Probability Distributions in Math Courses with the TI-92

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In this paper we report about a mathematics cours on an Austrian high school treating with probability distributions. All students in this course used a TI-92 during math classes, at home for doing exercises and during math tests. The TI-92 was used as a tool for calculations and illustrations making the concepts of probability distributions easy to understand for the students and problems to be solved in a very elementary way.

Introduction

We report about a mathematics course treating with probability distributions for a class (12 girls and 3 boys at the age of 17 / 18) at the Stiftsgymnasium Wilhering, which is a privat high school near Linz in Austria. The emphasis of the school and the interests of the students lie in the study of languages and arts. Since the pupils are not intrinsically motivated in mathematics, it is our aim, to make the concept of probability distributions easy to understand for the students by treating the distribution functions in a very elementary way. All pupils use a TI-92, which is a pocket calculator for symbolic and numerical manipulations and for graphical visualizations. The students use the TI-92 during math classes, at home for doing exercises and during math tests.

The TI-92 supports our goals quite well. We can use it for illustration (graphing the distribution functions), as a calculating aid (computing binomial coeffizients, integrals etc.), for modularizing (defining simple functions by the students) and for elementarization (e.g. solving problems within normal distributions via integrals without standardization, determining confidential intervals experimentally). An introduction to these concepts using the TI-92 can be found in [Aspetsberger, Schlöglhofer 96] and [Schmidt 96].

The treatment of probability theory and probability distributions is a central topic of the curriculum of Austrian highschools for students at the age of 17 and 18 (see [Bürger,

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Fischer, Malle 92], [Reichel, Müller, Hanisch, Laub 92] and [Szirucsek, Dinauer, Unfried, Schatzl 92]).

Frequency distributions

The goal of this section is to introduce the concept of probabiltiy distributions by treating frequency distributions. The students should learn that the shapes of the histogramms of frequency distributions become more similar to the shape of the respective probabiltiy distribution if we repeat an experiment for many times ("law of large numbers"). For this reason it is necessary that the students can analyze and compare many frequency distributions with growing numbers of experiments.

It is very time consuming to compute frequency tables by hand and to plot histogramms for large numbers of experimental values. On the other hand, if we are computing frequency distributions for few numbers only, major differences in the shapes of the histogramms may occur and no convergence can be observed.

In this situation computers can be used for doing simulations. Using the TI-92 we can compute frequency distributions and plot histogramms for large numbers of values and to repeat this procedure for many times very quickly.

In the following we demonstrate this process by simulating the example of throwing two dices and adding the number of the points of both dices. Which type of distribution will occur?

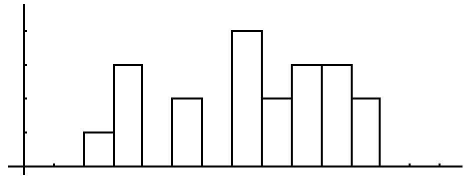
We start our investigations by computing a frequency table by hand. It is quite important to do the first steps by hand. The students learn who to compute frequency distributions and how to interpret the results of the TI-92. Handcalculations at the beginning often help students to obtain a better understanding of the problem type and make the methods for solving the problems more transparent.

Of course it would be preferable, to use two real dices and to throw them for e.g. 20 times. However, for a second attempt we can use the TI-92 for simulating this process. Using the internal function rand(6) we can compute integer random numbers from 1 to 6. The sum of points of two dices can be simulated by rand(6) + rand(6). Sequences of random numbers can be computed by the seq command in the following way: Entering Seq(rand(6)+rand(6),i,1,20) we obtain for example the following sequence of twenty numbers {5,3,5,7,9,10,8,11,8,10,7,7,7,2,3,9,10,3,9,11}. We will use the seq command latter on when simulating 100 or 200 throws of dices. The students are then familiar with this command and can concentrate on plotting histogramms. It is important that the students learn the commands stepwise and that there are not too many new features in one section.

