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The "New" Differential Equations and the "Old" Numerical Analysis

James P. Fink
Gettysburg College
Gettysburg, PA 17325
jfink@cs.gettysburg.edu

Introduction

Since the almost legendary *Calculus for a New Century* colloquium in 1987, there has been considerable discussion and change about the way we teach calculus. More recently, many of the same ideas and goals have had a major effect on the way we approach differential equations, but numerical analysis courses have so far escaped our attention. In this paper, we review the welcome changes that have occurred in the first undergraduate differential equations course and discuss the need for similar changes in a first numerical analysis course.

The "New" Differential Equations

We begin with a historical question: Where were you on Friday, March 13, 1964? (Yes, it was a Friday the 13th.) As it turned out, I was a first-year graduate student at Stanford University and just turned in my take-home final examination in ordinary differential equations. In that exam is a painstakingly hand-drawn phase-plane portrait for a certain system of nonlinear differential equations followed by the comments:

> "...tedious calculations are not included here..."
> "...this picture is highly qualitative..."

Thirty-four years of computer technology have sure changed all that! Nowadays, simple and affordable computer software has turned what used to be the exclusive domain of graduate differential equations courses into standard undergraduate fare and is the underlying reason for the recent transformation of undergraduate differential equations courses.

There are many goals for a modern undergraduate differential equations course, but they typically fall into the following three categories:

- Given today's technology, an emphasis on the tricks and techniques of an old differential equations course is no longer appropriate.

- The differential equations obtained from real-world applications are often nonlinear, and so numerical and qualitative techniques are more effective.

- Differential equations is one of the few undergraduate courses in which students can get a glimpse of contemporary mathematical research.

Of course, these goals can be implemented in a variety of ways. Here is an effective program advocated by many faculty who have been actively involved in the teaching of differential equations (for example, see [1] and [2]):

- Replace specialized techniques by a focus on the formulation of differential equations and the interpretation of solutions.
- Emphasize the qualitative approach by visualizing differential equations and their solutions.
- Use the computer with numerical techniques to graph solutions.
- Analyze any possible explicit solutions in light of qualitative and numerical information.
- Understand the geometry and long-term behavior of solutions.
- Adopt a modeling and a dynamical systems approach to differential equations.

The “Old” Numerical Analysis

The above discussion of undergraduate differential equations courses is neither new nor surprising. For the past several years, the Boston University Differential Equations Project and the Consortium for Ordinary Differential Equations Experiments (CODEE) have been centers of activity for rethinking the way we teach undergraduate differential equations, holding workshops, and maintaining Web sites and electronic discussion groups. The two textbooks [1] and [2] are products of those efforts.

What is surprising is how little effect these ideas and modern computer technology have had on the teaching of numerical analysis, a discipline which closely parallels the development of scientific computing. Perhaps I was merely uninformed, and so I approached colleagues across the country at meetings, called them on the telephone, and contacted them by e-mail in an attempt to learn what was happening. Unfortunately, this informal survey of numerical analysis courses and textbooks turned up little in the way of change and innovation. Indeed, the responses were downright discouraging:

- A disturbingly large number of faculty members said that “numerical analysis died” at their colleges due in part to the lack of a suitable textbook.
- Many went so far as to blame the current crop of textbooks for actually deterring the curious from further investigation of numerical analysis.
- A few suggested that the nature of the material itself gets in the way of innovation.

- In general, the attitude was that numerical analysis is regarded as unglamorous and is held in low esteem by many mathematicians, physicists, and computer scientists.

Well, given this lowly state of affairs, what can be done? The central theme which underlies the discussions I had with colleagues is:

> A first numerical analysis course should emphasize the *positive* (what can be done) instead of the *negative* (what can go wrong).

That is, numerical analysis should emphasize the study of algorithms and the power of what can be done when one goes beyond the need for analytic solutions; it should *not* be preoccupied with rounding error. Rounding error is important, but like the tricks and techniques for solving differential equations it is less appropriate today. Numerical analysis is concerned with much more than rounding error.

This past spring semester, I taught an undergraduate numerical analysis course and experimented with a variety of ideas and approaches. Needless to say, some worked; others did not. I am grateful to the students who willingly agreed to go along with my experiments and who equally willingly let me know what they thought. The "on-the-job" lessons they taught me bear mentioning.

- Don't start the course with floating-point arithmetic.

When floating-point arithmetic does come up, emphasize that it is an important topic, but it is not the central topic of numerical analysis. Instead, numerical analysis is principally concerned with the development of fast algorithms for problems that usually cannot be solved in closed form.

- Avoid the depressing expression "error analysis" when it is usually "convergence" that is the issue.

It is indeed depressing to devote most of a subject to a discussion of error. In the 5% of the time when we must talk about rounding error, we should call it "rounding error analysis" to make it clear what is being discussed.

- Keep the syllabus reasonable and accessible.

Numerical analysis often becomes a service course with too many topics being crammed into the syllabus. It is unfortunate that a student who will not be a pure mathematician spends as long on group theory as on all of algorithmic mathematics. Given this reality, however, it is better to do fewer topics well and give students the means to know what to do and where to go if they need to learn more.

- Do computer demonstrations in each class to illustrate key points.

This lesson may seem obvious, but it is one we as instructors do not always heed. Excellent software is readily available, and students respond much better to computer demonstrations.

Besides, it helps to keep them involved, interested—and awake. I have even begun to devote one class each week to a laboratory similar to what our colleagues in the sciences have done for years. The only difference is that our laboratory is a computer laboratory, and the environment is *Mathematica* and MATLAB.

- Finally, sprinkle in a bit of history.

From 1945 to 1998, computers have speeded up by a factor of 10^{10}, and algorithms have essentially kept pace. It is a historic development that shows no signs of slowing down and is leading to a transformation of the way we do science and engineering. Even better, it is happening in our lifetimes. The group theorists should be so lucky!

References

[1] *Differential Equations* by Paul Blanchard, Robert L. Devaney, and Glen R. Hall, Brooks/Cole Publishing Company, Pacific Grove, CA, 1998.

[2] *Differential Equations, A Modeling Perspective* by Robert L. Borrelli and Courtney S. Coleman, John Wiley & Sons, Inc., New York, 1998.