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Doing Advanced Mathematics with the TI-92 Plus Module

Carl Leinbach
Gettysburg College
Bert Waits
Ohio State University
leinbach@cs.gettysburg.edu
waitsb@math.ohio-state.edu

Description of the TI-92 Plus Module

The TI-92 Plus Module is a new microprocessor that will replace the present processor found in the rear of the standard TI-92. It is not a new calculator housing, it is a new processing unit and expanded memory for your present calculator. It is as if you took your Personal Computer and added a new Central Processing Unit and also increased the size of the computer's memory while retaining the computer's case, video unit, and mother board. Thus, you can greatly increase the power of your TI-92 without paying for a new housing and wondering what to do with your old TI-92. This, in itself, is a wonderful improvement. The upgrade cost is reduced by approximately 50%!

Here are a few technical details about the new module. First of all the TI-92 Plus has 256K of RAM as opposed to the 128K of RAM in the standard TI-92. The system, as can be expected, is larger – approximately 69K as opposed to the approximately 61K for the standard TI-92. However, the net result is that the user has 187K of RAM available for use as opposed to the 67K on the standard TI-92. If you consider what you are able to do on a TI-92 with 67K, think of what you can do with 120K more!

However, there are additional features to the TI-92 Plus. The first is that the operating system is stored using FLASH Memory. This is the same technology that is used in digital cameras to store pictures and delete or overwrite them after you have saved them. In the TI-92 Plus the operating system can be changed of updated by accessing a program over the World Wide Web or by having it shipped to you on a disk. Once again, you will not have to buy a new housing or even a new module as upgrades are developed. In addition, third party software developers can develop new programs or enhancements to the TI-92 Plus that can be purchased and added to your calculator.

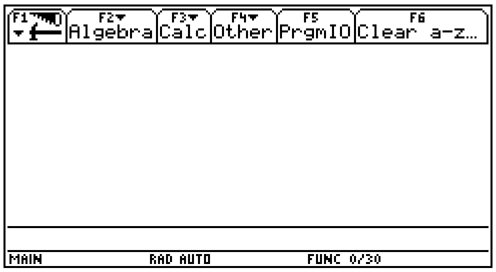
Another feature of the TI-92 Plus is that you have, in addition to the RAM, 384K of space to archive programs and your (non system) variables. This is a handy feature for programs that you want on your calculator, but use only on an occasional basis. You must transfer the program or variable back into RAM in order to use it, but it is on your calculator for use when you need it. The transfer from auxiliary storage to RAM is done in the VAR/LINK menu using the F1 key and the Archive Variable/Unarchive Variable option.

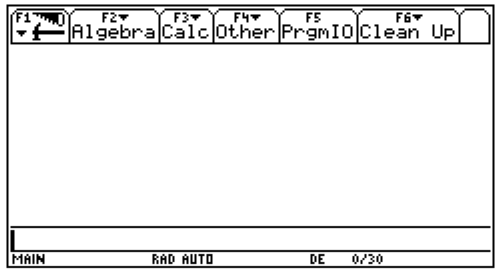
There are many other features to the TI-92 Plus module. They can be found by referring to the TI web site (http://www.ti.com/calc) or looking up the literature from Texas

Instruments. The technical features mentioned above are those that describe the most noticeable features of the Plus module.

How Do I Know If I am Working With a TI-92 Plus?

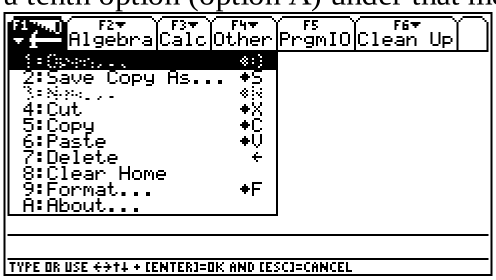
There are three ways to tell that you are working with a TI-92 Plus. The first way is the difference in the functionality of the F6 key. This is noticeable when you first turn on your TI-92 plus. On the left below is the standard TI-92 home screen. On the right is the TI-92 Plus home screen.

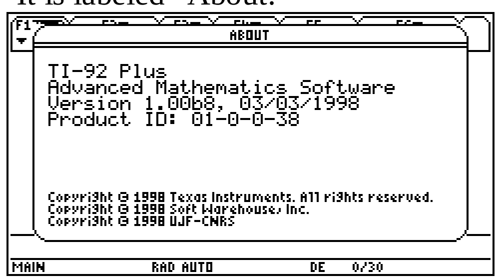




Notice that F6 on the TI-92 Plus home screen is labeled “Clean Up” while on the standard TI-92 home screen it is labeled “Clear a-z…”. The “Clear a-z…” is an option under the menu for F6 on the TI-92 Plus.

