The TI-92 and Linear Algebra: Less is More

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This workshop was intended to provide participants with experiences parallel to those of our students in five recent presentations of a sophomore-level linear algebra course in which the TI-92 was used as the main "technology component." The intended focal points of the workshop were to be the built-in matrix capabilities and symbolic algebra features of the TI-92, the use of specific programs, scripts and handouts used to extend these built-in facilities, a concurrent discussion on the use of technology and its impact on pedagogy and learning of this subject, and discussion of the comparative merits of the "big" CAS programs versus the calculators used in the workshop. Experiences with scripts which provide a measure of CAI for students, with program sharing, distribution, and demonstration using a TI Graph-Link and LCD panel were to be covered, as well. The thesis that "the TI-92 is enough" for technology support for this course was to be tested and demonstrated.

TI-92 Features for a Linear Algebra Course

Since rref(), the function which delivers the reduced row echelon form of a matrix, and most basic matrix operations come with any TI-92, support for a technology-based linear algebra course is at hand. Many of the advanced functions built into CAS software may be duplicated. Graphic capabilities of the TI-92, while powerful for a calculator, often fall short of software with more sophisticated display technology, but in this subject the disadvantages are not pronounced.

We have taught our sophomore level linear algebra course with a Calculus I prerequisite using the well-known text by Anton[1] and more recently with a text by Bretscher[2]. I first turned to the TI-92 for technology support when teaching a summer session off campus where there was no computer lab at hand. The setting was right for trying this minimal approach to technology in the math classroom. Most students had TI-82's or TI-85's and thus could relate to the syntax they saw on the screen when I made use of the TI-92. I followed the same approach the next time I taught the course, even though I had access to a computer lab. I balanced the alternatives and chose the option which would minimize the time learning technology in favor of getting further down the road on linear algebra content.

The subject of linear algebra by calculator was covered admirably for the HP calculator, the previous most powerful calculator, by Don LaTorre at Clemson. He taught the course in matrix-oriented linear algebra from mid-1989 using the HP-28S and HP48S and provided us with a book[3] which is a useful model to teachers who would be TI-92 users. Other program sources which convinced me to use the TI-92 augmented with useful programs are listed in the references. [4, 5, 6]

Recommendations of the Linear Algebra Curriculum Study Group[7] point toward a course based on matrix computations. Emphasis on coordinated laboratory experiences and the use of small group experiences are other current classroom approaches which seemed and proved to be accessible using the TI-92 as the platform of choice. Following are comments about the topics of the course with some comments on how the TI-92 may enter in. All programs, functions, and scripts alluded to were to be available for hands-on use at the workshop.

Basic TI-92 Operations

Some time must be spent on basic operations such as turning the calculator on and off, on keyboard use, on how results are viewed and saved, on basic syntax of arithmetic, and on some of the basics of algebra. Very important is time spent on the viewing environment – switching from one application or screen view to another – and recovering when something unexpected happens. For students with no prior computer experience or with limited calculator experience (they still exist in abundance), time spent here will pay off in saved time and reduced student frustration later.

Early on I spend a little time on writing, editing, and using a simple program. Also, I show a simple tutorial script with the split screen option. This can be a very powerful teaching-learning aid, and students with limited programming experience are more likely to develop a script to do something useful for themselves than to write a good program for similar purposes.

Built-in Vector and Matrix Operations

First, how does one enter matrices and vectors into a TI-92? One must discuss conventions of rows or columns here and address whether vectors are columns or rows or lists. It's good to explore how the TI-92 compares to other calculators and computer languages, as well. Matrix operations are really easier with the TI-92.

Many primitive functions of linear algebra which are supported in the more expensive computer algebra systems, such as the rank() function, may be missing. That's just fine when one is learning what is involved in the concept of rank. We want students to understand the concept, not just use the numbers spit out by the computer. Eigenvalues and eigenvectors are also not supported on the TI-92. (But they are on the Plus.) Later, it is easy to provide functions or scripts which do most any desired computation. Lots of features are added to the TI-92 Plus, and almost any feature of MATLAB could conceivably be added in the future if enough of us requested it or provided the code on the internet or to TI for distribution.

Row Operations and Linear Systems with the TI-92

The TI-92 has functions to solve a square system of simultaneous linear equations, to augment a matrix, and for the standard row operations. I use these little in my linear algebra course, preferring to concentrate on matrix representation of any linear system in matrix form. We push row reduction with emphasis on manual operations and use tags to indicate what operations were made.

We use a simple, widely available program called PIVOT in many courses. We have it for all TI calculators and for other brands, as well. Use of this program provides good drill on the steps necessary for solution of a system or inversion of a matrix by row operations with the calculations being performed by the calculator. The TI-92 version can be much more sophisticated and may be used to advantage in lecture demonstrations. I use a modification of a program of Ferrard[4], also called pivot(), which is integrated with a program which allows editing and step-wise viewing of results. Later, we use rref() just about exclusively to solve any linear system, not just consistent ones or those with unique solutions, for example, while studying the prominent linear subspaces associated with a linear transformation.

Bases and Linear Independence

It may simply be mentioned here that the basic ideas of linear independence and of basis - recast in terms of systems of equations - are approachable with the primitives provided. Later, after the ideas sink in, we use a program to make checking linear independence a no-brainer.

