BesselI}\left(-\frac{-\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{\alpha}\,g\,m0\right)\right)+2\left(m0^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)$$

$$\cos\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)16^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\Gamma\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)^2\Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{2\sqrt{\alpha}}\right)^2\Bigg($$

$$k\,v\_exp\,g\,\mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right) m0$$

$$-\alpha^{(3/2)}\,v\_exp^{(3/2)}\sqrt{k}\,\mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,(m0-\alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)} k^{(3/2)}\,g\,m0\sqrt{v\_exp}\Bigg/\Bigg($$

CONTS

$$\pi^2 \left(\left(\left(-\frac{k\, g\, m0}{\alpha^2} \right)^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \right)^2 \left(\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\, \mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) \alpha^2\, v\_exp \right.\right.$$

$$\left.\left. + \sqrt{k}\, \sqrt{v\_exp}\, \mathrm{BesselI}\left(-\frac{-\sqrt{\alpha} + 2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) \sqrt{\alpha}\, g\, m0 \right) \right) - \left(m0^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)$$

$$\cos\left(\frac{\pi\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right) 16^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \Gamma\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)^2 \Gamma\left(\frac{\sqrt{\alpha} + 2\sqrt{v\_exp}\, \sqrt{k}}{2\sqrt{\alpha}} \right)^2 \Bigg($$

$$k\, v\_exp\, g\, \mathrm{BesselI}\left(\frac{\sqrt{\alpha} + 2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) m0$$

$$-\alpha^{(3/2)}\, v\_exp^{(3/2)} \sqrt{k}\, \mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) \sqrt{-\frac{k\, g\, m0}{\alpha^2}}\, (m0 - \alpha\, t)^{\left(-\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} k\, g\, \alpha^{(3/2)}\, t \Bigg/ \Bigg(\pi^2$$

$$\left(\left(\left(-\frac{k\, g\, m0}{\alpha^2} \right)^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \right)^2 \left(\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\, \mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) \alpha^2\, v\_exp \right.\right.$$

$$\left.\left. + \sqrt{k}\, \sqrt{v\_exp}\, \mathrm{BesselI}\left(-\frac{-\sqrt{\alpha} + 2\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}} \right) \sqrt{\alpha}\, g\, m0 \right) \right) - 2 \left(m0^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)$$

$$\cos\left(\frac{\pi\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right) 16^{\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)} \Gamma\left(\frac{\sqrt{v\_exp}\, \sqrt{k}}{\sqrt{\alpha}} \right)^2 \Gamma\left(\frac{\sqrt{\alpha} + 2\sqrt{v\_exp}\, \sqrt{k}}{2\sqrt{\alpha}} \right)^2 \Bigg($$

CONTS

$$k\, v\_exp\, g\, \mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right) m0$$

$$-\alpha^{(3/2)}\, v\_exp^{(3/2)} \sqrt{k}\, \mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right)\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\,(m0-\alpha\, t)^{\left(-\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)} k^{(3/2)}\, g\, \alpha\, t\sqrt{v\_exp} \Bigg/ \Bigg($$

$$\pi^2\left(\left(-\frac{k\, g\, m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \left(\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\, \mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right)\alpha^2\, v\_exp\right.$$

$$\left.\left.+\sqrt{k}\sqrt{v\_exp}\, \mathrm{BesselI}\left(-\frac{-\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right)\sqrt{\alpha}\, g\, m0\right)\right)$$

$$\mathrm{hypergeom}\left([\ ], \left[-\frac{2(-\sqrt{\alpha}+\sqrt{v\_exp}\sqrt{k})}{\sqrt{\alpha}}\right], -\frac{k\, g\,(m0-\alpha\, t)}{\alpha^2}\right) \Bigg/ \Bigg(\Bigg(-\left(m0^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \sin\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)$$

$$\cos\left(\frac{\pi\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right) 16^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\, \Gamma\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)^2 \Gamma\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{2\sqrt{\alpha}}\right)^2 \Bigg($$

$$k\, v\_exp\, g\, \mathrm{BesselI}\left(\frac{\sqrt{\alpha}+2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right) m0$$

$$-\alpha^{(3/2)}\, v\_exp^{(3/2)} \sqrt{k}\, \mathrm{BesselI}\left(\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\right)\sqrt{-\frac{k\, g\, m0}{\alpha^2}}\Bigg)$$

$$\mathrm{hypergeom}\left([\ ], \left[\frac{\sqrt{\alpha} - 2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right], -\frac{k\,g\,(m0 - \alpha\,t)}{\alpha^2}\right)(m0 - \alpha\,t)^{\left(-\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)} \bigg/ \left(\pi^2\left(\left(-\frac{k\,g\,m0}{\alpha^2}\right)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right)^2 \left(\right.\right.$$

