Proceedings of the Third International DERIVE/TI-92 Conference

The Austrian TI-92 Project: A Preliminary Report

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Abstract

The project „New Technologies in Mathematics Education“ was started in Austrian high schools in 1997. It is a follow up of the „Austrian DERIVE Project (Symbolic Computation Systems in the Classroom)“ and it focuses on possible ways to use the TI-92 in mathematics education and its effects on classroom activities, learning processes, conceptual understanding, teacher in-service-training and the curriculum. The investigation involves 87 classes with more than 2000 students from different year groups, ranging in age from 13 to 17 in 56 schools spread across Austria. The teachers and the students are volunteers, hoping to develop new modern methods of teaching and learning mathematics.

In accordance with certain guidelines established by a central planning group year group coordinators had to moderate the writing of concepts to organize special sequences of lessons, so called observation windows.

The focus of the evaluation is mainly based on:

- didactical methods, mathematical contents and changes in curricula
- social structures, students activities and teacher-student interactivities
- parameters for successful use of a computer algebra calculator
- technology and exams
- the new situation of the teacher
- quality of hardware, software and manuals with regard to pedagogical requirements

As the analysis of the data has only just begun, it would not be appropriate to write in detail about the results at the moment. During the coming months further investigations will be carried out, in order especially to explore the interdependencies of didactical methods and the calculator on the one hand and student’s understanding and problem solving on the other.

Teacher training has been intensified with the support of the T^3 organization to give every Austrian mathematics teacher the opportunity to come to know the TI-92 and its possible uses in teaching mathematics.

1. Organization

1.1 ACDCA

ACDCA (Austrian Center for the Didactics of Computer Algebra) is an organization of educators and educational scientists founded in 1992 and headed by Dr. Helmut Heugl, school inspector for the region of Lower Austria and lecturer in the didactics of mathematics at the Technical University of Vienna. Other co-founders are Klaus Aspetzberger, Karl Fuchs, Walter Klinger, Bernhard Kutzler, Josef Lechner and Robert Nocker.

The aim is to focus and force activities and experiences in the use of CAS (mainly DERIVE) at Austrian high schools and to run field studies to develop didactical guidelines and tools. Parallel ACDCA activities were set up to enforce teacher training and supply materials to motivate more teachers to use technology in math education.

In 1993 the Austrian Ministry of Education and Culture entrusted ACDCA with support from the Center of School Development at Graz, Styria (Dr. Günther Grogger), to organize and execute the project "Symbolic Computation Systems in the Classroom". This project involved about 700 students and 28 teachers in 17 schools from 3 federal states of Austria and became known as the "Austrian DERIVE Project".

Methods and results have been internationally presented and published. Most complete references are:

- The International DERIVE Journal, 1996, Vol. 3, No. 1 (dealing exclusively with "The Austrian Research Project")
- Heugl, Klinger, Lechner, 1996, Mathematikunterricht mit Computeralgebra-Systemen, Addison-Wesley (in German only)

1.1 TI-92 Project

The project is run by ACDCA by order of the Austrian Federal Ministry of Education. It was initiated and is headed by Dr. Helmut Heugl and supported by the Austrian Center for School Development in Graz/Styria, Dr. Manfred Kronfellner (Technical University Vienna) and Klaus Aspetsberger (T^3 Austria).The TI-92 project involves more than eighty teachers and two thousand students from age 13 to 17 at 57 schools spread over all federal states of Austria. About 50% of the students are girls.
The teachers and the students are volunteers, hoping to develop new modern methods of teaching and learning mathematics. Teachers, students and parents took the decision to

participate jointly and were offered some financial support, to reduce the expense of the hardware, in the form of 400 free TI-92s from a central pool sponsored by the Austrian Federal Ministry of Education and TI-Austria. The teachers include some who already have a lot of experience in using Derive in the classroom and others without any previous practice of teaching with technological support.