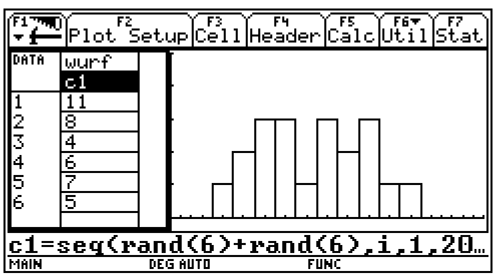
Now the students compute a frequency table for these results

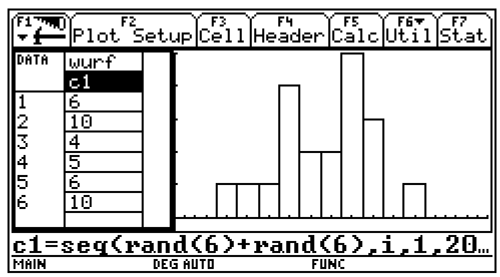
Sum of points	2	3	4	5	6	7	8	9	10	11	12
Frequency	1	3	0	2	0	4	2	3	3	2	0

and also plot the corresponding histogramm. By this example the students learn about the techniques of the problem and how to interpret the results of the TI-92 in the following section.

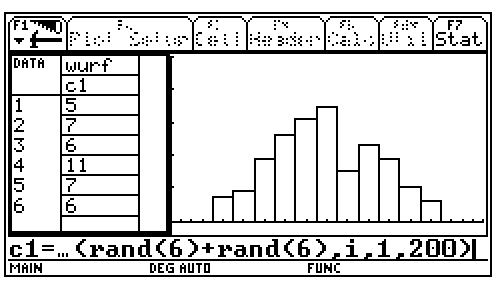


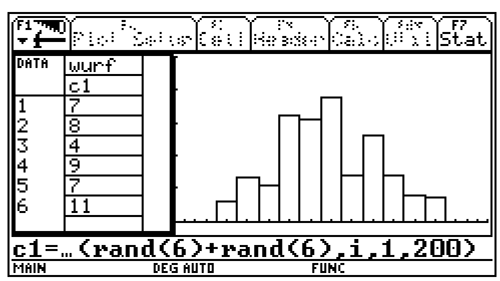
Now we simulate large numbers of throws of two dices using TI-92 DATA/MATRIX-Editor. For this reason we define in a new DATA/MATRIX-window the header of column c1 by c1 = seq(rand(6)+rand(6),i,1,20) producing a sequence of twenty items. For this sequence we now can compute the frequences and plot the histogramm automatically. Using split screens it is easy to change between the DATA/MATRIX window and the GRAPH window.





Repeating this process several times we can observe major differences in the shape of the different frequency distributions. This is due to the fact that we analyze sequences of 20 inputs only. If we compute and analyze sequences of 100 or 200 items the differences of the shapes of the various histogramms become less significant.

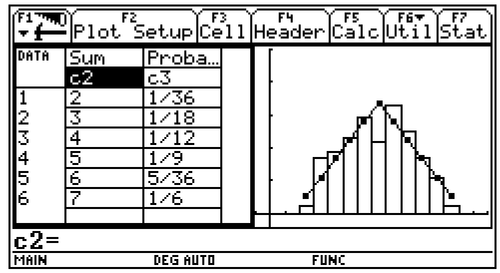




The students have the possibility to compute many different sequences with many items and to compare the shapes of the histogramms.

Now the question arises of how the histogramm will look like, if we throw the dices for "infinite times". This question forces us to consider the probability for the respective results of the experiments. Assuming an equal probability for all combinations of the two dices we obtain the following probability distribution for the sum of the points:

Sum of points	2	3	4	5	6	7	8	9	10	11	12
Probability	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36



For visualizing the probability distribution of this problem we enter the various values (2,..., 12) for the sum of the points of the two dices into column c2 and the respective probabilities of the probability distribution above into column c3. Since the histogramm presents the absolute frequencies we have to multiply the probabilities by the number of items in the sequence (e.g. 200). This can be managed in column c4 by entering c4 = c3*200. Now we can plot the graph of the probability distribution in the same GRAPH window together with the histogramm of the frequency distribution. For a better distinction we use an xy-line, which is not totaly correct, since our problem is a discrete one.

The students learn that the shape of the histogramms for frequency distributions converge to the graphs of probability distributions if we throw the dices for many times. However, it might be interesting that this concept of convergence is different of that one of real sequences.

