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Integrating DERIVE into Calculus Instruction

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Description of Experimental Course

The purpose of this study was to investigate the effects of a computer algebra system (DERIVE) on students' comprehension and computational skills in Brief Calculus, a first year course for non-mathematics majors.

The study took place at the University of Wisconsin-Eau Claire, which has an undergraduate enrollment of approximately 10,000. The experimental group totaled 62 students, all in the same section. The control group consisted of seven sections taught by four instructors. The instructor of the experimental group did not teach any of the control group sections.

The experimental section was specifically assigned as the computer-enhanced instructional group. All sections met for four hours of class time per week for sixteen weeks.

Those who did not want to participate had the right to do so. Any students who wished to no longer participate in the study had the option of withdrawing from the study at any time.

The experimental group enrollment was 76 students, with 62 in the experiment. Those not in the experiment consisted of repeats, those opting not to participate, and anyone with previous experience with DERIVE. The control group sample was 234 students.

The experimental group was to receive the normal content of the course. The experimental group was also required to learn the computer software DERIVE.

The experimental group met in a large lecture room with three large blackboards and a very large projection screen above the blackboards. A portable personal computer was connected to the display device, enabling the instructor to display soft copy on the projection screen above the blackboards.

During the semester the experimental class section used the software package DERIVE as an instructional tool and as a problem-solving device. It is important to note that there are other available computer algebra systems (CAS) that could be used in place of or in addition to DERIVE.

The experimental group instructor's main intent for implementing DERIVE into the instructional activity was to reinforce the idea that a preconceived, well thought-out, organized, and logical algorithm must be established before proceeding to solve any mathematical problem. If the student was unencumbered by the thought of complicated computations, then the preconceived algorithm would reinforce the concepts which applied to the solution of the problem. This algorithm must be established regardless of whether the problem is to be solved by hand or solved using DERIVE.

All too often in mathematics we observe students trying to solve problems in one large step without organizing a plan of smaller steps that will solve the problem. Student encounter frustration because they visualize an enormous, insurmountable obstruction. The format which DERIVE provides enhances the concept of a step-by-step approach to problem solving. Because DERIVE can only execute one command at a time, the student must establish a one-command-at-a-time (or one-step-at-a-time) algorithm to solve a mathematical problem. Thus, before attempting any problem solving, the student realizes that a logical plan must be established and the student does not feel the anxiety that is encountered when trying to solve a problem in one large, disorganized step. The experimental group's instructor, therefore, emphasized that to solve a problem by hand or by DERIVE, a plan must be established first.

To establish the concept of a pre-conceived algorithm, the instructor provided a variety of different methods for showing examples of problem solving. The classroom environment of three large blackboards with a large projection screen above the blackboards for displaying computer output provided the opportunity to display different methods of problem solving.

One method was to do a problem by hand on the blackboard. Then, on the blackboard beside the solution achieved by hand, the instructor would provide the DERIVE commands which correspond to each step of the solution. Next, the output from executing the DERIVE commands would be projected on the overhead screen. It would then be emphasized that the same algorithm that was preconceived to solve the problem by hand was used to solve the problem by DERIVE.

Another method employed was to first provide the commands to solve a problem by DERIVE and then show the output of the solution on the overhead. Then the problem was either done on the blackboard by hand or the problem was assigned to the students as homework to be done by hand. Thus a cycle of problem solving was established with a two-way "street" between each means of solving the problem.

Often handouts were presented that displayed problems solved by hand, the DERIVE commands used to solve the problem, and the DERIVE output of the solution to the problem. An example of a typical class handout output is to find dy/dx by implicit differentiation and to find the equation of the tangent line at the given point for $x^3 - xy + y^2 = 4$ at (0, -2). Also included in this handout was a graphical display of the solution.

The graphing ability of DERIVE by means of the [PLOT] command was utilized by the instructor to emphasize concepts and to give a visual display of solutions. The speed and accuracy at which the graphs can be displayed establishes more credibility and also appeals to those students with a more visual learning style.

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The instructor also used the computing ability of DERIVE to emphasize certain concepts. An example of this was to investigate $\lim_{x \to 1^-} \frac{x^3 - 1}{x - 1}$.

The instructor used DERIVE to evaluate the function for values that got increasingly closer to 1 from the left side. To emphasize the "exactness" of DERIVE, the instructor then compared evaluations of the function between different utilities. This comparison appears in Table 1.