CONTS

$$\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\,\mathrm{BesselI}\left(-\frac{2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\alpha^2\,v\_exp$$

$$\left.+\sqrt{k}\sqrt{v\_exp}\,\mathrm{BesselI}\left(-\frac{-\sqrt{\alpha} + 2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}, 2\sqrt{-\frac{k\,g\,m0}{\alpha^2}}\right)\sqrt{\alpha}\,g\,m0\right)\right)$$

$$\left.+\,\mathrm{hypergeom}\left([\ ], \left[\frac{\sqrt{\alpha} + 2\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right], -\frac{k\,g\,(m0 - \alpha\,t)}{\alpha^2}\right)(m0 - \alpha\,t)^{\left(\frac{\sqrt{v\_exp}\sqrt{k}}{\sqrt{\alpha}}\right)}\right) k\,(-\sqrt{\alpha} + 2\sqrt{v\_exp}\sqrt{k})\,(\sqrt{\alpha} + 2\sqrt{v\_exp}\sqrt{k})$$

$$\left.\sqrt{\alpha}\right)$$

```
> g:=9.8; v_exp:=2.79e3; alpha:=1.333e4; m0:=2.904e6;
```

$$g := 9.8$$

$$v\_exp := 2790.$$

$$\alpha := 13330.$$

$$m0 := 0.2904\ 10^7$$

```
> v;
```

$$0.008661336771\,(-0.3285774558\ 10^{10}\,k\,(0.2904\ 10^7 - 13330.\,t)^{(0.4574957109\,\sqrt{k})}$$

CONTS

$$+0.3006455534\ 10^{10}\ k^{(3/2)}\ (0.2904\ 10^{7}-13330.\ t)^{(0.4574957109\sqrt{k})}+0.1508242936\ 10^{8}\ k\ (0.2904\ 10^{7}-13330.\ t)^{(0.4574957109\sqrt{k})}$$

$$-0.1380029348\ 10^{8}\ k^{(3/2)}\ (0.2904\ 10^{7}-13330.\ t)^{(0.4574957109\sqrt{k})}\ t)$$

$$\mathrm{hypergeom}([\ ], [2.000000000+0.9149914218\sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^{7}-13330.\ t))\ \Big/\ ((-$$

$$(0.2904\ 10^{7^{(0.4574957109\sqrt{k})}})^{2}\ \sin(0.4574957109\ \pi\sqrt{k})\ \cos(0.4574957109\ \pi\sqrt{k})\ 16^{(0.4574957109\sqrt{k})}\ \Gamma(0.4574957109\sqrt{k})^{2}$$

$$\Gamma(0.5000000000+0.4574957110\sqrt{k})^{2}\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000+0.9149914221\sqrt{k}, 2\sqrt{-0.1601630715\ k})$$

$$-0.2268044279\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(0.9149914218\sqrt{k}, 2\sqrt{-0.1601630715\ k})\sqrt{-0.1601630715\ k})$$

$$\mathrm{hypergeom}([\ ], [1.000000000-0.9149914221\sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^{7}-13330.\ t))$$

$$(0.2904\ 10^{7}-13330.\ t)^{(-0.4574957109\sqrt{k})}\ \Big/\ (\pi^{2}\ ((-0.1601630715\ k)^{(0.4574957109\sqrt{k})})^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-0.1601630715\ k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k}, 2\sqrt{-0.1601630715\ k})$$

$$+0.1735560926\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(1.000000000-0.9149914221\sqrt{k}, 2\sqrt{-0.1601630715\ k})))+$$

$$\mathrm{hypergeom}([\ ], [1.000000000+0.9149914221\sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^{7}-13330.\ t))$$

$$(0.2904\ 10^{7}-13330.\ t)^{(0.4574957109\sqrt{k})})\ k\ (-115.4556192+105.6409012\sqrt{k})\ (115.4556192+105.6409012\sqrt{k}))+0.008661336771\ ($$