The common goals, contents and methods were developed together with nearly all the teachers involved in the project during a meeting in summer 1997. There is one category for each year group, i.e. for ages 13, 15, 16 and 17, each being headed by a year group coordinator.

In accordance with certain guidelines established by a central planning group they had to moderate the writing of concepts to organize special sequences of lessons, so called observation windows. Such sequences consist of two to ten lessons chosen from particular topics e.g. independent student work to discover trigonometric functions (sin/cos), modeling of growth processes, binomial distribution, introduction of the differential quotient, solving systems of linear equations, term manipulations etc.

The focus of the evaluation is mainly based on:

- didactical methods, mathematical contents and changes in curricula
- social structures, students activities and teacher-student interactivities
- parameters for successful use of a computer algebra calculator
- technology and exams
- the new situation of the teacher
- quality of hardware, software and manuals with regard to pedagogical requirements

Teacher training has been intensified with the support of the T^3 organisation to give every Austrian mathematics teacher the opportunity to come to know the TI-92 and possible uses in teaching mathematics.

2. Observation Windows (OW)

In accordance with certain guidelines established by a central planning group each group coordinator had to moderate the writing of concepts to organize special sequences of lessons, so called observation windows. Such sequences consist of some lessons to test certain methods of TI-92 usage.

They are embedded in frame topics, which are school math topics e.g. ratios, term manipulations, solving systems of linear equations, trigonometric functions, modelling of growth processes, binomial distribution, introduction of the differential quotient etc. Normally they include obligatory tests for all classes before and after every observation window phase.
Observation windows have previously been used in the Austrian Derive Project [Fuchs,1996, in: The Austrian DERIVE Project = DERIVE Journal, Vol.3, No.1, p.39-56]. The idea is to allow each individual teacher plan most of the years´ lessons

independently within a commonly developed time structure and to test special teaching/learning units within those observation windows.

It is typical for our observation windows to include mathematical contents and/or methods not common in traditional mathematics education, as they depend on the availability of suitable technology.

The following observation windows have been carried out (responsible coordinators in brackets):

2.1 Age 13 (Walter Klinger)

OW1: Direct and inverse proportionality (ratio)

Investigation of students behaviour when selecting models to describe relationships (direct, inverse or proof that none of them exists)

Hypothesis: The TI-92 helps the student to develop translation knowledge by using different representations (term, graph, table) simultaneously. This knowledge enables the students to use a broad variety of models to characterize given situations.

The following two goals were most important in the frame topic:
- Extending knowledge about properties of direct and inverse proportionality
- Development of strategies to prove or disprove the existence of such proportionality

These have been the central goals of the OW itself:
- Students should know and be able to use the connexion of term, graph and table.
- Students should be able to analyze given data to find existing properties
- Students should experience the variety of representations and solutions and be able to formulate adequate models.
- Students should be able to make decisions and analyze the chosen models.

OW2: Formula and term manipulations

Investigation on students' work with formula (deriving, testing and practicing) and term structures (expand, factor, substitute,...), e.g. binomial formula and areas of objects consisting of squares and rectangles.

Hypothesis: The TI-92 helps the student to explore term structures and develop manipulation skills and test strategies to prove validity and equivalences.

The following goals were most important:
- Student should be able to derive formula and describe them verbally.

- Students should recognize the structure of terms and be able to manipulate them on the basis of the recognized structure, this includes substitutions.
- Student should be able to recognize and prove equivalence of terms.
- Students should be able to test (validate) formula by using several different strategies.

The main method is independent students' work with working sheets. Most frequently used TI-92 commands are factor, expand, comDenom and the with-operator |.
To prove equivalence about ten different strategies have been developed by the students.

