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EMPHASIZING APPLICATIONS OVER COMPUTATIONS USING DERIVE FOR WINDOWS

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Introduction

One of the features of a computer algebra system such as DERIVE is the capability to empower students to solve problems which they might find too difficult to compute manually. This workshop at the Third International DERIVE and TI-92 Conference at Gettysburg College centered on projects in which the computations may inhibit student understanding of the conceptual mathematics. Some of the projects considered may require manipulation of large amounts of data, or use of many trigonometric function properties. Other projects may obfuscate the mathematics properties with lengthy algebra, matrix algebra, or calculus computations.

Although projects such as those outlined below include a level of computation or manipulation which can seem discouraging to our students, they can be used to demonstrate the power of mathematics. DERIVE can serve as a computing assistant in the solution of real applications in which the computations, although tedious and obtrusive, are an essential tool for problem description and solution.

Using projects which emphasize applications over computations place a greater burden on instructors than traditional pencil-and-paper projects. First, gathering and interpreting meaningful data requires an extensive time commitment. The teacher must create assignments that still require students to think, not just copy keystrokes and dump algebra expressions from the stack into the teacher inbox. Just as the teacher expends more effort creating projects, so must the student. Students who perform large calculations via computer algebra still need to understand the concepts which they are applying. In addition, reading and writing skills become even more critical, as students must be able to correctly interpret both the data which they use and the results they generate. Writing also provides a nice means of assessment of such projects. Finally, students need to be able to access the technology necessary for the projects. Access is indeed an issue for which hand-held, portable CAS such as the TI-92 have a definite advantage over a computer lab with a limited number of machines and limited access.

Outline of Projects

Each of the projects outlined below is available via email and adaptable to other CAS such as the TI-92.

1. **Chicago Weather**. This project is included in its entirety below. DERIVE is a useful tool as we scatter-plot data and fit appropriate curves to the data.

2. **Bacteria Growth**. This project models growth of e. coli bacteria in a test tube. DERIVE makes it possible to focus on the lag, log, and death phases of population growth. Without DERIVE's autoscale feature, students experience great difficulty "finding" the graph of the exponential curve.

3. **Modeling the AIDS Epidemic**. This study of actual AIDS case data allows the mathematics of exponential growth and curve fitting to serve student exploration of a current health issue. DERIVE allows the focus to shift away from the manipulations toward the interpretation of data.

4. **Resistance to Blood Flow**. A heart "by-pass" operation allows blood to flow past a blocked artery. The radii of the blood vessels dictates the appropriate angle for joining the vessels to minimize resistance to blood flow. The computations are nearly mind-numbing, with many constants, variables and ugly combinations of trigonometric functions. However, the mathematics involved is simple optimization techniques using the first derivative. DERIVE keeps track of the awkward functions and constants.

5. **Modular Arithmetic**. DERIVE can quickly and efficiently perform modular computations. A nice application of this is an extension of the Caesar Cipher, where messages are encoded by the algorithm $x \to ax + b \bmod 27$. This exploration can be used at different levels of student mathematics understanding, grade school to collegiate.

6. **Regression Analysis of Market Data**. DERIVE is of course extremely useful in fitting polynomials to curve data. One example of meaningful data is market sales, which can be of use to business as a predictor. A discussion of techniques which may be used to select the most appropriate curve as well as the matrix algebra computations which underlie the FIT algorithm allow the instructor to tailor such a project to student level.

Project on Chicago Weather

Goal: We study weather data for Chicago, Illinois. We will create a function that describes the temperature as a function of time, a second function to describe precipitation per month as a function of time and a third function which quantifies the amount of snow per month as a function of time. We will use these functions to calculate the total precipitation and total snowfall for a typical year in Chicago. The data source is the 115th Edition of Statistical Abstract of the United States 1995 published by the U.S. Department of Commerce.

Introduction: The function relating temperature to time is a periodic function which we could model with either a sine curve or a cosine curve. In this project, we will use a cosine curve. The graph below shows a cosine curve which is shifted up D units and to the right

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C units. The equation for this cosine curve is $y = A\cos(B(x - C)) + D$ (see Fig. 1 next page).