$$(0.9385607761\ 10^{10}\ (0.2904\ 10^{7^{(0.4574957109\sqrt{k})}})^{2}\ \sin(0.4574957109\ \pi\sqrt{k})\ \cos(0.4574957109\ \pi\sqrt{k})\ 16^{(0.4574957109\sqrt{k})}$$

$$\Gamma(0.4574957109\sqrt{k})^{2}\ \Gamma(0.5000000000+0.4574957110\sqrt{k})^{2}\ ($$

$$0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000+0.9149914221\sqrt{k}, 2\sqrt{-0.1601630715\ k})$$

$$-0.2268044279\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(0.9149914218\sqrt{k}, 2\sqrt{-0.1601630715\ k})\sqrt{-0.1601630715\ k})$$

$$(0.2904\ 10^{7}-13330.\ t)^{(-0.4574957109\sqrt{k})}\sqrt{k}\ \Big/\ (\pi^{2}\ ((-0.1601630715\ k)^{(0.4574957109\sqrt{k})})^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-0.1601630715\ k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k}, 2\sqrt{-0.1601630715\ k})$$

$$+0.1735560926\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(1.000000000-0.9149914221\sqrt{k}, 2\sqrt{-0.1601630715\ k})))-0.7857718124\ 10^{10}$$

$(0.2904\ 10^7)^{(0.4574957109 \sqrt{k})^2} \sin(0.4574957109\ \pi \sqrt{k}) \cos(0.4574957109\ \pi \sqrt{k})\ 16^{(0.4574957109 \sqrt{k})}\ \Gamma(0.4574957109 \sqrt{k})^2$ CONTS

$\Gamma(0.5000000000 + 0.4574957110 \sqrt{k})^2\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000 + 0.9149914221 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})$
$- 0.2268044279\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(0.9149914218 \sqrt{k}, 2 \sqrt{-0.1601630715\ k}) \sqrt{-0.1601630715\ k})$
$(0.2904\ 10^7 - 13330.\ t)^{(-0.4574957109 \sqrt{k})}\ k^{(3/2)} \Big/ (\pi^2 ((-0.1601630715\ k)^{(0.4574957109 \sqrt{k})})^2\ ($
$0.495752031\ 10^{12} \sqrt{-0.1601630715\ k}\ \mathrm{BesselI}(-0.9149914218 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})$
$+ 0.1735560926\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(1.000000000 - 0.9149914221 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})))$
$\mathrm{hypergeom}([\ ], [1.000000000 - 0.9149914221 \sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^7 - 13330.\ t)) +$
$(0.9385607761\ 10^{10}\ (0.2904\ 10^7 - 13330.\ t)^{(0.4574957109 \sqrt{k})} \sqrt{k} - 0.7857718124\ 10^{10}\ k^{(3/2)}\ (0.2904\ 10^7 - 13330.\ t)^{(0.4574957109 \sqrt{k})})$
$\mathrm{hypergeom}([\ ], [1.000000000 + 0.9149914221 \sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^7 - 13330.\ t)) + (0.3285774558\ 10^{10}$
$(0.2904\ 10^7)^{(0.4574957109 \sqrt{k})^2} \sin(0.4574957109\ \pi \sqrt{k}) \cos(0.4574957109\ \pi \sqrt{k})\ 16^{(0.4574957109 \sqrt{k})}\ \Gamma(0.4574957109 \sqrt{k})^2$
$\Gamma(0.5000000000 + 0.4574957110 \sqrt{k})^2\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000 + 0.9149914221 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})$
$- 0.2268044279\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(0.9149914218 \sqrt{k}, 2 \sqrt{-0.1601630715\ k}) \sqrt{-0.1601630715\ k})$
$(0.2904\ 10^7 - 13330.\ t)^{(-0.4574957109 \sqrt{k})}\ k \Big/ (\pi^2 ((-0.1601630715\ k)^{(0.4574957109 \sqrt{k})})^2\ ($
$0.495752031\ 10^{12} \sqrt{-0.1601630715\ k}\ \mathrm{BesselI}(-0.9149914218 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})$
$+ 0.1735560926\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(1.000000000 - 0.9149914221 \sqrt{k}, 2 \sqrt{-0.1601630715\ k}))) + 0.3006455534\ 10^{10}$
$(0.2904\ 10^7)^{(0.4574957109 \sqrt{k})^2} \sin(0.4574957109\ \pi \sqrt{k}) \cos(0.4574957109\ \pi \sqrt{k})\ 16^{(0.4574957109 \sqrt{k})}\ \Gamma(0.4574957109 \sqrt{k})^2$
$\Gamma(0.5000000000 + 0.4574957110 \sqrt{k})^2\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000 + 0.9149914221 \sqrt{k}, 2 \sqrt{-0.1601630715\ k})$
$- 0.2268044279\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(0.9149914218 \sqrt{k}, 2 \sqrt{-0.1601630715\ k}) \sqrt{-0.1601630715\ k})$
$(0.2904\ 10^7 - 13330.\ t)^{(-0.4574957109 \sqrt{k})}\ k^{(3/2)} \Big/ (\pi^2 ((-0.1601630715\ k)^{(0.4574957109 \sqrt{k})})^2\ ($