Another way to tell that you have a TI-92 Plus is to press F1. You will notice that there is a tenth option (option A) under that menu. It is labeled “About.”

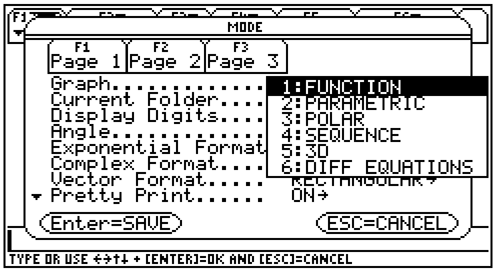


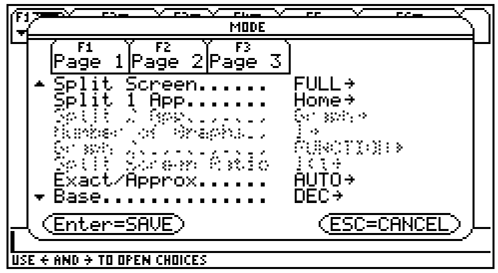


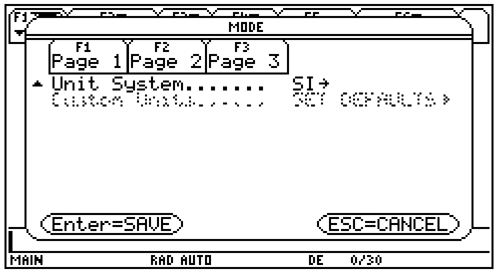
Choosing this option displays a screen that announces the fact that you have a TI-92 Plus with the Advanced Mathematics Software package. We will discuss this software in the remaining sections of this unit.

The third way to tell that you have a TI-92 Plus is to press the Mode button and note that there are three pages of options. These pages are shown at the top of the next page. Note that there is a sixth graphing option available. It is called Differential Equations and it has the same functionality as the numerical differential equations graphing option on the TI-86. We will discuss this option in much more detail later in this unit.

There are many other ways in which you can tell that your unit is a TI-92 Plus and not a standard TI-92. These include the MEM screen which advises you about the amount of memory that is in RAM as well as in the Archive section, the APPS screen that has some new applications, and the 3D Graphics Window. We will look at some of these options in our survey of the TI-92 and the Advanced Mathematics Package.



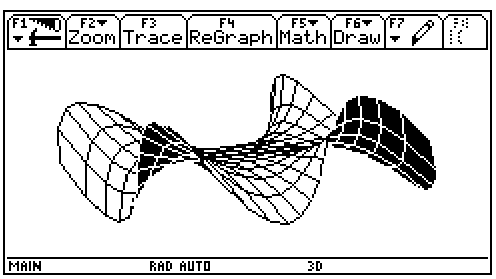


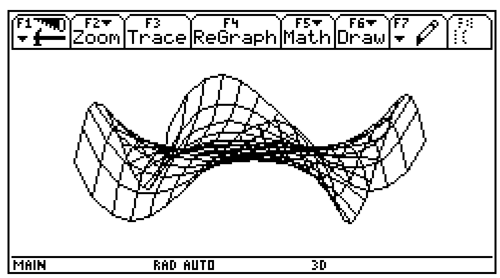


Improvements to 3-D Graphing

Rotating the Graph of a Surface

The TI-92 Plus makes the viewing of 3D surfaces even more exciting. It is possible to rotate the surface! This is done by pressing the cursor key. What really happens is that your viewing orbit rotates in the direction indicated by the cursor keys. You need to try it to believe it. Below are some views obtained by rotating the surface defined by the equation, $z = \dfrac{x^3y - y^3x}{390}$.





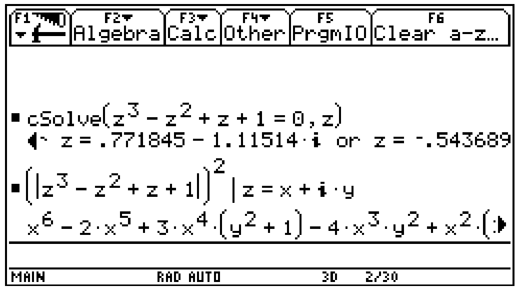
We can return to the original orientation of the surface by pressing 0 (zero) and we will see the graph of the surface as it was with our original window settings.

Visualizing the Zeros of a Complex Valued Function

Given a function, $f(z)$, of a complex variable, can we find the zeros of $f(z)$? If the function is a polynomial, such as

$$f(z) = z^3 - z^2 + z - 1,$$

it is possible to find the zeros using the **czeros(** or **csolve(** command on the TI-92. The screen shown on the following page illustrates this use of the TI-92.



The zeros are identified as {-.543689, .7771845 + 1.11514 *i*, .7771845 -1.11514 *i*} in approximate decimal form.