2.2 Age 15 (Otto Wurnig)

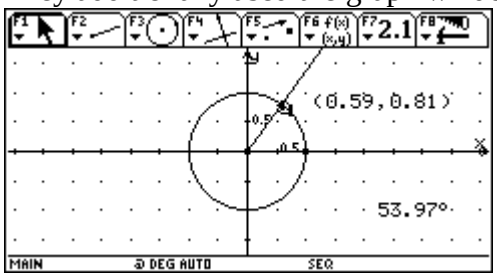
OW1: Pairs of simultaneous linear equations

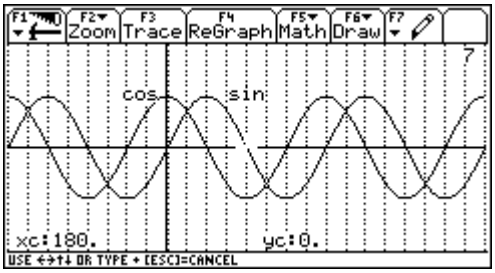
The students had to find and use different methods of solving simultaneous linear equations.

2.3 Age 16 (Robert Nocker)

OW1: Introduction of trigonometric functions sine and cosine

The basic idea was, that by moving a point on a unit circle in the Cabri window the students can independently develop the graph and a table of sine and cosine functions. They additionally used the graph window to explore their fundamental attributes.





OW2: Recursive (discrete) modelling of growth processes

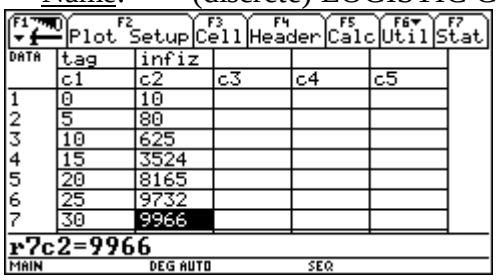
The main idea was to force independent students work by using the sequence mode of the TI-92 to develop and experiment with different discrete models of growth processes.
First the students had to work with linear and exponential functions. As those models are not completely new at age 16 (linear functions and interest account have been introduced at earlier stages, series have not been dealed jet), the teacher had to show them the way to use sequence mode of the TI-92, to hand out working sheets and to collect the results. Most students were able to independently find the characteristics of the models and from thence the following recursive equations:

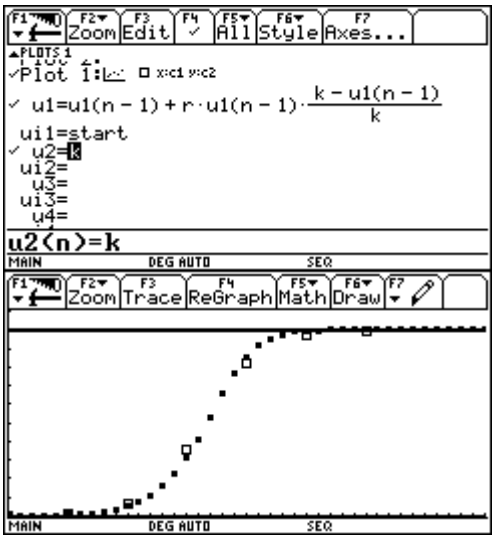
Equation:	u(n) = u(n-1) + k	k ... amount of growth
Name:	LINEAR GROWTH	
Equation:	u(n) = u(n-1) + r*u(n-1)	r ... rate of growth
Name:	EXPONENTIAL GROWTH	

The example shown down on Data/Matrix Editor screen deals with the spread of an infection on a limited population. At the begin the process can be described by an exponential function, but later on it becomes harder for the virus to find not jet infected persons and so the speed of diffusion slows with decreasing number of potential victims.

To describe this process we need a term of adjustment, reducing the rate of growth when approaching to the limiting value. In a discussion slightly directed by the teacher students developed the term (K-u(n-1))/K without using any further mathematical tools.