CONTS

$$0.495752031\ 10^{12}\sqrt{-0.1601630715\,k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$+\ 0.1735560926\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(1.000000000-0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k}))) - 0.1508242936\ 10^{8}$$

$$\left(0.2904\ 10^{7}{}^{(0.4574957109\sqrt{k})}\right)^{2}\sin(0.4574957109\,\pi\sqrt{k})\cos(0.4574957109\,\pi\sqrt{k})\ 16^{(0.4574957109\sqrt{k})}\ \Gamma(0.4574957109\sqrt{k})^{2}$$

$$\Gamma(0.5000000000+0.4574957110\sqrt{k})^{2}\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000+0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$-\ 0.2268044279\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})\sqrt{-0.1601630715\,k})$$

$$(0.2904\ 10^{7}-13330.\ t)^{(-0.4574957109\sqrt{k})}\ k\ t\ \Big/\ (\pi^{2}\left((-0.1601630715\,k)^{(0.4574957109\sqrt{k})}\right)^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-0.1601630715\,k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$+\ 0.1735560926\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(1.000000000-0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k}))) - 0.1380029348\ 10^{8}$$

$$\left(0.2904\ 10^{7}{}^{(0.4574957109\sqrt{k})}\right)^{2}\sin(0.4574957109\,\pi\sqrt{k})\cos(0.4574957109\,\pi\sqrt{k})\ 16^{(0.4574957109\sqrt{k})}\ \Gamma(0.4574957109\sqrt{k})^{2}$$

$$\Gamma(0.5000000000+0.4574957110\sqrt{k})^{2}\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000+0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$-\ 0.2268044279\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})\sqrt{-0.1601630715\,k})$$

$$(0.2904\ 10^{7}-13330.\ t)^{(-0.4574957109\sqrt{k})}\ k^{(3/2)}\ t\ \Big/\ (\pi^{2}\left((-0.1601630715\,k)^{(0.4574957109\sqrt{k})}\right)^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-0.1601630715\,k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$+\ 0.1735560926\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(1.000000000-0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k})))$$

$$\mathrm{hypergeom}([\ ],\ [2.000000000-0.9149914218\sqrt{k}],\ -0.5515257284\ 10^{-7}\ k\,(0.2904\ 10^{7}-13330.\ t)))\ \Big/\ ((-$$

$$\left(0.2904\ 10^{7}{}^{(0.4574957109\sqrt{k})}\right)^{2}\sin(0.4574957109\,\pi\sqrt{k})\cos(0.4574957109\,\pi\sqrt{k})\ 16^{(0.4574957109\sqrt{k})}\ \Gamma(0.4574957109\sqrt{k})^{2}$$

$$\Gamma(0.5000000000+0.4574957110\sqrt{k})^{2}\ (0.79401168\ 10^{11}\ k\ \mathrm{BesselI}(1.000000000+0.9149914221\sqrt{k},\ 2\sqrt{-0.1601630715\,k})$$

$$-\ 0.2268044279\ 10^{12}\sqrt{k}\ \mathrm{BesselI}(0.9149914218\sqrt{k},\ 2\sqrt{-0.1601630715\,k})\sqrt{-0.1601630715\,k})$$

$$\mathrm{hypergeom}([\ ],\ [1.000000000-0.9149914221\sqrt{k}],\ -0.5515257284\ 10^{-7}\ k\,(0.2904\ 10^{7}-13330.\ t))$$