We also see the horizontal line, $y = D$, which bisects the curve between the high point and low point. Notice that the distance between the curve maximum *y* and *D* equals the distance between the minimum *y* and *D;* this distance is called the amplitude, $|A|$. We can state this mathematically as $|y_{max} - D| = |y_{min} - D| = |A|$.

Studying the curve behavior from left to right, notice the first high point (for positive *x*) is at $x = C$. We choose *A* positive since we want this graph to be a cosine curve shifted right *C* units with a maximum at $x = C$. The cosine curve repeats values for intervals of fixed size along the *x* axis, that is we see the same shape between any two high points, or between any two low points, or between any two points where $y = D$ crosses the function, and so on. This characteristic means we have a periodic function. Investigating the nature of periodicity, we have

$$\begin{aligned}\cos(B(x + 2\pi/B)) &= \cos(Bx + 2\pi)\\ &= \cos(Bx)\cos(2\pi) - \sin(Bx)\sin(2\pi)\\ &= \cos(Bx)\end{aligned}$$

Hence, the period of a function is $2\pi/B$, and can be deduced by computing the horizontal distance between subsequent maximum (or minimum) values. Knowledge of the period allows us to compute the value of *B*.

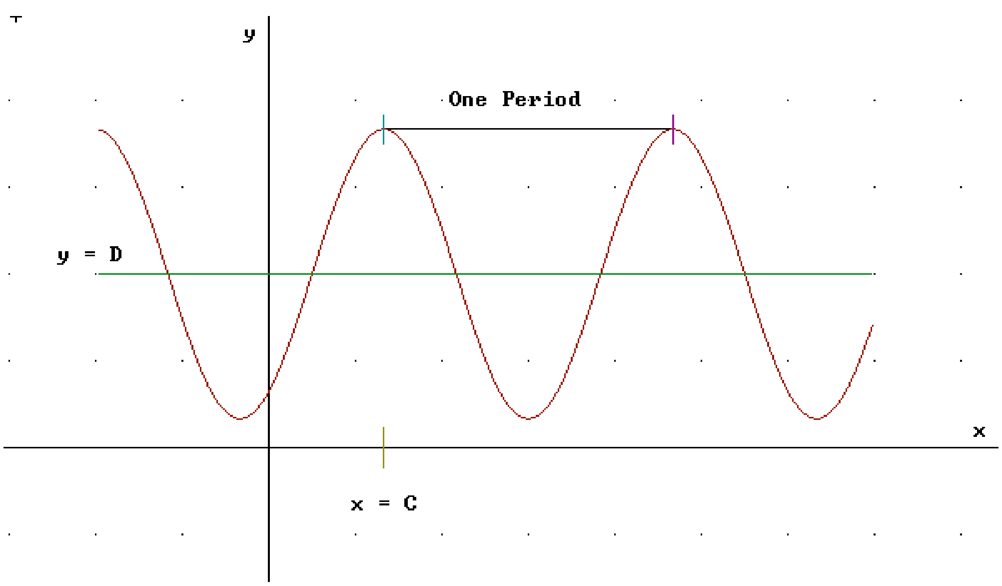

Fig. 1 $y = A\cos(B(x - C)) + D$

Before we continue, be certain you understand how to write a cosine function based on this data. Write the equation for a cosine function which has a maximum at (4,11) and a minimum at (9,1) (and no extrema between these points). Complete the following and then write the function:

A = __________ Period = __________ $B = 2\pi$/(period) = __________

C = horizontal shift = __________ D = vertical shift = __________

y = _________________________

Outline of Procedure: We begin our study with an exploration of temperature data, then precipitation data and then snowfall. Before you begin the procedures, change the following setting: Declare, Algebra State, Output, Notation, *Decimal*

1. **Temperature**

The data presented below represents (time in months, AVERAGE temperature in Fahrenheit). For the x values, $x = 0$ is January 1 and $x = 0.5$ is January 15. As another example, $x = 5.5$ represents June 15. Use DERIVE to Author and plot the points:

(0.5, 21.0) , (1.5, 25.4) , (2.5, 37.2) , (3.5, 48.6) , (4.5, 58.9) , (5.5, 68.6) ,
(6.5, 73.2) , (7.5, 71.7) , (8.5, 64.4) , (9.5, 52.8) , (10.5, 40.0) , (11.5, 26.6)

Does the plot of data points suggest a cosine curve is feasible? Use the information presented in the introduction to write a cosine curve to fit these data.

