Secondary school calculus problems on the TI 92.

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Maximum/minimum problems using the symbolic calculator and the geometry application.

Reference : Swokowski, Analyse 5ème édition, De Boeck Université, Bruxelles,1993.

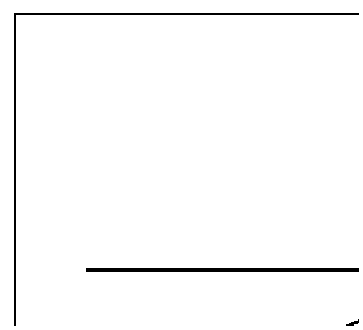
In the fifth year of secondary school (age 16-17), students are asked to solve minimum/maximum problems as applications of derivatives.
With the TI 92, we proceed through the following steps :

- determination of the solution using the symbolic calculator to perform the computations
- simulation of the problem with the geometry application to check if the solution is exact.

Let us give two examples :

First example.

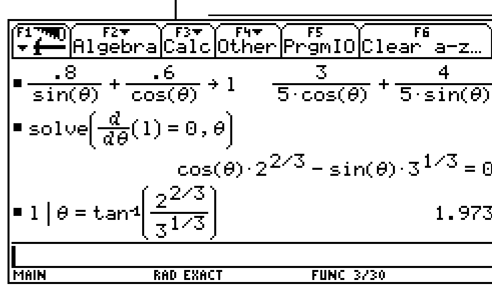
What is the maximum length of a metallic ladder which has to be transported horizontally from the point A to the point C of the corridor ABC assuming that parts AB and BC are respectively 0.8 m and 0.6 m wide (do not take into account the thickness of the ladder).



Let L be the length of the ladder.

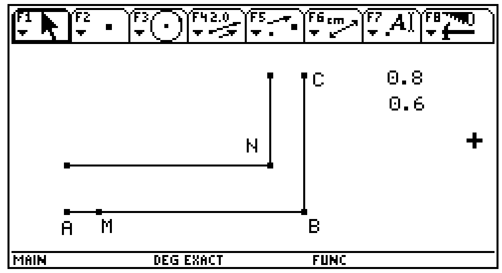
$$L = |MN| + |NP| = \frac{0.8}{\sin\theta} + \frac{0.6}{\cos\theta}$$

Determination of the solution.

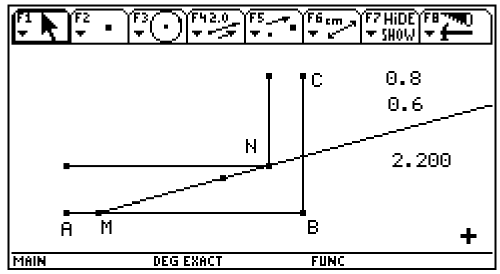


Using the symbolic calculator to make the computations one can easily find that the maximum length is given by $L = 1.973$

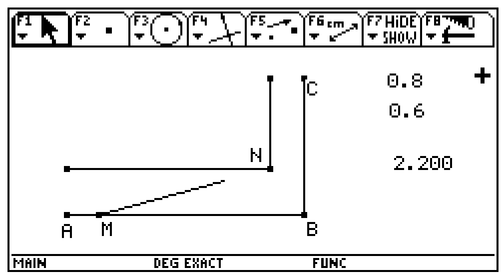
Simulation of the problem to check whether the solution is correct or not.



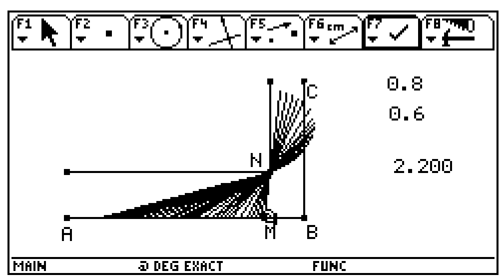
Draw the corridor and choose a point M on AB



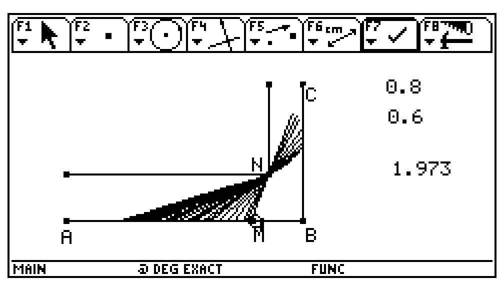
Edit a number x (for instance 2.200).
Transfer the corresponding measure on the ray [MN.
Let X be the point such as |MX| = x.