$(0.2904\ 10^7 - 13330.\ t)^{(-0.4574957109\sqrt{k})} \Big/ (\pi^2 ((-0.1601630715\ k)^{(0.4574957109\sqrt{k})})^2\ ($
$0.495752031\ 10^{12} \sqrt{-0.1601630715\ k}\ \mathrm{BesselI}(-0.9149914218\sqrt{k}, 2\sqrt{-0.1601630715\ k})$
$+ 0.1735560926\ 10^{12} \sqrt{k}\ \mathrm{BesselI}(1.000000000 - 0.9149914221\sqrt{k}, 2\sqrt{-0.1601630715\ k}))) +$
$\mathrm{hypergeom}([\ ], [1.000000000 + 0.9149914221\sqrt{k}], -0.5515257284\ 10^{-7}\ k\ (0.2904\ 10^7 - 13330.\ t))$
$(0.2904\ 10^7 - 13330.\ t)^{(0.4574957109\sqrt{k})})\ k\ (-115.4556192 + 105.6409012\sqrt{k})\ (115.4556192 + 105.6409012\sqrt{k}))$

> k:=Cx*rho*A/2;

$$k := \frac{Cx\ \rho\ A}{2}$$

CONTS

La valeur pour k est issue d'un document où je ne trouve pas la fusée mais qui me permet de fixer une valeur approximative pour k. Un mélange du long cylindre et du cône...

> Cx:=0.75; A:=Pi*(5.05)^2; rho:=1.293;

$$Cx := 0.75$$

$$A := 25.5025\ \pi$$

$$\rho := 1.293$$

> k;

$$12.36552469\ \pi$$

> v;

$0.0007004423175\ (-0.4063032644\ 10^{11}\ \pi\ (0.2904\ 10^7 - 13330.\ t)^{(1.608767564\sqrt{\pi})}$
$+ 0.1307295025\ 10^{12}\ \pi^{(3/2)}\ (0.2904\ 10^7 - 13330.\ t)^{(1.608767564\sqrt{\pi})} + 0.1865021527\ 10^9\ \pi\ (0.2904\ 10^7 - 13330.\ t)^{(1.608767564\sqrt{\pi})}\ t$
$- 0.6000772272\ 10^9\ \pi^{(3/2)}\ (0.2904\ 10^7 - 13330.\ t)^{(1.608767564\sqrt{\pi})}\ t)$

CONTS

$$\mathrm{hypergeom}([\ ], [2.000000000 + 3.217535128\sqrt{\pi}], -0.6819905014\ 10^{-6}\ \pi\,(0.2904\ 10^{7} - 13330.\ t)) \Big/ ((-$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000 + 1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000 + 3.217535127\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})$$

$$-\ 0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi})$$

$$\mathrm{hypergeom}([\ ], [1.000000000 - 3.217535127\sqrt{\pi}], -0.6819905014\ 10^{-6}\ \pi\,(0.2904\ 10^{7} - 13330.\ t))$$

$$(0.2904\ 10^{7} - 13330.\ t)^{(-1.608767564\sqrt{\pi})} \Big/ (\pi^{2}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})$$

$$+\ 0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000 - 3.217535127\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi}))) +$$

$$\mathrm{hypergeom}([\ ], [1.000000000 + 3.217535127\sqrt{\pi}], -0.6819905014\ 10^{-6}\ \pi\,(0.2904\ 10^{7} - 13330.\ t))$$

$$(0.2904\ 10^{7} - 13330.\ t)^{(1.608767564\sqrt{\pi})})\ \pi\,(-115.4556192 + 371.4825104\sqrt{\pi})\,(115.4556192 + 371.4825104\sqrt{\pi})) + 0.0007004423175$$

$$((0.3300415932\ 10^{11}\ (0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}$$

$$\Gamma(1.608767564\sqrt{\pi})^{2}\ \Gamma(0.5000000000 + 1.608767564\sqrt{\pi})^{2}\ ($$

$$0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000 + 3.217535127\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})$$

$$-\ 0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi})\ (0.2904\ 10^{7} - 13330.\ t)^{(-1.608767564\sqrt{\pi})}$$

$$\Big/ (\pi^{(3/2)}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})$$

$$+\ 0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000 - 3.217535127\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi}))) - 0.3416766252\ 10^{12}$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000 + 1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000 + 3.217535127\sqrt{\pi}, 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi}\ (0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}$$

$$\Big/\ (\sqrt{\pi}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ (0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+\ 0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi}))))$$

$$\mathrm{hypergeom}([\ ],\ [1.000000000-3.217535127\sqrt{\pi}],\ -0.6819905014\ 10^{-6}\ \pi\ (0.2904\ 10^{7}-13330.\ t))+$$