Binomial distributions

Bernoulli trials

We are investigating experiments repeating n independent trials with exactly two outcomes. The probabilily of each outcome is exactly the same for each trial. The probability that an event will occur on any trial is p. The probality that an event will occur exactly k times on n trials is given by

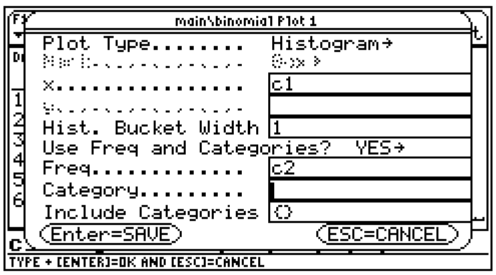
$$P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$$

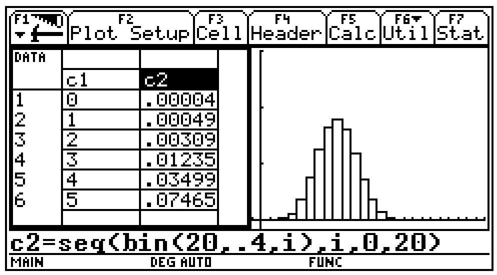
Working on binomial distributions we use the TI-92 as a calculating aid for computing the binomial coefficients $\binom{n}{k}$. Using the internal function nCr(n,k) of the TI-92 we can compute the binomial coefficient. For determining special values of the binomial distribution function $P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}$ the students define a simple function bin(n,p,k) = nCr(n,k)*p^k*(1-p)^(n-k). This function can be used for defining a new function binom(n,p,a,b) = Σ(bin(n,p,k),k,a,b) for computing probabilities like $P(a \le X \le b)$. Solving typical problems within Bernoulli experiments the students can concentrate on modelling the problems and determining important parameters. The computations of the special probabilities like $P(5 \le X \le 12)$ for a binomial distribution with $n = 20$ and $p = 0.4$ are delegated to the TI-92 via a function call binom(20,0.4,5,12). The detious tasks of determining certain values of the binomial density function require most of the time in traditional math courses.

Graphs of binomial distributions

Visualization of graphs is one of the most popular application field of computers within math education. The permanent availability of plots in the graph window of the TI-92 allows students to solve many problems by analyzing graphs. How can we plot histogramms of binomial distributions?

There are two possibilities to produce histogramms for a binomial distribution (e.g. $n = 20$, $p = 0.4$). In the first one we generate two sequences within the DATA/MATRIX-editor. In the first column we compute the various values of the variable X by the sequence c1 = seq(i,i,0,20). In the second column we compute the according probabilities $P(X = k)$ using the user-defined function bin(n,p,k) from above: c2 = seq(bin(20,0.4,i),i,0,20). In the Plot Setup window we define a histogram with x-values in column c1 and frequences in column c2.

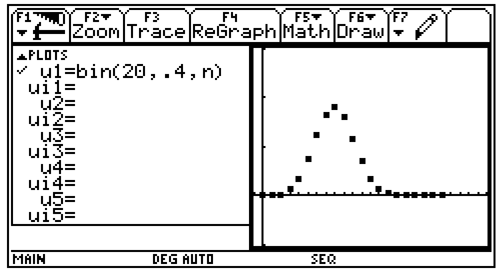




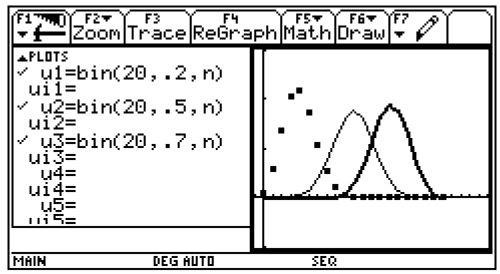
Obtaining nice pictures similar to that ones in math textbooks is one of the advantages of this method. In this representation of binomial distributions the students can illustrate certain probabilities by coloring apropriate areas. This relation between areas and probabilities is also important for normal distributions.

On the other hand it is rather complicated to produce these histograms. Especially when the students should find out the relations between the parameters n,p and the shapes of the graphs of binomial distributions experimentally, an easier method would be preferable.

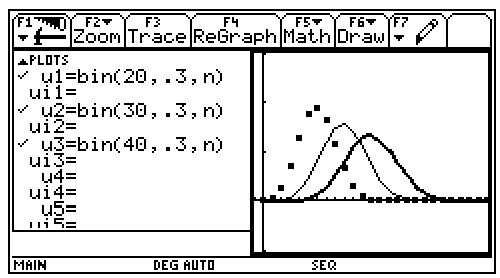
We can also plot graphs illustrating binomial distributions entering bin(20,0.4,n) for sequence u1(n) in the Y= window. For entering and plotting sequences the mode of graphs has to be changed to SEQUENCE by pressing the MODE button. Sequences are plotted using dots or little squares in GRAPH windows.