Hide some of the constructions



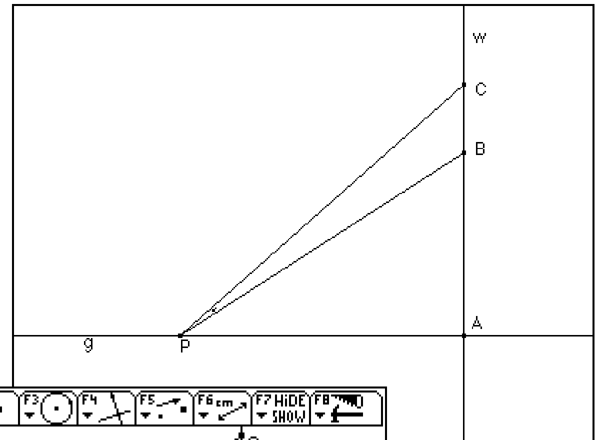
Check if the ladder goes through : ask the trace of the ladder and move point M on AB



Edit the number x (take x = 1.973).
Check if the ladder goes through by moving point M.
This time it works !

Possible development : take the thickness of the ladder into account.

<u>Second example</u>.

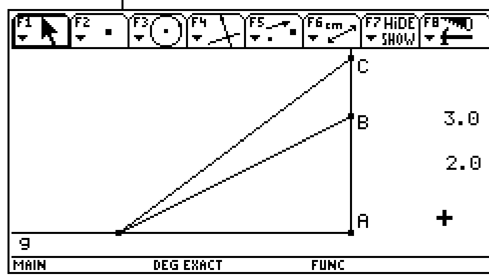


How far should you stand from a wall on which a vertical screen is hanged, to see this screen under the greatest possible angle ?

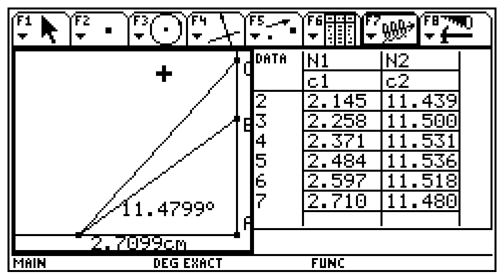
The screen [BC] is hanged on the wall w .
B and C are respectively 3 and 2 m high.

Simulation of the problem.

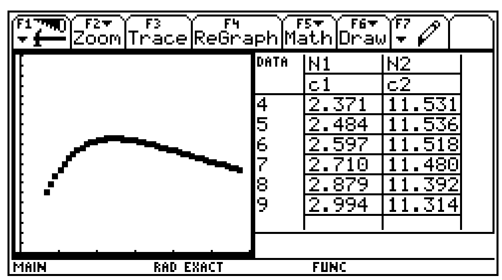
Draw the ray [AC, the points B and C, the segment g.
Take a point P on segment g.



Measure the distance |PA| and the angle ([PB, [PC).
Collect the distance in c1 and the angle in c2 and view the data.
Animate point P.



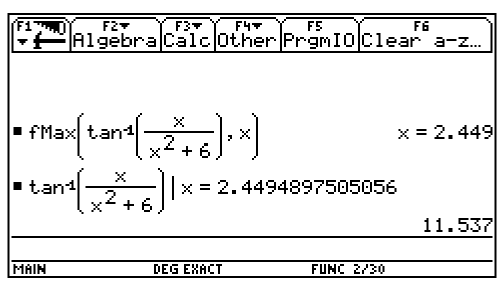
Graph the angle c2 in function of distance c1.
We have to find the equation of this curve and its maximum.



Computation of the maximum.

It is easy to show that $c2 = \tan^{-1} \dfrac{c1}{c1^2 + 6}$

Compute the maximum of this function with the symbolic calculator.



Construction of the graphs of basic trigonometric functions with the geometry application.