$$(0.3300415932\ 10^{11}\ (0.2904\ 10^{7}-13330.\ t)^{(1.608767564\sqrt{\pi})}\sqrt{\pi}-0.3416766252\ 10^{12}\ \pi^{(3/2)}\ (0.2904\ 10^{7}-13330.\ t)^{(1.608767564\sqrt{\pi})})$$

$$\mathrm{hypergeom}([\ ],\ [1.000000000+3.217535127\sqrt{\pi}],\ -0.6819905014\ 10^{-6}\ \pi\ (0.2904\ 10^{7}-13330.\ t))+(0.4063032644\ 10^{11}$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000+1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000+3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi}\ (0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}$$

$$\Big/\ (\pi\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ (0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+\ 0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})))+0.1307295025\ 10^{12}$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000+1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000+3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi}\ (0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}$$

$$\Big/\ (\sqrt{\pi}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ (0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+\ 0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})))-0.1865021527\ 10^{9}$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000+1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000+3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi}\ (0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}\ t$$

$$\Big/\ (\pi\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ (0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})))-0.6000772272\ 10^{9}$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000+1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000+3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi}\ (0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}\ t$$

$$\Big/\ (\sqrt{\pi}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ (0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi}))))$$

$$\mathrm{hypergeom}([\ ],\ [2.000000000-3.217535128\sqrt{\pi}],\ -0.6819905014\ 10^{-6}\ \pi\ (0.2904\ 10^{7}-13330.\ t)))\ \Big/\ ((-$$

$$(0.2904\ 10^{7^{(1.608767564\sqrt{\pi})}})^{2}\ \sin(1.608767564\ \pi^{(3/2)})\ \cos(1.608767564\ \pi^{(3/2)})\ 16^{(1.608767564\sqrt{\pi})}\ \Gamma(1.608767564\sqrt{\pi})^{2}$$

$$\Gamma(0.5000000000+1.608767564\sqrt{\pi})^{2}\ (0.9818371034\ 10^{12}\ \pi\ \mathrm{BesselI}(1.000000000+3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$-0.7975497872\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})\sqrt{-1.980500416\ \pi})$$

$$\mathrm{hypergeom}([\ ],\ [1.000000000-3.217535127\sqrt{\pi}],\ -0.6819905014\ 10^{-6}\ \pi\ (0.2904\ 10^{7}-13330.\ t))$$

$$(0.2904\ 10^{7}-13330.\ t)^{(-1.608767564\sqrt{\pi})}\ \Big/\ (\pi^{2}\ ((-1.980500416\ \pi)^{(1.608767564\sqrt{\pi})})^{2}\ ($$

$$0.495752031\ 10^{12}\sqrt{-1.980500416\ \pi}\ \mathrm{BesselI}(-3.217535128\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})$$

$$+0.6103038908\ 10^{12}\sqrt{\pi}\ \mathrm{BesselI}(1.000000000-3.217535127\sqrt{\pi},\ 2\sqrt{-1.980500416\ \pi})))+$$

$$\mathrm{hypergeom}([\ ],\ [1.000000000+3.217535127\sqrt{\pi}],\ -0.6819905014\ 10^{-6}\ \pi\ (0.2904\ 10^{7}-13330.\ t))$$

