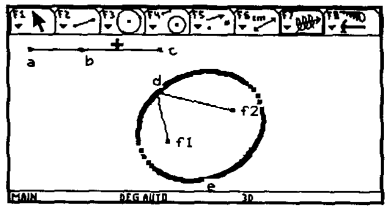
Contents

A GEOMETRIC CONSTRUCTION OF AN ELLIPSE USING A TI-92

Stuart Moskowitz
Humboldt State University
sml4@axe.humboldt.edu



The following exploration builds an ellipse based on the geometric definition:
An ellipse is the set of all points whose distance from two given points remains constant.

Detailed keystroke instructions are given for each step in italics. If you know how to use the construction tools, you can skip the italicized text. If you do not know how to use them, ***read each instruction completely before attempting the construction.***

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1. Open a new geometry session on your TI-92. *Press* [APPS] [8] [3] *to open a new session. Press* [∇] *to move the cursor to the Variable line. Type a new name, then press* [ENTER] *twice.*

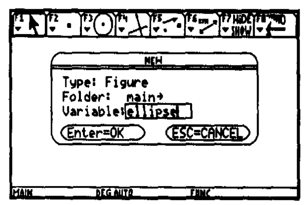


FIGURE 1

2. Construct two points; these will be the foci of the ellipse. *Activate the Point construction tool by pressing* [F2]*, then select* **1:Point** *by pressing* [1]*. Move the cursor (it will look like a pencil) with the big arrow key at the top right of the keyboard. pressing* [ENTER] *will leave a point at the cursor location. Label the points immediately after constructing each one by simply typing* **F1** *and* **F2**. *If it is not labeled immediately after it is constructed, it can be labeled later by activating the* **F7:Label** *tool.*

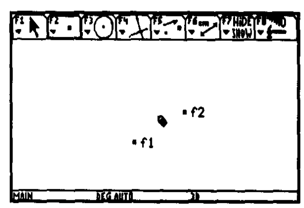


FIGURE 2

3. Construct segment **ac** near the top of the screen. *Select the segment tool by pressing* [F2]*, then select* **5:Segment** *by pressing* [5]*. Move the cursor near a corner of the screen; begin the segment by pressing* [ENTER]*. Immediately type* [A] *to label this as point* **a**. *Move the cursor; point* **a** *remains fixed, while the other endpoint will follow the cursor. Press* [ENTER] *to fix point* **c** *and complete the segment. Immediately press* [C] *to label this endpoint.*

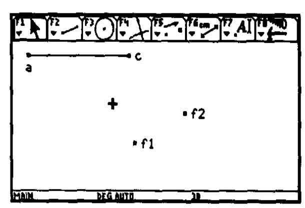


FIGURE 3

If labels obscure other objects, they can be moved with the "grab" key (the "hand" key). With the **F1:Pointer** tool active, move the cursor until the screen prompt says **THIS LABEL** then press and hold the "grab" key while pressing the arrow keys.

4. Construct point **b** on segment **ac**. *Select the* **Point on Object** *tool from the* **F2** *menu. Move the cursor near segment* **ac**. *When the screen prompt says* **ON THIS SEGMENT**, *press* $\boxed{\text{ENTER}}$. Immediately press $\boxed{\text{B}}$ to label the point.

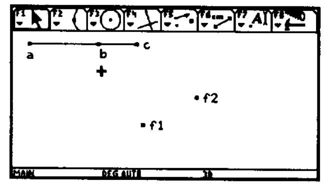


FIGURE 4

NOTE: Regardless of **b**'s position on segment **ac**, the length of **ac** is constant, and:

$$m(\mathbf{ab}) + m(\mathbf{bc}) = m(\mathbf{ac})$$

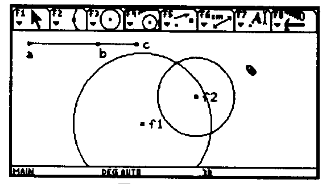


FIGURE 5

Drag point **b** along segment **ac**. The circles change size, but their radii stay the same length as **ab** and **bc**.