The trigonometry curriculum of the fourth year of secondary school (age 15-16) contains among other topics :

the definition of the radian,
the definitions of the sine, cosine, tangent as well as the cosecant,
secant, cotangent of an oriented angle in standard position,
the graphs of sin x, cos x, tg x, cosec x, sec x, cotg x.

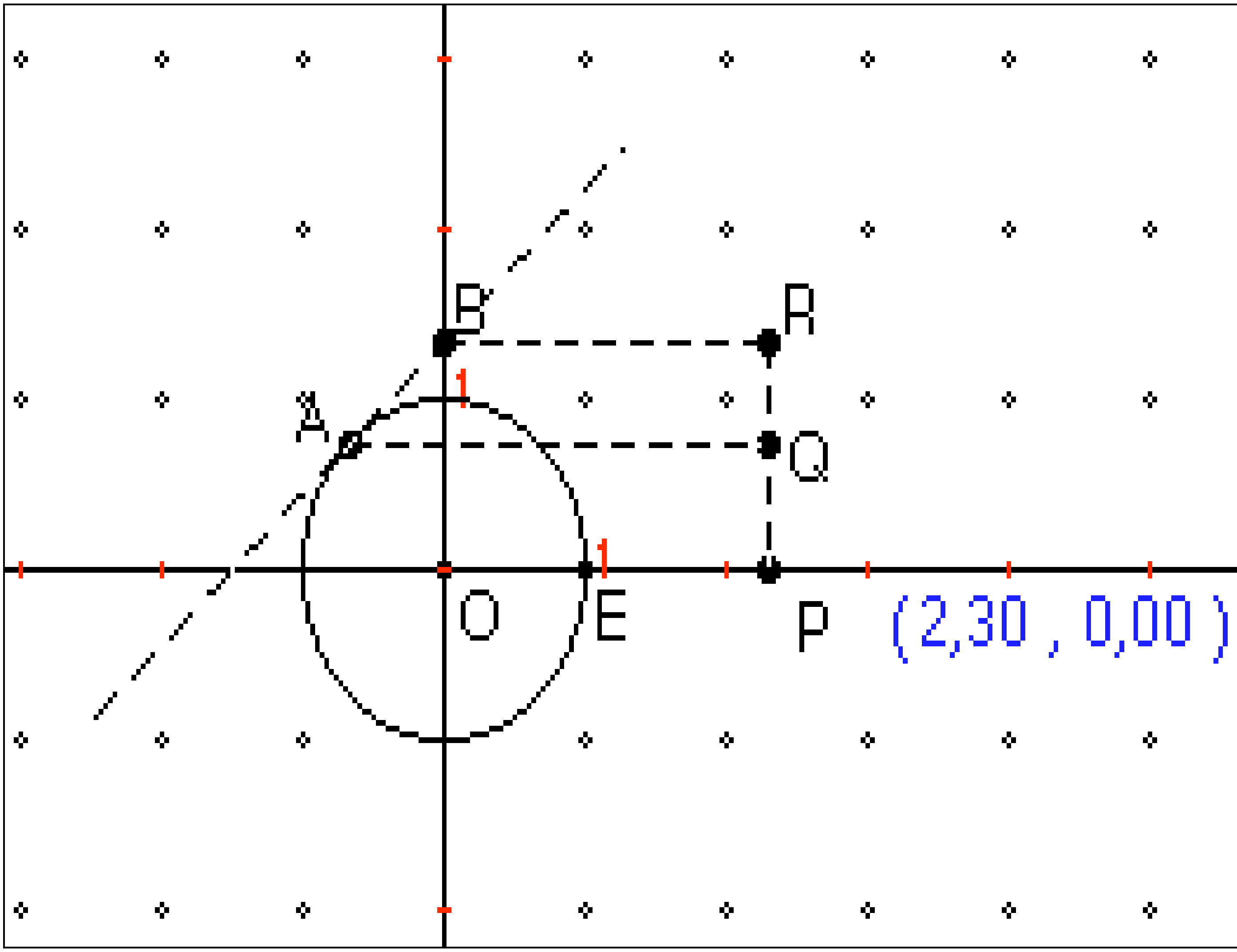
The geometry application of the TI 92 is an excellent tool to generate these graphs geometrically.

