

WORKSHOP: An Introduction to Automatic Theorem Proving in Geometry and Automatic Search of Geometric Loci with DERIVE 5¹

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Introduction

There are two methods of Automatic Theorem Proving in Geometry: the Gröbner bases method and Wu's pseudo-remainder method. The idea in the behind both methods is this: some hypotheses and a thesis are given. Instead of working with a geometric drawing of the configuration and reasoning "mathematically", the hypotheses and the thesis are, respectively, translated into a set of polynomials and (usually just) a polynomial by using co-ordinates. Then the thesis will follow from the hypotheses if and only if the (polynomial) thesis is an algebraic combination of the (polynomial) hypotheses. In algebraic terminology: if and only if the (polynomial) thesis belongs to the ideal generated by the (polynomial) hypotheses.

Therefore, in the previous approach, what has to be proved (the thesis) has to be known in advance. We have used "known" here in the sense that its statement is precisely given in advance to the prove. If the thesis has been proved before, or at least it is suspected to be true, an automatic proof can be attempted.

Details about these methods can be found for instance in Roanes-Macías & Roanes-Lozano: *Nuevas Tecnologías en Geometría*, Ed. Complutense, 1994; Buchberger: *Applications of Gröbner Bases in non-linear Computational Geometry* (in Rice (ed.): *Mathematical Aspects of Scientific Software*, Springer-Verlag, 1988) and Chou: *Mechanical Theorem Proving*, Reidel, 1988.

Some years ago we showed how to use DERIVE to prove Geometric Theorems by adapting Wu's method. To achieve this we implemented the operation "pseudo-division" of polynomials in DERIVE. An explanation of the method, together with the code and some examples can be found in *The International Derive Journal* (Vol. 3, No. 2, 1996, pages 67-82) and also in the *Proceedings of the International Derive and TI-92 Conference 1996* (pages 404-419). The goal was to automate this polynomial operation in DERIVE, in order to apply Wu's Automatic Theorem Proving method.

Prof. Tomás Recio has enlarged the field of application of these techniques by showing how the conditions of the thesis can be completed during the automatic proving process (Recio: *Cálculo simbólico y geométrico*, Ed. Síntesis, 1998).

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At the Spanish Educational Session of IMACS-ACA 1999 Conference, that took place at Madrid last June, we presented a proof using pseudodivisions of a theorem related to a geometric locus (Bol. Soc. "Puig Adam", No. 53, Oct. 1999, pages 67-77). This geometric locus was found by Prof. Miguel de Guzmán (the result is published in *American Mathematical Monthly*, June 1999). The theorem generalises a result of Jakob Steiner (1833), that is itself a generalisation of the well-known Simson-Wallace's Theorem (the projections of the vertices of the triangle from any point on the circumscribed circumference on the lines containing the sides of the triangle are always collinear). Let us observe that the pseudo-divisions process determined the shape of the curve where the solutions lied and also that all the points in that curve were really solutions.

So we have extended the applications of these techniques in another way, so that they can be applied to find an unknown geometric locus. Therefore different problems within the Automatic Theorem Proving area can be treated. Instead of just proving theses like:

- the lines ... are concurrent
 - the equations of the geometric locus of the points that satisfy ... is
- we can automatically solve (including a formal proof) questions like:
- which are the equations of the geometric locus of the points that satisfy...?

This workshop will begin by introducing the standard Gröbner bases and Wu's pseudo-remainder methods in selected examples (DERIVE-5 new non-linear polynomial systems solving allows to use the first method and the new programming facilities of DERIVE-5 makes the second method far more comfortable to use than with previous versions of DERIVE). Afterwards the automatic search of geometric loci will be introduced through other classical examples. The workshop will be organised as a sequence of short explanations followed by extensive practice in the examples proposed. The workshop is self-contained. No algebraic special background is required (these ideas are presented in such a way that can be understood, in the simplest cases, by secondary-school students with little algebraic knowledge --only the usual polynomial division). Nevertheless, at least an intermediate level in DERIVE is strongly recommended.