Equation: u(n) = u(n-1) + r* u(n-1)((K-u(n-1))/K)
r ... rate of growth K ... limiting value, capacity, saturation-point
Name: (discrete) LOGISTIC GROWTH





Now the student could experiment with different parameters (esp. r) to find a good fitting model.
We did not deal further on with the problem of "best fit", but the motivation to work on this point later on is high among the students.

The continuos model: $f(x) = \frac{A}{1+Be^{-kx}}$ can be found and investigated later on, when calculus tools are available (in Austria at age 17 and 18), esp. differential equation mode when TI-92 Plus will be available. We did not want to use those tools as completely black boxes but preferred to answer certain questions by working with tables, e.g. point of inflection by searching the maximum of a table produced by the differences of two consecutive values.

2.4 age 17 (Alfred Eisler)

OW1: Introduction of the derivative

Hypothesis: CAS supports the development of basic conceptions (derivative as local rate of change, derivative as slope of the tangent to the graph of a function) by offering multiple representations (numerical, symbolic, graphical).

First average and instantaneous velocity at free fall is investigated by a naive approach using the difference quotient (average rate of change). It had to be worked on the following questions:

What happens with smaller time intervals?
What happens, if the two time points meet?
What happens then with the average velocity?

Students experience that calculation then fails. Then they had to try different approaches, approach with graph and approach with table. The results are inconsistent.

To solve the problem simplify was used as „dirty“ trick (TI-92 does it regardless to the domain!) and then worked with the limes to get the differential quotient (local rate of change). In a last step the concept was formulated and used for general functions

OW2: Binomial distribution

Hypotheses:
- CAS allows to calculate with bigger collectives
- the graphical features lead to deeper understanding by having a better representation of the stochastic distribution, so that concepts such as expected value, variance, standard deviation can be acquired by calculating many examples and having a basis for a more detailed understanding.

contents:
- Bernoulli experiment
- concept of binomial distribution
- expected value of binomial distribution
- variance and standard deviation

From the starting example of throwing dice the definitions of Bernoulli experiment and binomial distribution ware developed and then the properties have been investigated by working on several examples of binomial distributed processes. Finally the concepts have been theoretically founded and in some cases enlarged (e.g. expected value: estimate -> „normal“(common) mean -> weighted mean). Bnpsum-histograms (Aspetzberger, 1996, The Derive International Journal, p. 65) have been used.

3. First Results

The following points are results of working groups of project teachers. Their task was to give structured feedback and rating on their experiences they had so far.

3.1 Technical

- very few technical problems, some problems with downloaded software (games).
- sometimes teachers are very stressed because of handling problems (esp. in big classes).
- the manual is not adequate for use in classroom situations, students in general did not experience it to be very helpful.
- the overhead panel is an important accessory.
- an immediate print feature is missed.
- the display is sometimes to small (terms cannot be seen completely, split screen often not efficient).
- we would like to have a way to connect a TI-92 with the graph link cable to Derive running on a PC.

3.2 Organizational

- students as tutors - a normal situation
- continuos use of the TI-92 is necessary
- acquisition of handling and mathematical knowledge should not occur simultaneously
- lack of adequate teaching and learning materials

3.3 Pedagogical

- the TI-92 is very helpful for students to check results (e.g. by different representations)
- a lot more exercises and more applications can be done in the same amount of time.
- more real-life examples can be done, sometimes resulting in a more demanding situation for teachers and students
- the linear input mode and the multiple representation features are helpful to develop structural knowledge
- the graph window is essential to discover characteristics of functions
- increased focus on explanation and interpretation in exam situations, marks didn't change in general
- students should still be able to do standard tasks by hand
- students should always give reasons for their decisions (e.g. chosen model, method of solving an (system of) equation)
- variables are used more often
- the lessons become more students and action orientated (teacher as organizer and trouble shooter)

From Window Shuttle Principle to Multiple Representation Principle)

DERIVE offered the feature to have several windows on the screen simultaneously, used by a method we called the window shuttle principle, that means to have different prototypes of the same object in different windows visible at the same time.