Let us show this in the case of <u>sin x /cosec x</u> :

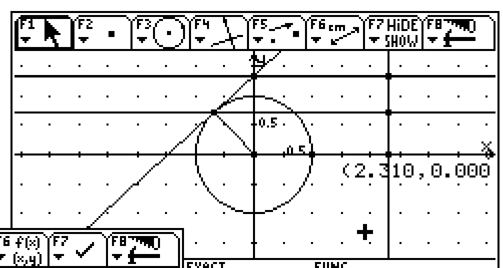
Draw the trigonometric circle (center O, origin E) and choose a point P on the x-axis. Determine the x-coordinate x of this point and draw the point A of the trigonometric circle that corresponds to the angle in standard position measuring x radians.
The y-coordinate of A is sinx.

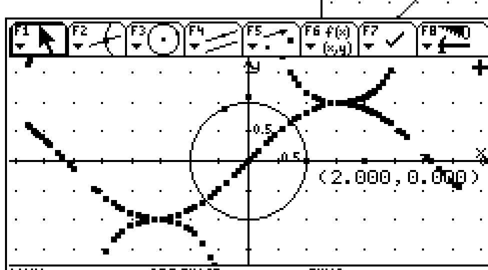
Let B be the intersection point of the tangent to the circle in A with the y - axis. It is easy to show that the ordinate of B is $\frac{1}{\sin x}$. Construct the point Q $(x, \sin x)$ and R $\left(x, \frac{1}{\sin x}\right)$.

Contents



Hide some constructions, ask to keep the trace of Q and R. Move P on the x-axis.
This generates the graphs of sin x and cosec x.





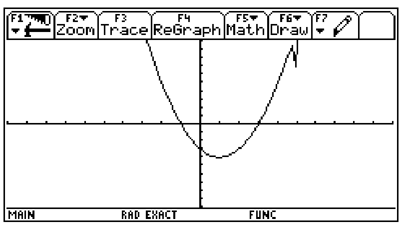
Tricky and challenging graphs.

References : - Paul Drijvers, Neem de grafiek over, NW, Tijdschrift voor Nederland Wiskundeonderwijs, juni 1995.
- Jos Willems, Uitwiskeling 9/1 (1992), 5-7.

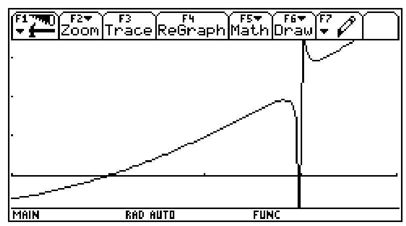
The purpose of the following exercises is to train students
- to have a critical look at the graphs appearing on their calculator
- to put together the behavior of the graph and the algebraic properties of the function.

1° *Sketch the graph of* $f(x) = \dfrac{5x^3 - 35x^2 + 35x + 76}{5(x-5)}$

In a standard window

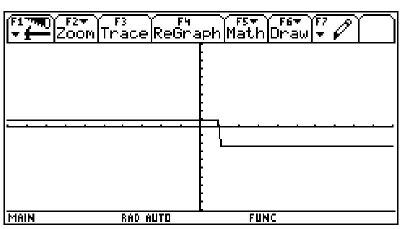


After determination of the domain and



extrema (detail of the graph)

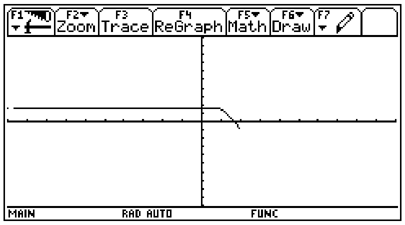
2° *Sketch the graph of* $f(x) = \tan^{-1}\dfrac{1+x}{1-x} - \tan^{-1} x$



In a standard window

It is easy to come to the conclusion that f(x) is a constant function which is not defined for $x = 1$. But why does this constant differ when $x > 1$ and $x < 1$?

3° *Sketch the graph of* $f(x) = \sin^{-1}(1-x) + \sin^{-1}\sqrt{2x - x^2}$



There is a problem of domain !
Moreover, for many students, it is not easy to understand why the function is not constant everywhere in the domain

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