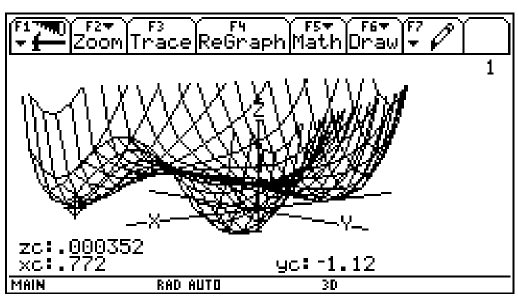
Is it possible to see these zeros graphically. In this case a theorem from complex variable theory comes to our rescue. This theorem involves the absolute value of a complex variable,

$$|z| = \sqrt{z\bar{z}} = \sqrt{x^2 + y^2} \quad \text{where } z = x + iy.$$

The theorem states that $|f(z_0)| = 0$ for some z_0 if and only if $f(z_0) = 0$. This is a very straightforward theorem to prove and we suggest that you do it (the margin of these notes is sufficient room.) Actually, the statement of the theorem can be extended to say: $|f(z_0)|^2=0$ for some z_0 if and only if $f(z_0) = 0$.
This is the form that we use.

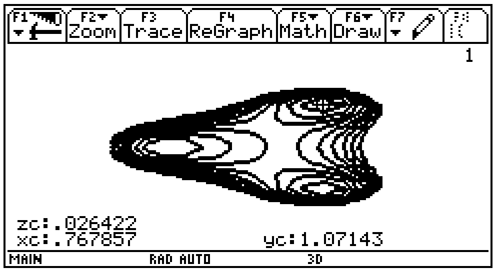
On the next line of the window shown above we evaluate $|f(z_0)|^2$. Note the local substitution of $z = x + iy$. This results in a sixth degree polynomial in x and y. We copy this expression and paste it in the **Y=** editor (of course, the mode has been set to 3-D.)

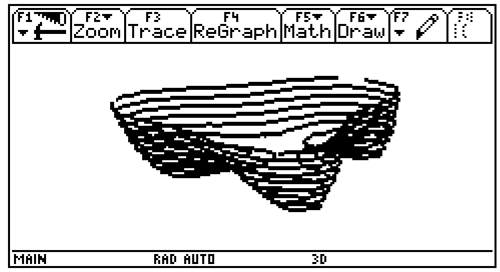
Although the expression is rather long and unwieldy, the TI-92 handles the graphing quite well as is seen below.



In order to emphasize the location of the zeros a trace was done and the cursor positioned at (.772, - 1.12). This point, of course corresponds to the complex value of .772 - *i*1.12.

A few comments about the graph. First of all, the window was set to [-1.5,1.5] by [-2,2] by [-.5, 5] and $\theta = 50^o$ $\varphi = 70^o$. As usual, we chose 20 *x* and *y* grids. The main comment is, however, that we chose the "wire frame" mode for the plot. There are times when this mode simply allows you to see more of the graph that highlights the points in which you have an interest. In this case, this mode allows you to more easily see the three zeros. The viewing may be made more easy by rotating the graph on the TI-92 Plus. But even more spectacular is the option that allows you to see the contour levels as shown in the figure below. Plotting in this





mode is very slow, so there is a need to be patient.

Using the F3 key to trace the contour map (it actually traces the wire frame graph which is not visible) we get an approximation to the zero in the first quadrant. The graph on the left is the contour plot rotated to $\theta = 70^o$, $\varphi = 50^o$, and $\psi = 0^o$.

Exercise:

1. Choose another polynomial function of a complex variable. Locate the zeros of the function algebraically and graphically.

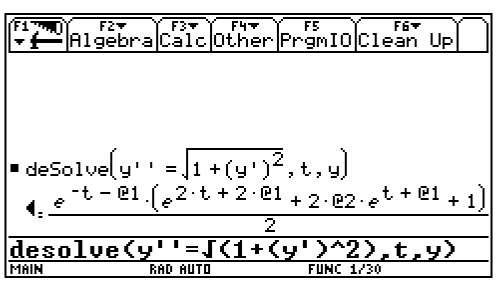
Differential Equations on the TI-92 Plus

Exploring The Simple Pendulum

An important feature of the new Advanced Math Package is the ability to analytically solve first and second order Ode's. Before exploring the simple pendulum, we will illustrate the use of this feature. The new command is **desolve(** . The prime, ′, is typed using the 2-nd b key stroke combination.