Table 1: COMPARATIVE CALCULATIONS

x	f(x) CASIO FX-85	f(x) TI-82	f(x) DERIVE
.9999	2.997001	2.997001	2.997001
.99999	3.0	2.999997	2.999997000001
.999999		2.9999997	2.99999970000001
.9999999		3.0	2.9999999700000001

The instructor continued to evaluate the function up to f(.999999999999999999) and still received output from DERIVE of a value less than 3. This is very helpful in the concept that the function, as x approaches 1 from the left, "draws near" but does not equal the value of 3.

In order to communicate effectively in class, on written assignments, quizzes and tests, a DERIVE language was established between the instructor and the students. Typically, the upper case letter that represents each DERIVE command, was used to indicate the particular command. For example, A is the [Author] command and M,S would indicate [Manage][Substitute].

The students had the opportunity to use DERIVE through various activities. Approximately every two weeks the students were assigned a five-problem DERIVE project. These problems were assigned from the current material being covered and were typically the hardest problems in the problem section from the textbook. The students produced hard copy of the solutions to the problems and also were asked to show the plan that was used to solve the problem on the same page just to the right of the output.

The main purpose of the five-problem projects was to reinforce the concept of a preconceived algorithm. Since the students can only execute one DERIVE command at a time, a step-by-step algorithm must be followed to successfully solve the problem. The students then carry over the step-by-step approach to solving problems by hand. The concepts, therefore, were being emphasized, not the computational process.

Another purpose of these projects was to instill in students a feeling of success by having them solve "harder" problems without being encumbered by the anxiety of possibly making algebraic and computational errors. If the students happen to get an incorrect answer, they are assured that the wrong answer is not due to an algebraic or

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computational error. Thus the reason for the error probably lies in an incorrect algorithm. This assists the students in accepting the concept that a correct algorithm is essential to the successful completion of the problem.

The students also were required on quizzes and tests to indicate the DERIVE commands they would use to solve a particular problem. A DERIVE reference sheet with all appropriate commands was available to the students. Occasionally the quiz or test problem to be solved by DERIVE was similar to a problem to be solved by hand. On one such occasion, a comparison was made between the answer generated by hand and the answers generated by DERIVE. The answers were categorized as right, wrong, or left blank. A matrix of the results is shown in Table 2.

TABLE 2: COMPARISON OF SOLUTIONS BY HAND V.S. DERIVE

		BY HAND			
		RIGHT	WRONG	BLANK	TOTAL
	RIGHT	20	10	0	30
DERIVE	WRONG	1	24	2	27
	BLANK	0	3	5	8
	TOTAL	21	37	7	65

Thirty students had the correct answer by DERIVE, while 21 students did the problem correctly by hand. It is interesting to note that 10 students had the correct answer by DERIVE but incorrectly generated an answer to the question by hand. In most of these ten cases, the hand calculation had an algebraic or computational error. Only one student did the problem correctly by hand and incorrectly by DERIVE.

For the most part, once the students learned how to use the software, most of them enjoyed the opportunity to utilize DERIVE as a problem-solving tool.

In summary, the intent of the experimental group's instructor was to reinforce concepts by [1] graphical, analytical and numerical computer demonstrations in class, and [2] unburdening the students from computational anxiety in order to reinforce conceptual understanding through formulation of a step-by-step problem solving algorithm.

Description of Study

Hypothesis

The intent of this research activity is to investigate whether implementing DERIVE into the study of calculus will develop better conceptual understanding without loss of computational ability.

H1: Students in the experimental group will perform significantly higher on the conceptual part of the posttest than the control sections.

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H2: Students in the experimental group will show no significant difference on the computational part of the posttest from the control sections.

Description of Subjects

Eight sections of Brief Calculus were available to students at the University of Wisconsin-Eau Claire during the Fall 1994 semester. At the beginning of the semester, seven sections each had approximately 35 students registered and one section, the experimental group, had approximately 75 students enrolled.

The instructor of the experimental group did not teach any of the other seven sections. Those sections were taught by four different faculty members, with three people each teaching two sections and one person teaching one section. All instructors agreed to participate in the experiment.

Text

All sections of the course used the same text, Brief Calculus with Applications, alternate third edition by Roland Larson, Robert Hostetler and Bruce Edwards (D.C. Heath and Company, 1991).