Matrices and Linear Transformations and Functions of Matrices

Students need help in understanding the geometry or linear transformations. I use a collection of small graphic functions which demonstrate rigid motions. Affine transformations and fractals are a fascinating side trip. A program which demonstrates the idea of an eigenvector also helps here, helping to firm up the idea of a matrix as a transformation. The TI-92 will support computation of simple functions of matrices, so verification of the Cayley-Hamilton Theorem is easy and may be addressed as a discovery exercise.

Orthogonalization, Matrix Factorizations and Canonical Representations

None of these are explicitly supported on the TI-92, but their computation and demonstration is easy. LU, Cholesky, and QR factorization are readily done. I have used scripts, homework projects, and short programs, to cover these topics and the Gram-Schmidt process. Manual verifications of orthogonality and use of simple functions to do vector projections are useful here.

Eigenvalues, Eigenvectors, and Diagonalization

With a technology approach, I am sure that almost all students understand the diagonalization of a matrix and the "Spectral Theorem" for symmetric matrices. That is to say, they know that it works, can cite the theorem, and can do the appropriate computations to verify their hunches or assertions. This is more success than I had before, probably including when I first studied the topic myself. It is easy to think that this is because students are able to see concrete results arise as a result of their own inquiries and verifications which were unfettered by computational drudgery. The next teacher who wants to do the Spectral Theorem or Diagonalization to them, maybe even in a differential equations class, will have been well-served. I'm not very worried that my students, most of whom are now computer science and engineering aspirants, haven't mastered a formal proof which I first studied in graduate school.

Vector Spaces and the Fundamental Theorem of Linear Algebra

It is at this topic that I am most pleased with advantages gained from technology in my linear algebra courses. It seems to me that the concepts involved in kernel or null space, transpose, image, row and column spaces, and "perp" of a vector space are great first steps to the type of abstract math which emphsizes structure over individual instance. The fact that these all relate to solutions of linear systems, using ideas which students have seen repeatedly in earlier courses back to their first exposure to algebra, and ideas which have such tight relationships as expressed in Strang's formulation, the Fundamental Theorem of Linear Algebra[8], is icing on the first course cake. This may be exploited by requiring students to compute various components such as dimensions or bases of the four subspaces involved and to represent their relationships diagrammatically. Several such problems will make some of the Fundamental results obvious or at least very plausible. Again, a script or even a program may automate the process. Like many others, I have planned to get to Singular Value Decomposition and haven't made it yet, but the TI-92 will be there to help when I do get there.

Numerical Analysis and Accuracy Considerations

I've done very little in my courses in this area, chiefly due to time limitations. However, it is clear that a device with twelve digit computations can be valuable here. I usually give a few demos of numerical techniques. A warning: some examples I have tried and which were done on a computer with only 6D accuracy weren't badly behaved enough to misbehave on the TI-92.

Applications

We're at the end and, as too often happens, give short shrift to applications. Curve fitting and least squares problems are quite accessible with the TI-92 and relate to some of the theoretical topics mentioned above. Other applications are in the Notes cited[6], or the

ATLAST[9] book, or roll your own using any number of other readily available texts or articles. The TI-92 may not be appropriate for large, sparse matrices.

And More....?

What have we left out, besides all the details? Whatever favorite or essential topic for your first linear algebra course didn't make this account probably also may be computed or demonstrated on a TI-92, and now on a TI-89. Prove me wrong! These little machines aren't perfect, but they do show that sometimes, Less is More.

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[1] Anton, H. , Elementary Linear Algebra, Seventh Edition, , John Wiley & Sons, New York, (1994)

[2] Bretscher, O., Linear Algebra with Applications, Prentice-Hall, Upper Saddle River, N.J. (1997

[3] La Torre, D.R., Calculator Enhancement for Linear Algebra: A Manual of Applications Using the HP-48S and HP-28S, Saunders College Publishing, Fort Worth, (1992)

[4] Ferrari, Jean-Michel, TI-92: les programmes!, Dunod/Texas Instruments France, (1996) This book contains a very comprehensive custom menu of programs which is very useful in any linear algebra course. A translation is forthcoming. Contact gdorner@imaxx.net.

[5] See the programs which may be downloaded from www.derive.com. A review is at www.imaxx.net/~gdorner/ti92_paper. (Many of the functions there are incorporated in the TI-92 Plus, but the materials are interesting and of use for all serious users of the TI-92.)

[6] Leinbach, C., Editor, et al, Notes for Summer 1996 Calculus Short Course, CAS-CALC, Enhancing the Teaching and Learning of Calculus With Computer Algebra System, Technology Short Course Program, The Ohio State University (1996)

[7] Carlson, C., Johnson, C.R., Lay, D.C., Porter, A.D., The Linear Algebra Curriculum Study Group recommendations for a first course in linear algebra, College Mathematics Journal 24 (1993), 41-46. This also is reprinted in the Resources book listed in the following reference.

[8] Strang, G., The Fundamental Theorem of Linear Algebra, pp. 213-221, reprinted in: Carlson, C., Johnson, C.R., Lay, D.C., Porter, A.D., Watkins, A.E., Watkins, W., Editors, Resources for Teaching Linear Algebra, MAA Notes, vol. 42, The Mathematical Association of America, (1997)

[9] Leon, S., Herman, E., Faulkenberry, R., ATLAST, Computer Exercises for Linear Algebra, Prentice Hall., Upper Saddle River, N.J. (1996