1. Solve the following nonlinear second order ODE.

$$y'' = \sqrt{1 + (y')^2}$$

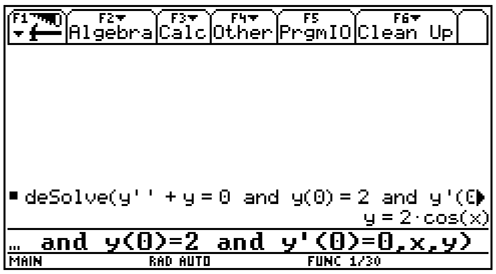


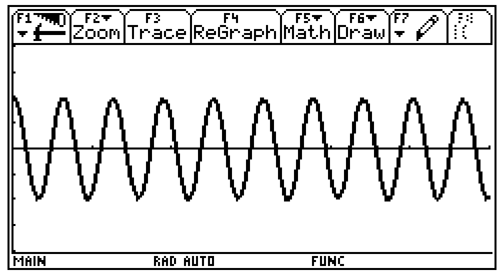
The TI-92 uses the notation @1 to denote an arbitrary constant. Subsequent constants will be denoted by @2, @3, etc. Note this solution has two arbitrary constants, @1 and @2.

Now, we explore the simple pendulum, begin with the undamped free vibrations case:

$$y'' + y = 0$$
$$y(0) = 2;\ y'(0) = 0$$

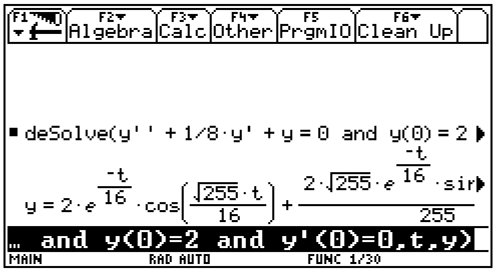
The initial conditions are added to the original equation by using an **and** connective.

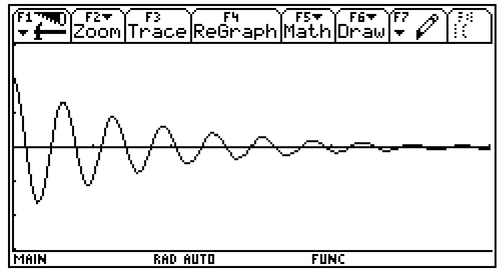




This result is, of course, not spectacular and the equation is easy to solve using paper and pencil, but it is the start of our exploration. Let's add a damping factor of $\frac{1}{8}y'$ to the original equation.

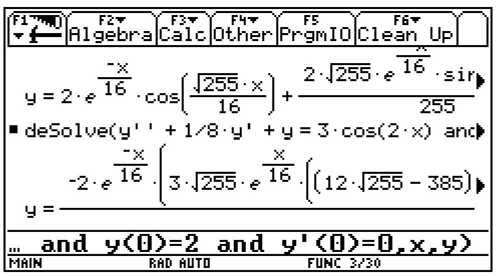
$$y'' + \frac{1}{8}y' + y = 0$$
$$y(0) = 2; y'(0) = 0$$

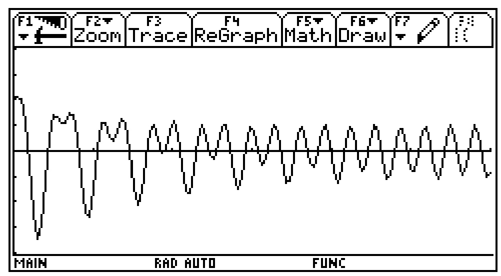




This was a slightly more formidable symbolic task! Now we consider the more general forced vibration with damping, first we consider the case where the forcing function is does not have the same period as the basic free vibration

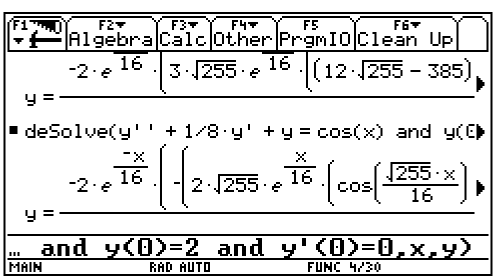
$$y'' + \frac{1}{8}y' + y = 3\cos 2t$$
$$y(0) = 2; y'(0) = 0$$

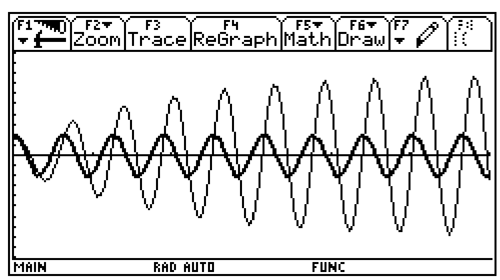




What appears to be happening in this case? Now choose a forcing function that has the same period as the basic free vibration.

$$y'' + \frac{1}{8}y' + y = \cos t$$
$$y(0) = 2; y'(0) = 0$$





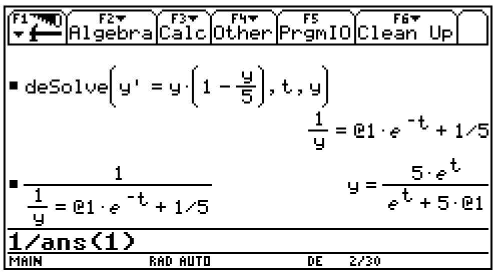
This is certainly a different behavior than we observed with the previous forcing function. Because we had to increase the range on the y-axis, we drew the graph of the solution to the free vibration problem (in thick plot style) on the same axis.