DERIVE

DERIVE is a computer algebra system authored by Albert Rich, Joan Rich, and David Stoutmeyer. It is published by Soft Warehouse, 3660 Waialae Avenue, Suite 304, Honolulu, Hawaii 96816. As described by the authors in the user manual, "Making mathematics more exciting and enjoyable is the driving force behind the development of the DERIVE program. The system is designed to eliminate the drudgery of performing long tedious mathematical calculations. This gives you the freedom to explore different approaches to problems -- approaches that you would probably not consider if you had to do the calculations by hand."

DERIVE is very user-friendly, with the appropriate menu for each step displayed at the bottom of the screen. The screen itself can be divided into two or more sections to display one algebra screen and one or more graphing screens. The program allows for input in typically the same manner that a user is accustomed to writing expressions or equations.

The range of its capabilities include algebra, trigonometry, calculus, vectors, and matrices. It can perform symbolically or analytically. The numeric computations can be either exact or approximate.
All commands needed for a calculus course are included. This includes evaluation limits, computing derivatives, implicit differentiation, and definite or indefinite integrals.

Description of Research Method

Both groups studied Brief Calculus, a requirement for business, health and some science majors. The pretest was Intermediate Algebra Skills, purchased from Educational Testing Services. The posttest was authored in cooperation from the participating instructors. The instructors each submitted several questions pertaining to the material covered in the course. From the question pool, the instructors then selected 30 questions which they mutually agreed upon for the posttest. The posttest had 20 conceptual questions and 20 computational questions.

Enrollment into the different sections was random. The use of DERIVE in the experimental section was not advertised. The instructor of the experimental group had previously used DERIVE in the same course, therefore students may have had prior knowledge that DERIVE would be utilized in this section. The students, at the time they were registering, had no knowledge that an experiment was going to be conducted.

Both groups were informed of the experiment during the first meeting of the class. They were then asked to sign a consent form. All students took the pretest during the second meeting of the class. The students who agreed to participate in the experiment had their scores recorded.

The posttest was administered following 16 weeks of instruction during the final exam period.

Relationship of Research Method to Hypothesis

The objective of this study was to address the use of computer assisted instruction, in the form of a particular computer algebra system, DERIVE, within the context of a full semester calculus course at the University of Wisconsin-Eau Claire in Eau Claire, Wisconsin with college students who were enrolled in Brief Calculus in Fall Semester 1994. The researcher wanted to find out whether using the symbolic algebra package would improve the conceptual understanding in Brief Calculus.

Instrumentation

The pretest was Intermediate Algebra Skills purchased from Educational Testing Services, Princeton, New Jersey. The test was intended to measure algebra skills. The posttest was a common final exam authored by the participating instructors. Data regarding reliability and validity was not available.

Procedure for Collecting Data

The tests were administered by the individual section instructors. The pretest was given out before any instruction and the posttest was administered after the end of the last class of the semester. The pretest and posttest were machine scored. The results were

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tabulated by the researcher.

Procedure for Analysis of Data

The collated data was analyzed by computer to calculate means and standard deviations, as well as the Wilcoxen rank sum test (a nonparametric test for independent sampling) to find any differences between the means for (a) the pretest of Group A and the pretest for Group B, and for (b) the posttest for Group A and the posttest for Group B. All Wilcoxen tests were calculated at the 0.05% level of significance.

Limitations

Given the history of the experiment, the maturation of subjects, the lack of validity and reliability of pretest and posttest, statistical regression, differential selection of students, selection-maturation intersection and experimenter effects, this study has limitations.

These limitations need to be taken into consideration when analyzing the results and making generalizations.

Results

Pretest

The purpose of the pretest was to ensure that no significant difference in mathematical ability existed between the experimental group and the control group at the beginning of the study. The instrument use to measure mathematical ability was "Intermediate Algebra Skills." The pretest was given during the first week of instruction.

> Ho: The mean of the experimental group is equal to the mean of the control group.
> Ha: The mean of the control group will be significantly different from the mean of the control group.

The mean for the experimental group was 21.977 (out of possible 30) correct answers. The mean for the control group was 22.109 correct answers. The standard deviations were 4.055 for the experimental group and 4.032 for the control group.

The results on the pretest of the control group were then compared to the results of the experimental group using a two-tailed t-test for independent groups at the .05 level of significance.