5. Construct circles with centers at **fl** and **f2** so that circle **fl** has radius **ab** and circle **f2** has radius **bc**. *Activate the* **F4: Compass** *tool*. (*The Compass tool constructs a circle at a specified center, using a given radius.*) *Specify radius* **ab** *by pointing at* **a** *pressing* $\boxed{\text{ENTER}}$, *then pointing at* **b** *and pressing* $\boxed{\text{ENTER}}$. *Be sure prompt says* **THIS POINT** *each time*. *Then specify the center by pointing cursor at* **f1**. & *pressing* $\boxed{\text{ENTER}}$. *Repeat to construct circle* **f2**.

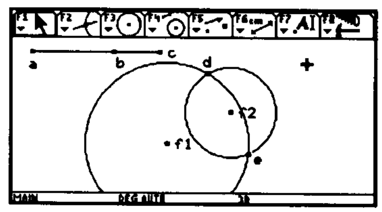


FIGURE 6

6. Construct points **d** and **e** at the intersections of the circles. *Activate the* **F2:Intersection Point** *tool*. *Move the cursor until the prompt says* **POINT AT THIS INTERSECTION**, *then press* $\boxed{\text{ENTER}}$. Type the labels immediately after each point is constructed.

Since the distance from **d** to **fl** and from **d** to **f2** equals the lengths of **ab** and **bc** respectively, the sum of which is constant, then, as **b** is dragged along **ac**, the path of **d** will move along an ellipse with foci at **fl** and **f2**. The same is true for **e**.

7. Draw the ellipse by tracing the paths of **d** and **e**. *Activate the* **F7:TraceOn/Off** *tool, then select points* **d** and **e**. *When selected, the points will be flashing* . Trace the ellipse by either grabbing and dragging point **b** along **ac** or by animating point **b**. *The animation tool is found in the* **F 7** *menu*. *After activating* **F7:Animation**, *grab* (*with the hand key*) *point* **b**, *then move the cursor in the opposite direction of the path* **b** *will follow* (*see figure* 7). *Let go of both keys simultaneously and watch as the TI-92 traces the path of the ellipse*

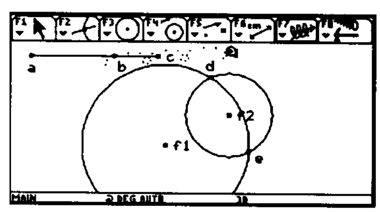


FIGURE 7

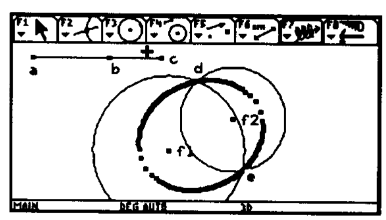


FIGURE 8

The first figure on page one of this manuscript has the segments from each focus to point d constructed (with the F2:Segment tool) and the circles hidden (with the F7:Hide/Show tool). Regardless of the position of d,

$$m(\mathbf{f_1\, d}) + m(\mathbf{f_2\, d}) = m(\mathbf{ab}) + m(\mathbf{bc}) = m(\mathbf{ab})$$

EXTENSIONS

1. Find an analytic definition for this ellipse.

2. Transfer points **a**, **b**, and **c**, onto a longer segment **pq** (*use the* **F4: Redefine Point** *tool*). Increase the distance between **fl** and **f2** Then watch what happens as point **b** is dragged or animated on segment **pq**!

CONNECTING THE GEOMETRIC AND ANALYTIC DEFINITIONS OF A PARABOLA USING THE TI-92

Stuart Moskowitz

The new TI-92 is the first piece of technology that combines into one unit interactive geometry software with symbolic algebra and statistics capabilities. In the following investigation, a parabola is constructed based on its geometric definition. Then, after collecting coordinates from the parabola and transferring them into the data editor, an algebraic equation can be calculated using the quadratic regression function.