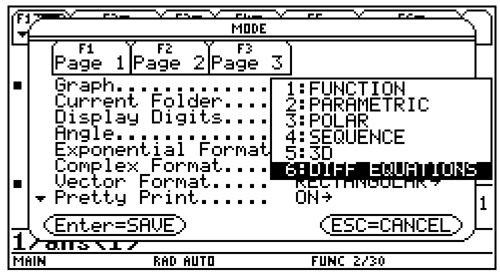
A Numerical/ Graphical Exploration of Logistic Growth

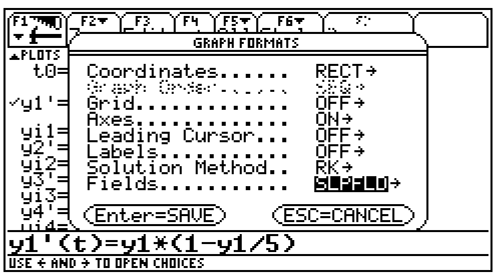
For the previous example we did our analysis by observing the behavior of some particular solutions. For many examples we can learn more about the phenomenon described by the model by using graphical methods. The TI-92 Plus provides this ability.

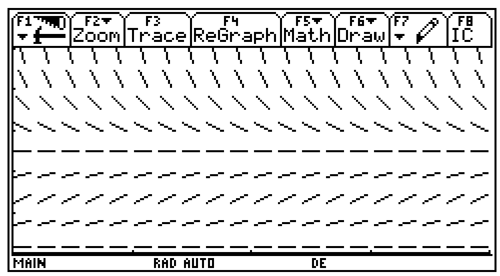
Consider a standard logistic equation for a population of animals developing in an environment having limited resources. The following first order differential equation describes this process.

$$y' = y(1 - \frac{y}{5})$$



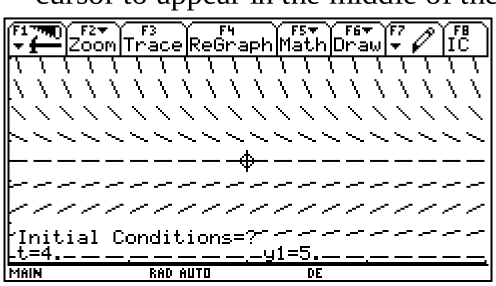


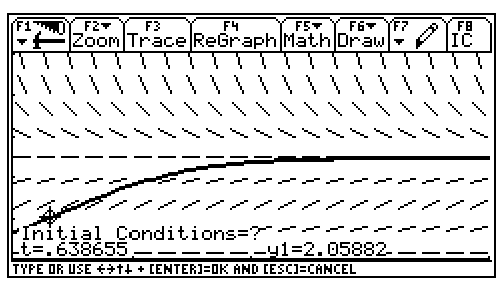




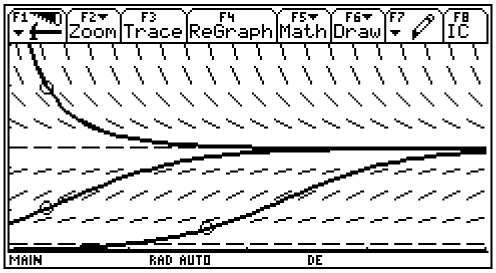
In the panel at the upper left we see the analytic solution. The next panel over shows the mode selection screen. Note the new graphing mode: **6: DIFF EQUATIONS.** This mode is similar to the one found on the TI-85 and TI-86. The panel on the lower right is from the Y= editor it is the Format screen (option 9) under the F1 pull down menu. The fourth panel is the result of these choices for our differential equation with $0 \leq t \leq 8$ and $0 \leq y \leq 10$. Notice that for $y = 5$ we have a horizontal slope. This is the carrying capacity for the environment.