Now it is quite easy for the students to vary the parameter p (e.g. p = 0.2, p = 0.5, p = 0.7 ...) observing how the shape of the graphs are changing. The "hill" of the distribution moves to the right-hand side if we increase the values for p. For a better distinction of the different graphs it is possible to change the style of plotting (line, dot, square, thick).



For the next investigation we change the values for n (e.g. n = 20, n = 30, n = 40 ...) for a fixed parameter p (e.g. p = 0.3). Now the "hills" become lower but wider for increasing values for n.



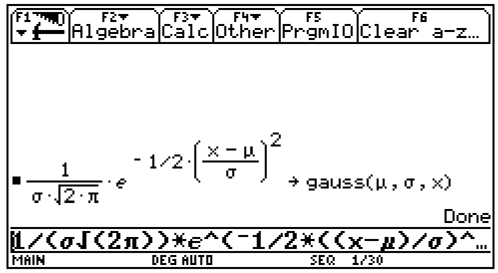
Confidential intervals

It is possible to determine confidential intervals experimentally variing the parameter p for the unknown probability. Thus we can introduce and compute these intervals very elementary. The TI-92 allows to handle realistic values for the ocurring parameters. There are no restrictions to special values of n similar to the use of tables.

Normal distributions

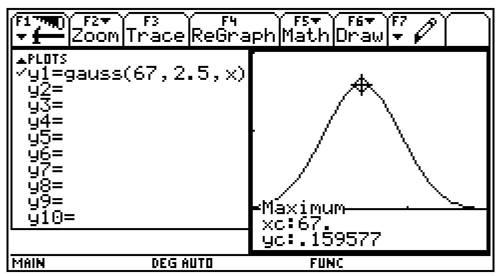
In contrast to the binomial distribution, which is a discrete distribution, the normal distribution is a continous one. Normal distributions with mean μ and standard deviation σ can be described by the following probability density function:

$$\varphi(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$



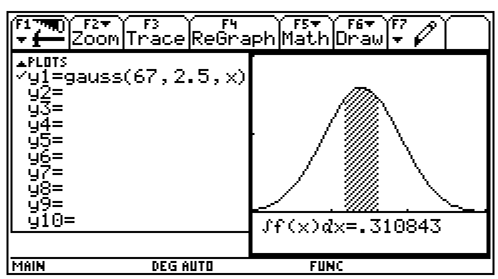
For an easy access to this probability density function we define a function gauss(μ,σ,x).

Many human characteristics such as height and I.Q. are distributed throughout a population according a normal distribution. Galton recorded the heights of 8585 men in Great Britain and found a mean μ = 67 inches with a standard deviation σ = 2.5 inches (see [Kelly 97]).

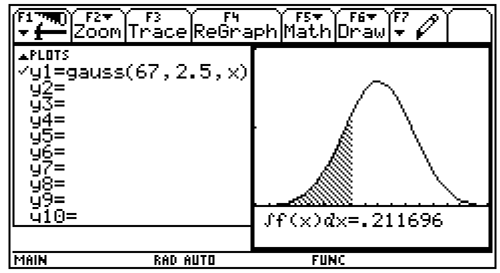


For a first investigation we plot the density function for the normal distribution above. We observe that the maximum height is at $x = 67$, which is the mean value of x.

Plotting density functions for different normal distributions the students can oberserve how the shape of graphs depends on the mean value μ and the standard deviation σ.



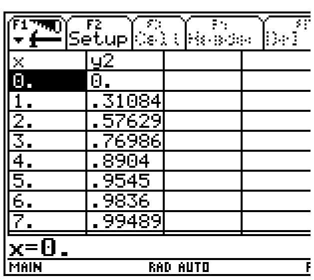
Now we can solve simple problems like „How many percent of men in Great Britain have a heigth between 66 inches and 68 inches“, i.e. to compute the probability of $P(66 \le X \le 68)$, where X denotes the height of the persons. It is possible to solve this problem in the GRAPH window directly. Therefore we choose the integral command in the Math menu F5. The area below the graph is shaded in the respective interval and the value of the integral is indicated at the bottom of the GRAPH window. However we can solve this problem in the HOME window, too. For this purpose we have to enter nInt(gauss(67,2.5,x),x,66,68) for computing the integral numerically.