> The geometric definition: *A parabola is the set of all points in a plane equidistant from a given fixed point (focus) and a given fixed line (directrix).*
>
> The algebraic (analytic) equation, with focus at (O, p), and directrix at $y = -p$:
>
> $$x^2 = 4py \qquad \text{or} \qquad y = \frac{.25x^2}{p}$$

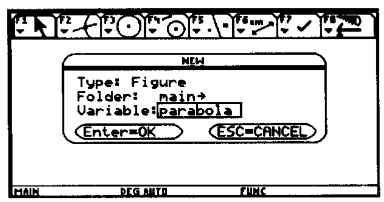


figure 1

Open a new geometry window on the TI-92. A variable name must be given prior to opening the new window. (figure 1)

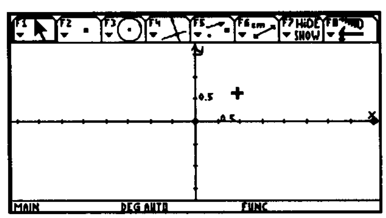


figure 2

Display rectangular axes. This is accomplished by going to the **Geometry Format** menu, the eighth choice under the $\boxed{\text{F8}}$ File toolbar menu, or by pressing ♦**F**. (figure 2)

For this construction, choose p = 1. Therefore, we need to construct the focus at (0, 1) and the directrix as the line y = −1.

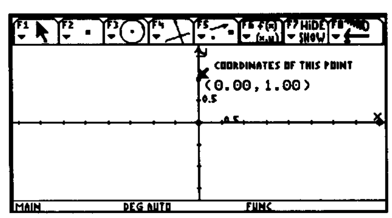


figure 3

Construct a point on the y-axis at (0, 1) by selecting **3:Point on Object** from the $\boxed{\text{F2}}$ **Points and Lines** toolbar menu. Display the coordinates by selecting the **4:Label** option from the $\boxed{\text{F7}}$ **Display** menu. The point can be dragged to (0, 1) by pressing and holding the **Drag** key (the little hand at thetop left of the calculator). This will be the focus. (figure 3)

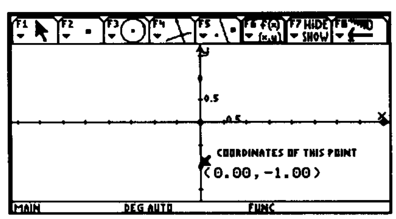


figure 4

Construct a second point at (0, −1). Do this by reflecting the focus across the x-axis with the **4:Reflection** option found within the $\boxed{\text{F5}}$ **Transformations** menu. (figure 4)

Use the **l:Perpendicular Line** option in the F4 Constructions menu to draw a line perpendicular to the *y*-axis and passing through the point at (0, –1). This line, with the equation $y = -1$, will be the directrix. (figure 5)

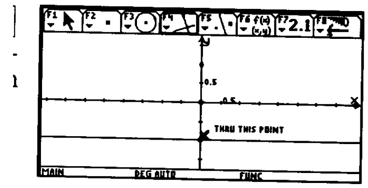


figure 5

With the **2:Point on Object** tool in the F2 **Points and Lines** menu, construct a point anywhere on the *y*-axis. Draw a line through this point perpendicular to the *y*-axis. In figure 6, at right, this new point (**b**), the point (**a**) at (0, –1), the directrix, and the focus are labeled on the TI-92 using the **4:Label** tool from the F7 **Display** menu

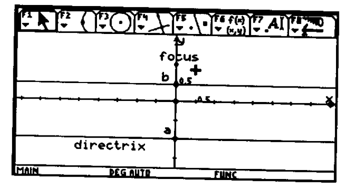


figure 6

Construct a circle with its center at the focus and its radius equal to the length of segment **ab**. Do this by first selecting the **8:Compass** option from within the F4 **Constructions** menu. Define the radius by selecting points **a** and **b**. Then select the focus to be the center of the circle. (figure 7)