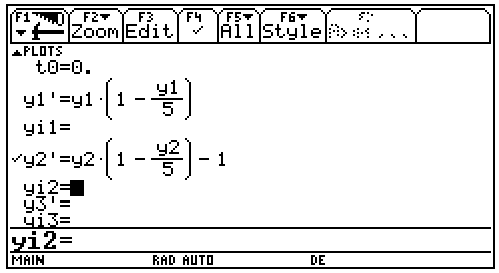
Now we can explore the model. Pressing F8 (IC for Initial Conditions) causes the cursor to appear in the middle of the screen

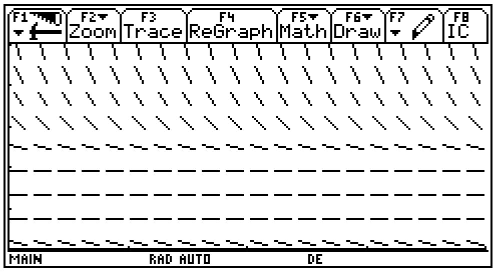


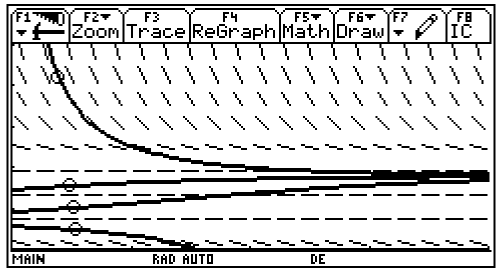


Move the cursor to a desired position and press the ENTER key. A curve corresponding to a solution is then drawn through this point. The user has a choice of numerical methods that are used to derive this solution, either Euler's Method or one of the Runge-Kutta methods. The choice for this example is Runge-Kutta.





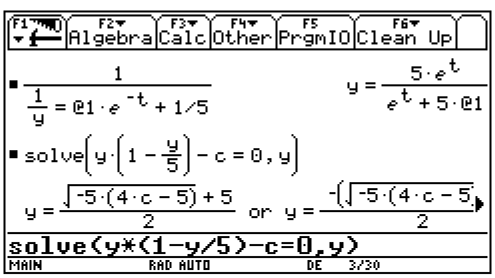


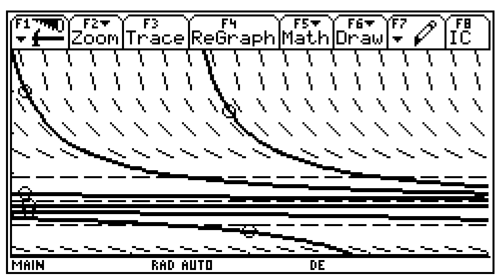


The panel at the upper right shows a modification of our model to include harvesting (hunting, fishing, etc.) of the animal population. The panel at the lower left shows a different slope field towards the bottom of the screen. Exploration with different sets of initial conditions shows that there are two equilibria for this model: one stable and one unstable. The larger of the two is the stable one. It behaves, as does the equilibrium value for the model without harvesting, albeit that it is a lower value. The other equilibrium value has the property that if the population is greater than this value; it tends towards the stable equilibrium. If the population falls below this value it dies out. We can solve for these equilibrium values by solving the quadratic equation

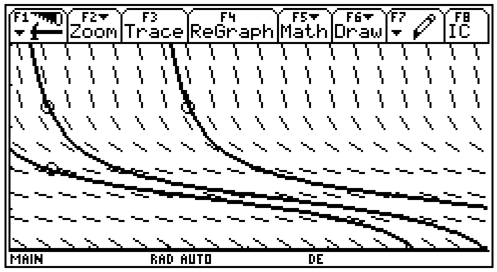
$$y*(1 - y/5) - c = 0$$

for y. In a bit of calculator over kill we show this solution below on the right.



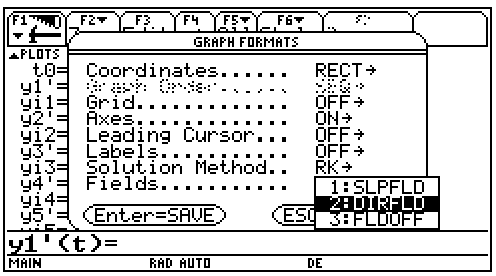


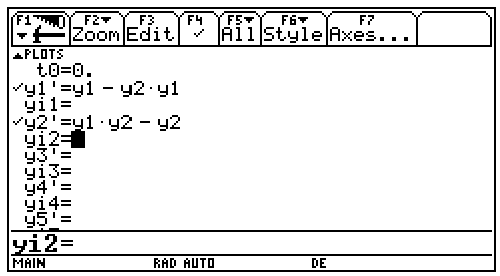
When c = 1.25 we have a rather strange behavior caused by the upper and lower limits coinciding. Finally we investigate what happens when the roots of the quadratic become imaginary. We choose c = 1.5 and extended the t-axis to t = 12. Observe this behavior. Note that no matter what initial conditions we choose the population becomes extinct. What does this say about the care that needs to be taken in setting the harvesting rate?



A System of First Order Equations

Of course no exploration of differential equations would be complete unless we had a way of observing the phase plane for a system of differential equations. This is one of the exciting features of the TI-86 calculator. It is just as exciting on the TI-92 Plus. We change the option under Format in the F1 pull down menu to Direction Field as shown below.