Typical examples like „How many persons have a height less than 65 inches?“ would require an lower bound of -∞. We cannot choose this value in the GRAPH window. If we select the left border of the window, e.g. 58, we obtain 0.211696, which is slightly less than the actual probability of 0.211855. We can compute this value in the HOME window by evaluating the expression 0.5-nInt(gauss(67,2.5,x),x,65,67) using the fact that the integral running from -∞ to 67 must yield 0.5. However, it is not wise to compute the integral from -∞ to 65 directly. This process is very time consuming and approximation will lead to dubios accuracy.

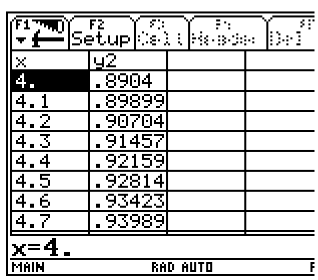
„In which interval symmetrical around 67 are the heights of 90% of men in Great Britain?“ We cannot solve this problem straight forward in the graph window, because we do not know the borders of the integral. There would be the possibility to solve the following equation nSolve(nInt(gauss(67,2.5,x),x,67-c,67+c)=0.9,c) in the HOME window numerically. However this method is very time consuming. Instead of this we compute a table for the different probabilities varying the variable *c*. Therefore we had to enter the function nInt(gauss(67,2.5,k),k,67-x,67+x) to an internal function e.g. y2(x). Since we want to vary the left and right border of the interval we have to change the names of the variables. We rename the integral variable to *k* and the variable describing the interval width to *x*.

Now we can experiment in the table window and find out that the value for *x* is in the interval [4;5] and further in the interval [4.1;4.2] and finally within [4.11;4.12].



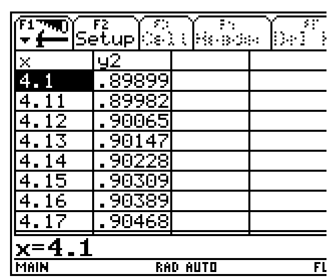
x	y2
0.	0.
1.	.31084
2.	.57629
3.	.76986
4.	.8904
5.	.9545
6.	.9836
7.	.99489

x=0.



x	y2
4.	.8904
4.1	.89899
4.2	.90704
4.3	.91457
4.4	.92159
4.5	.92814
4.6	.93423
4.7	.93989

x=4.



x	y2
4.1	.89899
4.11	.89982
4.12	.90065
4.13	.90147
4.14	.90228
4.15	.90309
4.16	.90389
4.17	.90468

x=4.1

Since the computations are quite time consuming, the students should enter reasonable values in the TableStart field of the Table Setup menu. Therefore, they should know that about 95% of the heights of all men are in an interval of [μ-2σ;μ+2σ], where μ is the mean and σ is the standard deviation of the normal distribution. So the students should expect a result near *x* = 5.

We can solve the problems above by using integrals. We do not need any transformation of the problem to a standardized normal distribution. The transformation of the data and the use of tables in traditional math courses is a technical handicap deflecting from the actual problems. It is very illustrative to graph the density functions of normal distributions and to determine the interesting integrals directly in the graph window.

Summary

Computations within probability distributions are quite time consuming. Especially simulating stochastic experiments is nearly impossible to be done by hand. We can use the TI-92 as a calculating aid computing sequences of random numbers and graphing histogramms.

The TI-92 is also helpful for visualization. We can plot the histogramms of binomial distributions investigating experimentally how the shapes of the graphs depend on the parameters *n* and *p*.

Finally, the aspect of elementarization is quite important. Delegating the enormous calculating amount to the TI-92 the students can try to find confidential intervals by computing tables and searching for suited probabilities *p*. By this method the concept of confidential intervals becomes more transparent than by using expressions for computing the left and the right border of the interval.

For normal distributions especially the possibility of determining probabilities by integrals directly is very illustrative and can be visualized and computed within the GRAPH window. This is a sample for elementarization too, because the concept of integrals is quite familiar to the students especially for determining areas below graphs. Here we use the TI-92 as a calculating aid, for visualization and for elementarization.

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