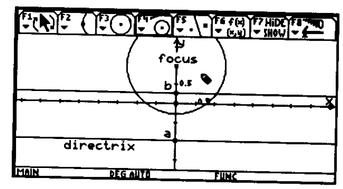


figure 7

Define a point at each intersection of the circle and the line through point **b**. Do this with the **3:Intersection point** tool in the F2 **Points and Lines** menu. (figure 8)

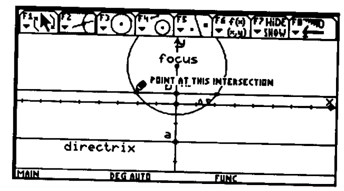


figure 8

Note: If you now drag point **b** along the *y*-axis (using the grab key), the circle will grow or shrink depending on the length of segment **ab**.

Lines **a** and **b** are parallel; the distance between them is always the length of segment **ab**. The radius of the circle is also defined as the length of segment **ab**. Therefore the points of intersection between the circle and line **b** are equidistant from the focus and the directrix.

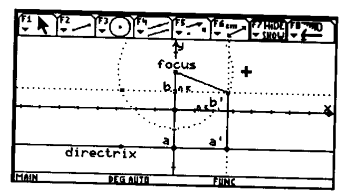
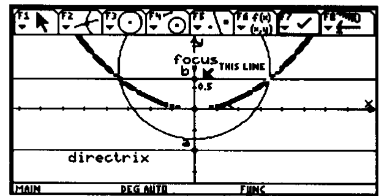


figure 9

To help visualize this concept, two segments have been constructed in figure 9 at right. Since the first segment is the radius of the circle, and the second segment is perpendicular to the directrix between lines **a** and **b**, and their endpoints meet at one of the points of intersection between the circle and line **b**, then the lengths of the segments are equal. (The circles are partially hidden in figure 9 by using the **l:Hide/Show** tool from the F7 **Display** menu.)

Choose the **2:TraceOn/Off** option from the F7 **Display** menu. Then select both of the points of intersection between line **b** and the circle. They should be flashing on the display screen.
Drag point **b** along the *y*-axis. The path of the two points of intersection will be traced, as seen in figure 10. The path is a parabola.



THE PARABOLA!
A GEOMETRIC CONSTRUCTION
figure 10

Unique to the TI-92 is the ability to combine the interactive geometry software with its data collection and statistics capabilities. This makes it possible to determine algebraic equations for many curves defined geometrically.

Figure 11 shows the screen of the TI-92 split into a left and right half. On the left is the geometry screen with the circles hidden, and the segments shown (as described in figure 9). On the right is the data editor. The screen is split by selecting the **B:DataView** option from the F8 File menu.

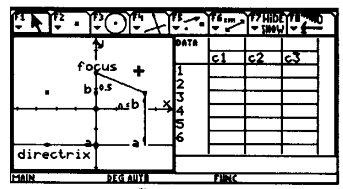


figure 11

Display the coordinates of one of the points of intersection of the circle and line **b**. This is done with the **5:Equation & Coordinates** option in the F6 **Measurement** menu. In figure 12 the selected point is at (1.24, 0.41).

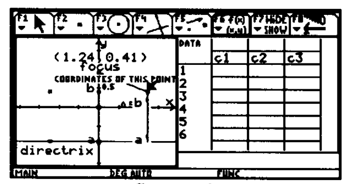


figure 12

We need to transfer these coordinates from the geometry screen to the data editor. Define the entries to be transferred using the **2:Define Entry** option of the **7:Collect Data** tool from the F6 Measurement menu. The first number defined is stored to list cl. The second number defined is stored to list c2. After defining each entry, press ♦**D** on the keyboard of the TI-92. The coordinates are transferred into the data editor (figure 13)

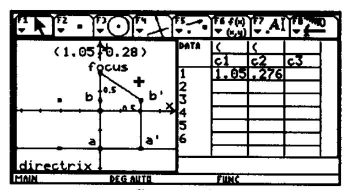


figure 13

As was done in figure 10, draw the parabola by dragging point **b** with the grab key. But this time stop periodically and press ♦**D** to collect more data. In figure 14, data was collected five times, and five sets of coordinates have been transferred to lists cl and
c2.