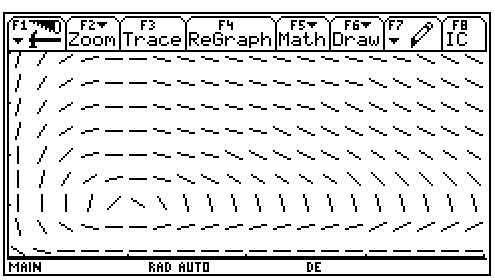


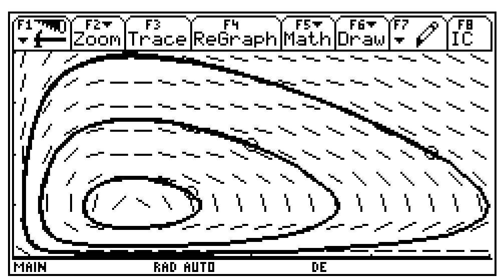
We briefly explore this option by looking at a version of the prey /predator model

$$y_1' = y_1 - y_1 y_2$$

$$y_2' = y_1 y_2 - y_2$$

We use the window $0 \le y_1 \le 4$ and $0 \le y_2 \le 4$ with $0 \le t \le 10$.





Using the TI-92 plus with its Advanced Math Package, it is possible to teach a truly modern course in ordinary differential equations that encourages both analytical and graphical exploration of the phenomena described by the equations.

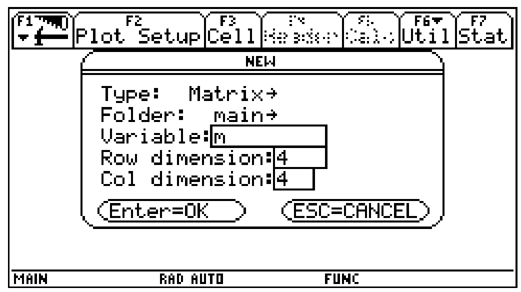
Linear Algebra on the TI-92 Plus

The TI-92 Plus has the capacity to do both numerical and symbolic matrix manipulations. We will illustrate some of the basics of these matrix manipulation capabilities.

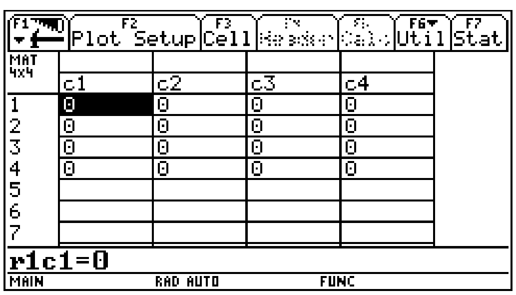
A Review of the Matrix Operations found on the TI-92

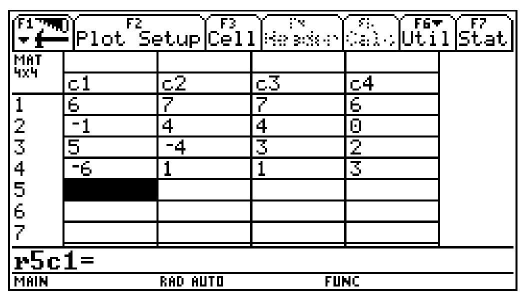
Entering Matrices

To enter a matrix choose option 5 under the APPS menu. Indicate that you are entering a "New" matrix . Fill in the next screen with the following information: the fact that you are creating a matrix; the name of the matrix; and the size of the matrix.



This will result in a screen showing a matrix of the appropriate size that is filled with zeros. You can then fill in the matrix with the values (either numerical or variable) that you desire.





The matrix may also be entered from the command line in a method that is reminiscent of the style of some numerical linear algebra packages.

[6, 7, 7, 6; -1, 4, 4, 0; 5, -4, 3, 2; -6, 1, 1, 3] -> m

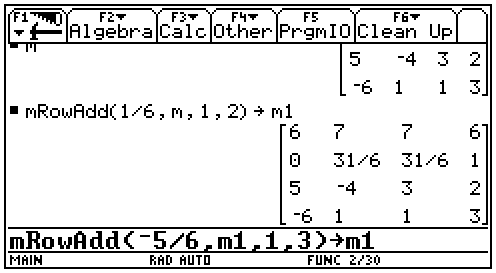
Note that we are using a numerical example here, but we may also have symbolic entries in the matrix.