The results indicate that there is no significant difference between the two groups in terms of mathematical ability at the beginning of the instruction period ($p = 0.85$, $p > 0.05$, therefore cannot reject null hypothesis).

Once the two groups were equated for algebra knowledge, the research questions could

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then be addressed.

Conceptual part of posttest

The first research question addresses if students in the experimental group will perform significantly higher on the conceptual part of the posttest than the control group.

Ho: The mean score of the experimental group on the conceptual part of the posttest will equal the mean score of the control group.
Ha: The mean score of the experimental group on the conceptual part of the posttest will be significantly higher than the mean score of the control group.

The posttest was given during final exam week following sixteen weeks of instruction. The posttest consisted of twenty conceptual questions and twenty computational questions. These questions were taken from a pool that was generated by the participating instructors. All participating instructors agreed upon the final content of the exam. No computers were available to any groups for use in the final exam. All conditions for taking the posttest were equal for all of the groups.

The mean for the experimental group on the conceptual part of the posttest was 12.568 (out of 20). The mean for the control group on the conceptual part of the posttest was 11.80. The respective standard deviations were 2.256 and 3.151.

The p-value of 0.035 ($p<0.05$) indicates that the null hypothesis can be rejected and that the experimental group scored significantly higher on the conceptual part of the posttest than the control group.

Computational part of posttest

The second research question addresses if students in the experimental group will show no significant difference on the computational part of the posttest than the control group.

Ho: The mean score of the experimental group on the computational part of the posttest will equal the mean score of the control group.
Ha: The mean score of the experimental group on the computational part of the posttest will be significantly higher than the mean score of the control group.

The mean for the experimental group on the computational part of the posttest was 13.866 (out of possible 20). The mean for the control group was 14.539 . The standard deviations were 3.029 and 3.520 respectively.
The p-value of 0.22 indicates that the null hypothesis cannot be rejected. Thus the mean scores of the experimental group are equal to the mean scores of the control group on the computational part of the posttest.

Conclusions

The intent of this research was to investigate whether implementing the computer algebra system DERIVE into a calculus class would develop a better conceptual understanding without loss of computational ability. Although the results were marginal, the study did indicate better conceptual understanding by the group using DERIVE without loss of pen-and-paper computational ability. This was accomplished in the experimental group by using DERIVE as an instructional tool in class to emphasize concepts and problem-solving, and also by letting students solve more complex, real-life problems using DERIVE. These more difficult calculus problems came in the form of six projects of five problems each. The instructor emphasized that in order to solve the problems, a step-by-step plan must be developed that utilizes the appropriate calculus concepts.

The very defined, step-by-step format of DERIVE required the students to decide which operations to perform in order to successfully find a solution. These decisions of what steps to proceed with require from the students a basic understanding of the mathematical concepts that are inherent to solving calculus-related problems. The step-by-step format allows students to realize that solving mathematical problems does not require one giant "miracle" step from initial problem to solution. What DERIVE provides is the opportunity to implement a series of small steps, each one simplistic in its nature, that -- when put together in an algorithm -- will solve complex problems.

Many students, when they enter the study of calculus, are accustomed to applying only one algebraic of trigonometric concept to problem solving, or they combine many simple concepts into one large step to solve a problem. At the calculus level, they are unable to approach complex problem-solving in a manner that necessitates separating out different concepts into a one-application-at-a-time procedure. Computer algebra systems, such as DERIVE, can ease the apprehension of doing a long step-by-step procedure by providing a friendly format to accommodate such a task, and also by reducing tedious calculations and eliminating algebraic and computational errors.

It is essential to note that in order to solve calculus problems using any computer algebra system, a basic understanding of the mathematical concepts must be attained before deciding what operations to perform -- the computer will not do this for you.

Another factor which increases conceptual understanding is the graphing ability of DERIVE. The geometrical and analytical connections made by the instructor during class, and also by the student while problem solving, help reinforce the mathematical concepts.

In class the instructor can use the plotting ability to show secants, tangents, area representations, etc. All concepts of analytical geometry can be facilitated in a manner that exudes more credibility because they are plotted accurately. The simultaneous displays on the algebra screen and the plot screen can enable the student to make connections between the two. An example is finding the equation of the tangent line to a given curve at a given point. On the algebra screen the step-by-step procedure for finding the equation to the tangent line can be presented. Then on the plot screen, the curve can

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be drawn and then the tangent line. With the free-floating cursor, the point of tangency can be explored. Of course this can be done by hand on the blackboard, but this researcher believes that the computer approach extends itself to more learning styles [which is a possible area for new research].