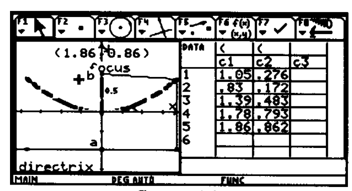


figure 14

With the Data Editor selected (rather than the Geometry screen), define a scatter plot by selecting F2 **Plot Setup**, then selecting Fl **Define**. Figure 15 shows the window that appears after the F1 **Define** option is selected. Plot Type, Mark, and lists for *x*, and *y*, have been defined.

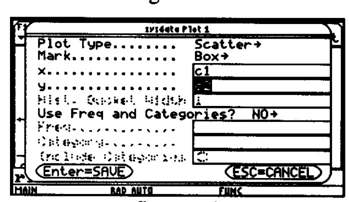


figure 15

Figure 16 shows what has been defined for each of nine possible statistical data plots. At this time only plot 1 is defined.

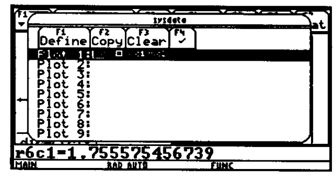


figure 16

Pressing ♦**R [GRAPH]** will draw the scatter plot. Since the TI-92 is still in split screen mode, the scatter plot appears in the right half of the screen. (figure 17)

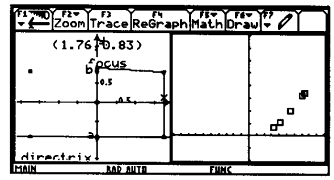


figure 17

Figure 18 displays the Calculate window, accessed by choosing $\boxed{\text{F5}}$ **Calc** from the Data Editor window. The type chosen for this set of data is Quadratic Regression (since we are seeking the equation for a parabola). Note the line reading "Store RegEQ to
yl(x)" When the regression analysis is complete, the equation
will automatically be transferred to yl(x).

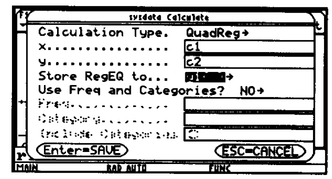


figure 18

The results of the regression analysis are displayed in figure 19. The values for b and c are close enough to zero, that we can substitute zero without any significant loss of accuracy. Therefore the equation generated is:

$$y = .25x^2 + 0x + 0$$

or

$$y = .25x^2 \ .$$

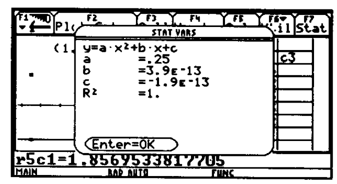


figure 19

Since we chose p = 1 for the example, this equation matches the equation given at the beginning of the demonstration.

Figure 20 shows, in the right half of the screen, the graph of $y = .25x^2$, and on the left, the parabola as traced in the geometry window. They do not appear to be the same parabola, due to the different viewing windows for each half. But tracing on the graph on the right, will reveal that all data points will match those on the left.

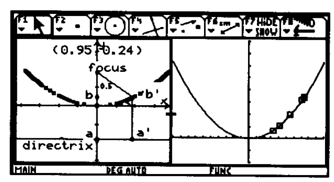


figure 20

The twenty figures in this demonstration provide a summary of the steps required to construct and analyze the parabola on the TI-92. They cannot replace actually constructing the parabola firsthand. These screen captures only hint at the power of watching the effects of dragging and tracing points in the geometry window. You must do the activity yourself to fully see and benefit from the capabilities of this new technology.