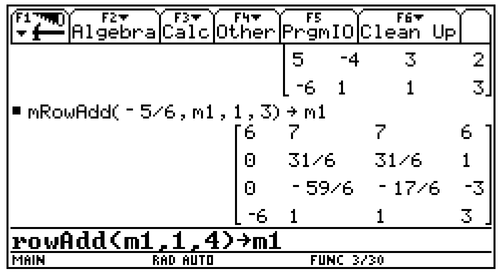
Matrix Functions

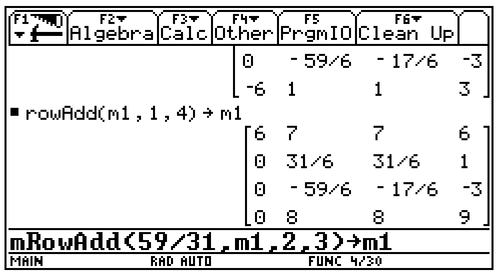
The TI-92 has many functions that allow you to manipulate matrices and vectors. For example, you can step your class through the row reduction of a matrix using the **-** (negation), **mRow( ),mRowAdd( ),rowAdd( )**, and **rowSwap( )** functions.

The determinant of a matrix is found using the **det( )** function. $m^{\mathbf{T}}$ yields the transpose of m, and m^{-1} the inverse (if det(m) ≠ 0). The n by n identity matrix is entered by typing: identity(n). Of course, n must have a specific numerical value.

For the matrix m that you entered, find $m^{\mathbf{T}}$**, det(**m**)**, and m^{-1}. Now use the elementary row operations to row reduce the matrix, m, to an upper triangular matrix. The following screens show the process applied to the first column of the matrix.

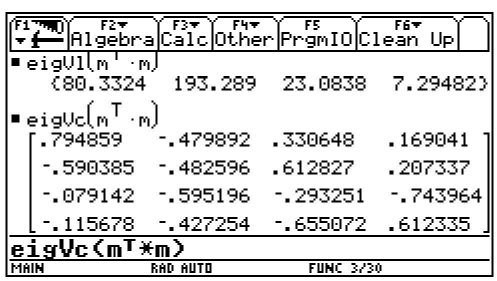




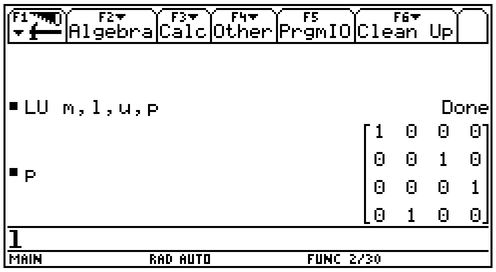


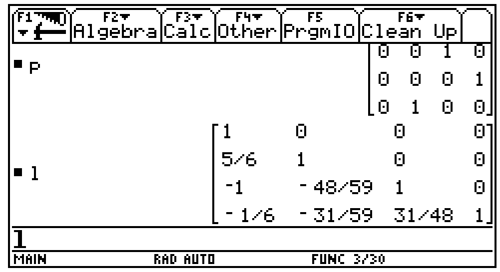
Matrix Functions available only on the TI-92 Plus

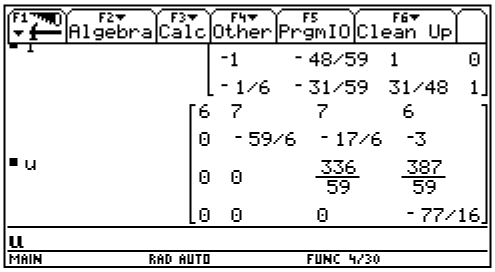
The TI-92 Plus has enhanced its linear algebra capabilities to a great extent. It has incorporated many of the numerical linear algebra functions of the TI-85 and 86. For example, the following screen illustrates the method for finding the eigen values and eigenvectors for a matrix. We used $m^{T}m$ in order to have real results that would fit nicely on our display screen. However, the routines will find complex eigenvalues and eigenvectors.

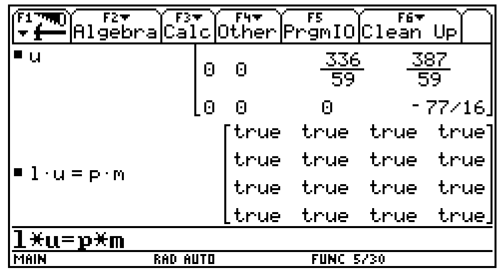


There are also two factorizations for square matrices on the TI-92 Plus. The first is the familiar LU factorization that finds matrices L, U, and P such that L*U = P*m, where P is a permutation matrix. The following four screens show the result of this factorization on the matrix, m.









It is also possible to find the QR factorization of the matrix, m. For this factorization Q is an orthonormal matrix and R is an upper triangular matrix that have the property, QR = m. We once again illustrate this for our matrix m.

For the display shown below the calculations were done in exact mode and the display of Q and R were done using approximate mode. In general, the calculation of the Qr factorization is faster if it is done in approximate mode. It is recommended that approximate mode be used for any large matrix.