As a student, the plotting ability of DERIVE can provide mental images of solutions to real-life problems, especially in the area of optimization. All too often we find problems in textbooks which have simple coefficients that are based in a small grid centered at the origin. The plotting capabilities (along with the algebra capabilities) of DERIVE allow students to view graphs whose main attention is far from the origin. This expands the imagination and opens up new worlds for discovery.

Besides conceptual understanding, this study also addressed the effect of DERIVE upon computational and manipulative ability. Although the experimental group was required to use DERIVE exclusively on some of their tests and assignments, computation by hand was not totally ignored in the class. Throughout the semester the students had the option of doing most of the regular homework by hand or using DERIVE. The chapter tests were generally pen-and-paper exams. Therefore, it is not surprising that no significant difference was found between the groups on the computational part of the posttest.

But as more intense implementation of computer algebra systems is incorporated into calculus classes, concerns will arise as to the traditional development of computational skills. The same concerns we feel when we see students doing simple multiplication with a hand-held calculator are going to rise when we see students doing simple factorizations and derivatives with computer algebra systems. With this new technology will there be changes in the curriculum and in the development of a basic "math foundation"? Will changes occur in the building of skills and understandings to accommodate the influence of technology?

The effective use of computer algebra systems are going to require the understanding of mathematical concepts. The removal of the burden of computational and algebraic errors will enable students to cross a threshold into successful real-life problem solving. The technology shall be viewed as a tool designed to give students the opportunity to explore and investigate.

The difficulty with drawing conclusions based on a simple observational study such as this is that a statistically significant difference could possibly be due to other factors than the method of instruction. In this case, one of the major factors is the involvement of multiple instructors. The instructor of the experimental group did not also teach any of the control group, and the control group consisted of seven sections taught by four instructors. Other factors, such as hardware problems with DERIVE throughout the semester, also should be considered.

Recommendations

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The recommendations I present are two-fold:

1. The implementation of computer algebra systems, and other technology, into educational activities should be considered as positive.
2. More research must be undertaken in order to understand the implications of implementing technology into the study of mathematics.

In 1989 the NCTM recommended that "appropriate technology should be available for classroom instruction." One interpretation of "appropriate" is "beneficial to the students." These benefits may be short-term, such as a one-day unit enhanced by technology, or long-term, such as a four-year college education with a planned curriculum replete with numerous technologies. This study, although full of limitations, did agree with Kulik's meta-analyses of the early 1980's that computer-based education does make a small, but significant, contribution to achievement. In this study it was the use of DERIVE on the conceptual understanding of college students in a first-year calculus course.

Computer algebra systems certainly do have advantages to both students and educators. Unburdening of computational and algebraic errors, ability to solve more difficult and complex application problems, and the concentration on concepts are a few advantages to the student. Development of exploratory tasks, enrichment of presentations, and measurement of student conceptual understanding are a few advantages to the educator.

We all know that our world is changing faster than ever due to the increased development of technology. In almost every area of life -- home, business, medicine, sciences, recreation -- technology is having a positive impact. This is also true in education. We must realize what this positive impact may be to our students and implement the technology in an appropriate manner.

There is no doubt that many competent people within the educational field have valid concerns about the ramifications of computer enhanced education. In the mathematical area, a large part of the concern is focused on the development of traditional computational and manipulative skills. This is one area where more research is needed to fully understand the impact of technology on these basic skills. Not only is research needed in our discipline, but also in every aspect of technology and education. As the potential for implementing technology into the classroom expands, so does the need for clear and decisive research.

** What are the short-term effects?
** What are the long-term effects?
** What is the best medium [or media] for a particular subject?
** What is the best environment?
** What type of student benefits most from a particular technology?
** How do textbooks interact with technology?
** Should curricula change to deal with less emphasis on "pen and paper"?

These are just a few of the questions that research will address in the near future. We are just beginning to understand the impact of computers on education. AS the technology increases, the impact will increase. I believe as teachers we need to do further investigation into these impacts so we can understand the benefits which may exist in the classroom. Finally, and most important, we have to address how these benefits of technology will affect the broader implications on the total curriculum.