A = __________ Period = __________ $B = 2\pi$/(period) = __________

C = horizontal shift = __________ D = vertical shift = __________

y = _________________________

(Make a note of the stack number of this function.) Use the CHI function to plot the curve for one period, which is one year, or twelve months.

2. **Precipitation**

We continue our weather project with precipitation data. The data presented below represents (time in months, precipitation per month in inches). Decide what kinds of precipitation (rain, snow, sleet, etc) are likely to occur in given months in Chicago. Author the points, create a second graphing window, and plot these points.

(0.5, 1.5) , (1.5, 1.4) , (2.5, 2.7) , (3.5, 3.6) , (4.5, 3.3) , (5.5, 3.8) ,
(6.5, 3.7) , (7.5, 4.2) , (8.5, 3.8) , (9.5, 2.4) , (10.5, 2.9), (11.5, 2.5)

Does the plot of data points suggest a cosine curve is feasible? What about a step function? Does a parabola look better? Use the DERIVE Fit Operator to fit a quadratic function to this data. Round and record the coefficients to two decimal places. (Author Fit([x, ax^2+bx+c], matrix of data points) and Simplify Approximate).

y = ____________________
(Make a note of the stack number of this function.) Use the CHI function to plot the curve over one year (twelve months).

This quadratic function representing precipitation calculates precipitation per month. It is the rate of change, a derivative. Use your knowledge of integration to set up a definite integral representing total precipitation per year in Chicago.

Definite Integral__________ Total precipitation__________(units! 2 decimal places)

3. **Snowfall**

The dependent variable for the third set of data represents inches of snow per month. Author and plot these points in a new graphing window:

(0.5, 10.1) , (1.5, 8.1) , (2.5, 7.0) , (3.5, 1.7) , (4.5, 0.1) , (5.5, 0) ,
(6.5, 0) , (7.5, 0) , (8.5, 0) , (9.5, 0.4) , (10.5, 1.9) , (11.5, 8.3)

A useful rule of thumb is that, near freezing temperature, 10 inches of snow melts to approximately 1 inch of water. Hence, this data can be considered a rescaling of data similar to the precipitation data. This implies that a curve similar to that used for precipitation should be fit to the months where there is snow. Use FIT to find a parabola which closely fits the data for months in which snow falls.

y = ___________________

Our last task is to estimate the annual total snowfall in Chicago. Integrate to determine annual snowfall.

Definite Integral:_________ Annual Snowfall:__________

Return to your graph of the average temperature function and graph $y = 32$ on the same plot. How many times does the line $y = 32$ intersect the function? Use the mouse to click on the intersection points and estimate the coordinates; convert these to calendar days. How could you explain that there is measurable snow after dates in which the average temperature is 32°?

Ideas for further discussion:

1. Discuss semilog and log-log graphs to determine if quadratic fit is appropriate for precipitation and snowfall graphs. For advanced students, discuss the algebraic process which DERIVE uses in the FIT algorithm (least squares analysis).

2. Using the given rule of thumb, convert the snowfall inches to inches of water. Predict the resulting graph by algebraically rescaling the curve. Water content of snow depends on the temperature that prevails while it is snowing. Research the conversion of snowfall to water at other temperatures, such as the heavy, slushy snow which falls in above-freezing temperatures and the light, fluffy accumulation of snow that falls at low temperatures.

3. Model other phenomena such as tides, hours of daylight, city electrical consumption, etc. What are some phenomena which can be modeled with a sinusoidal curve? The definite integral serves to measure accumulation in this project. What accumulations can you compute?