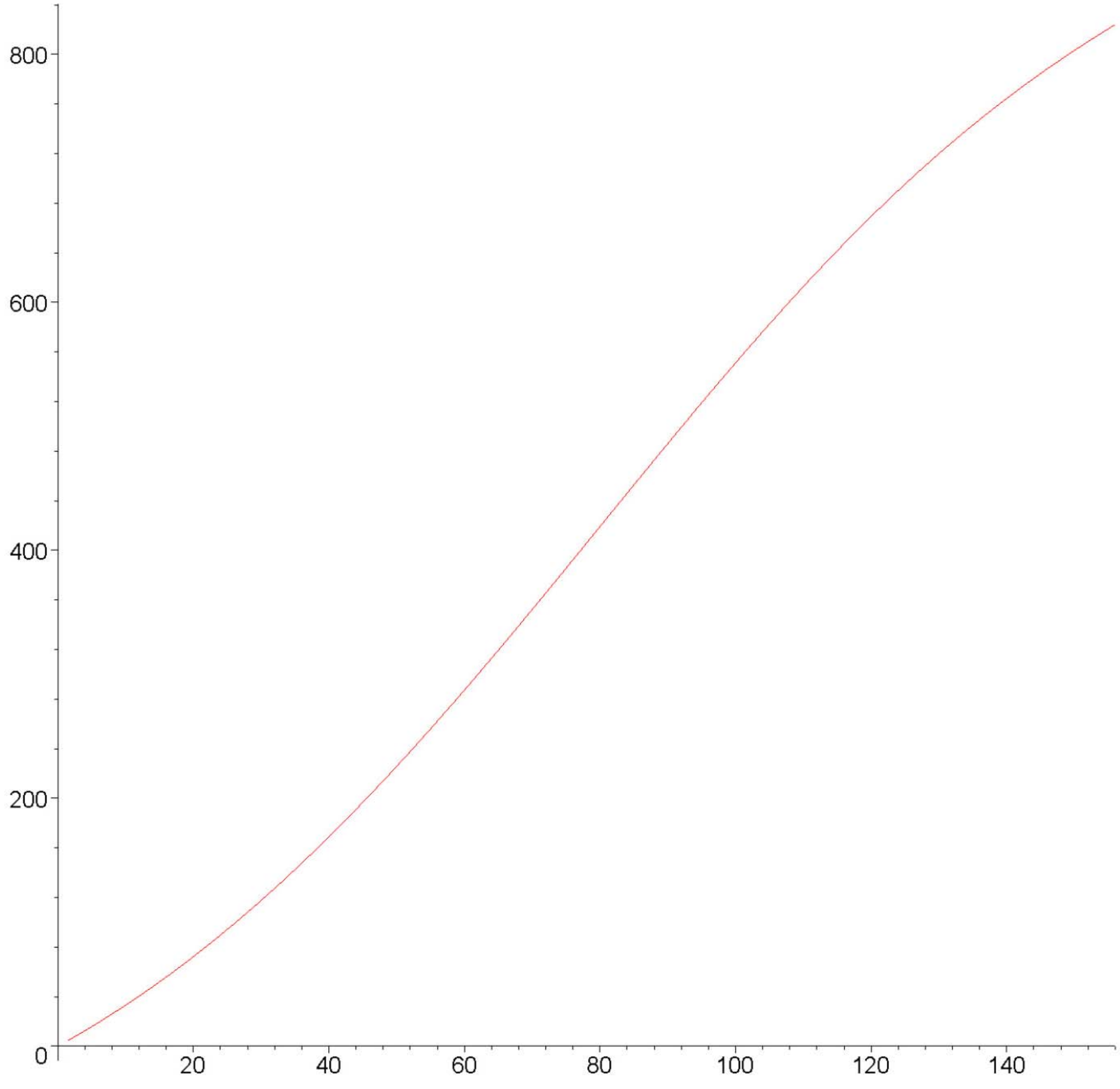
$$(0.2904\ 10^{7}-13330.\ t)^{(1.608767564\sqrt{\pi})}\ )\ \pi\ (-115.4556192+371.4825104\sqrt{\pi})\ (115.4556192+371.4825104\sqrt{\pi}))$$

```
> evalf(subs(t=156,v));
```

$$824.0190062 + 0.1708933552\ 10^{-6}\ I$$

```
> plot(v,t=0..156);
```

CONTS



CONTS

154,9	820,3706	0,1	0,336168
155	820,7068	0,1	0,335704
155,1	821,0425	0,1	0,335241
155,2	821,3777	0,1	0,334779
155,3	821,7125	0,1	0,334318
155,4	822,0468	0,1	0,333858
155,5	822,3807	0,1	0,333399
155,6	822,7141	0,1	0,332941
155,7	823,047	0,1	0,332485
155,8	823,3795	0,1	0,332029
155,9	823,7115	0,1	0,331574
156	824,0431		

CONTS

CONTS

Arguments and observations…

- These situation create a stimulating diversion from normal class work.
- Mathematicians could argue that the study of these subjects needs a serious theoretical basis.
- What’s left after ?

CONTS

From the students standpoint…

- Most students describe these situations as very stimulating.
- Some of them explore for more.
- Others follow projects related to some of these subjects.
- Some question their math teachers to better understand.

Conclusion

CONTS

- These situations have improved this preparatory course (the advanced physics course) by enlarging its span.
- Putting the emphasis on the use of these mathematical curiosities as tools for solving real situations, its stimulates the interest on better understanding them.
- They give the students an idea of what's left to discover…
- They are fun for the teacher…

CONTS

Thank you !