The TI-92 offers in many cases a lot of different representations for an object or a process. For an example we can use different representations to investigate certain properties of growth processes as mentioned above. We can model such a process by using function or sequence or parametric mode each time focusing on slight different properties of the process. The TI-92 Plus will additionally offer a differential equation mode. We can visualize the growth process using tables, graphs, web plots, soon (Plus) slope fields, phase diagrams and so on.

As the smaller screen of TI-92 - compared to Derive on a PC - the original window shuttle principle is very limited. Therefore it has changed to a Multiple Representation Principle, no more focusing on several different windows visible simultaneously, but on using different representations for different reasons.

This principle is supported by the simple ways to change representations on the TI-92 by pressing one or at most a few keys, thereby the producing of some very important representations as tables or some special function plots (e.g. web plot) needed some programming knowledge using Derive.

The Multiple Representation Principle can be used for several goals:

- to focus on certain properties by using adequate representations
- to investigate properties of by offering additional representations to enlarge the knowledge about step by step
- the change of the representations can often lead to a deeper insight in the differences and the associations between certain concepts/processes/models, that seem to be equivalent or without any connections when using just one representation
- to offer certain types of learners their suiting representation to understand the concept or model (e.g. some types of students prefer tables to graphs for investigating functions)

Module Principle

The principles used in the Austrian DERIVE Project (White Box/Black Box Principle, Black Box/White Box Principle, Module Principle) are still topical using the TI-92. The Module Principle gets more and more important as the TI-92 offers some fine features as extended variable and function definition management, programming and even programming within text modules.

When teachers and students are familiar with the machine they experience programming features to be very helpful. Indeed there are big differences between certain classes and teachers depending on school type, size of classes and individual teaching styles, mainly in preferring of the three different types of modules and the different way to use them (white or black box):

- built-in modules (e.g. solve function)
- modules created teacher or downloaded modules packages extending TI-92s CAS software
- modules created by students

We plan to set a focal point on investigating learning processes using modules, mainly on the process of programming modules by students to enlarge and deepen the insight to mathematical concepts.

The Module Principle and examples for practical operation has been presented at Nashville in March 1998 by Helmut Heugl. (Helmut Heugl, Modular thinking and learning - a new quality or a problem for mathematics education, Paper presented at the 10th Annual International T3 Conference, Nashville, USA, March 1998)

4. Future Plans

4.1 Evaluation

After one year of running the TI-92 project we have collected a lot of material, feedback, tests, concepts, ideas etc. 1998/99 will be spent analyzing that material and preparing the final report. Most work will be done by the central planning group, the extern evaluation will be done by the Center of School Development at Graz, Styria, headed by Dr. Guenther Grogger.

Nevertheless the use of the TI-92 in the classes will continue as they advance to a further stage and build on the experiences of the first year. The highest age group (17 a.) will have to pass final exams (matura) using the TI-92, an essential situation for the success of the whole project.

Additionally some special investigations are planned, which could not be realized this year as we had to concentrate our (human) resources on organizing classroom work and teacher training.

4.2 Interaction studies

In co-operation with the Center for School Development, classroom observations should supply quantitative data about the quality and quantity of teacher-student and student-student interaction in classroom situations when using the TI-92. One of the reasons not

to doing this in the first year of the project was to allow teachers and their classes develop their particular teaching style before observing them.

4.3 Interpretative studies

On the basis of the results of the first year, especially the test results, interpretative studies should bring light onto students strategies. Primary goals are to investigate the ways students use the facilities of multiple representations and their thoughts when working with modules.

4.4 Conference in 1999

In the summer of 1999 there will be a conference dealing with computer algebra and mathematics education at Hagenberg, Austria, organized by Bruno Buchberger and ACDCA. By this time the first year of the Austrian TI-92 project will be evaluated and we are looking forward to presenting our results in a more comprehensive manner at this conference.