Contents

Using the TI-92 and TI-92 Plus to Explore Derivatives, Riemann Sums, and Differential Equations with Symbolic Manipulation, Interactive Geometry, Scripts, Regression, and Slope Fields

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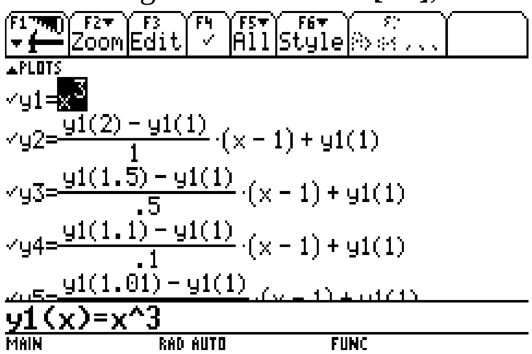
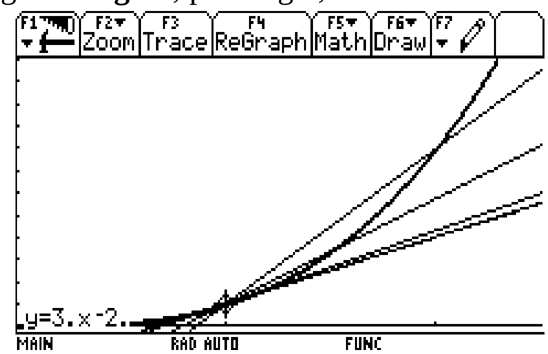
A Sequence of Secant Lines

To introduce the concept of derivative, we usually examine a sequence of secant lines to a curve through a fixed point. For example, secants to $\mathbf{y = x^3}$ through P_0P_i where $P_0 = 1$ and $P_i =$ 2, 1.5, 1.1, and 1.01. Notice that the slope of the secant line changes less between the latter lines drawn thus motivating the concept of a "limiting" position which we call the tangent line. The graph below shows the functions, y1 through y5, and the tangent line to $\mathbf{y = x^3}$ at $\mathbf{x = 1}$ in the window **[0, 2.5] x [-.2, 12]**. The tangent line is drawn using the **Math** menu **[F5]**, selecting **A:Tangent**, pressing **1**, then **ENTER**.



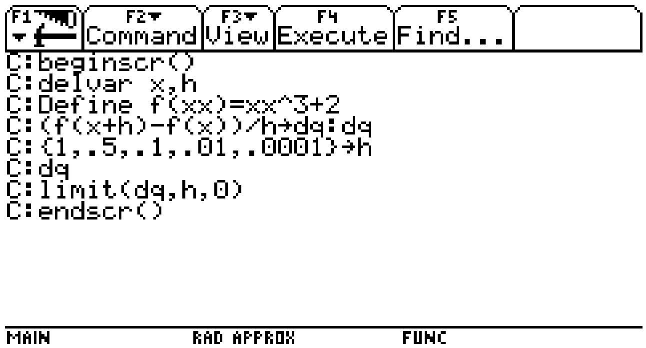


These functions and window settings can be saved as a graphical database [**secxcub**] to recall the graph on the left without having to re-enter everything. From either the **GRAPH** window or the **Y =** window, go to **TOOLS** [F1], press **2:Save Copy As** . . . , use **Type:GDB,** name the Variable: **secxcub**, then **ENTER** to save this graphical database.

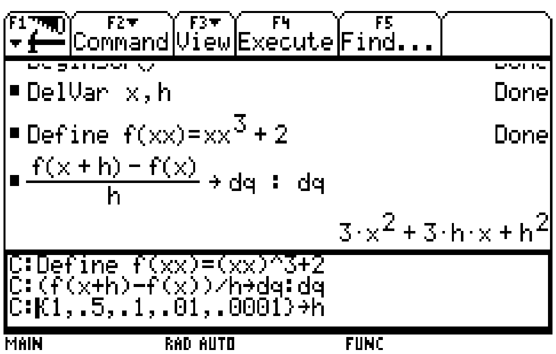
We can see a dynamic view of these secant lines approaching the tangent line by using the **CyclePic** command on the TI-92. Save the graphs of y1 and y2 as pic1, y1 and y3 as pic2, . . ., y1 and y5 as pic 4, and y1 and its tangent as pic5. **CyclePic "pic",5,.5,4** will cycle through the five saved pictures, showing each for half a second, for four cycles. Saving pictures is memory intensive; you can't store many of these cycles on your calculator. But you can save cycles as groups on a computer using TI's GraphLink. To save the graphs of y1 and y2, check those two functions [F4] on the **Y =** screen [♦W], **GRAPH** [♦R] to confirm the results, go to the **TOOLS** menu [F1], **2:Save Copy As** . . ., use **Type:Picture** [right arrow, press 2], name the Variable: **pic1**, **ENTER** to save this picture. Repeat through each pair through **pic4**. For the tangent line, **graph** $\mathbf{y = \sin (x)}$ by checking [F4] **y1** only, choose the **Math** menu [F5], select **A:Tangent**, press **0**, then **ENTER** to draw the tangent line at x=0, and save as **pic5**. By changing the function in **y1** (and probably the window), you can draw secant lines to your new function through $P_0 = 1$. As a second example, consider secant lines to $\mathbf{y = \sin (x)}$ through P_0P_i where $P_0 = 0$, and $P_i = \pi/2, \pi/4, \pi/8$, and then $\pi/16$.

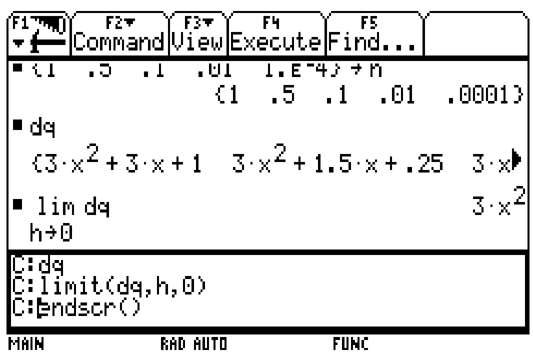
Using a Script to Generate Difference Quotients and Derivatives

One of the most powerful new features on the TI-92 is the text editor used to save work on the home screen that you may want to do repeatedly as a script. The commands are saved but not the results. To illustrate, let's examine the script **deriv** below. It defines a function **f(xx)**, computes the difference quotient **dq** at any point x for several values of h, then computes its limit as h → 0: the derivative f '(x). The "C:" at the beginning of a line indicates that it is a command to be executed. The lines beginscr( ) and endscr( ) call short programs that split the screen and then return to the text editor.



To view this script (if you have linked it) press **APPS**, **9:Text Editor**, **2:Open**, down arrow to **Variable:**, right arrow to see a list of scripts, down arrow to **deriv,** press **ENTER**. To execute the script, up arrow to the first line, press **Execute** [F4] repeatedly. The screen will split: the home screen is in its upper two-thirds showing each command as it is executed; the text window is in the lower third. We see the difference quotient in terms of h, evaluated for several values of h, and its limit f '(x).





By replacing f(xx) by sin (xx) or xx^n, and then re-executing the script, we can see the difference quotients and derivative of other functions. Impressive?

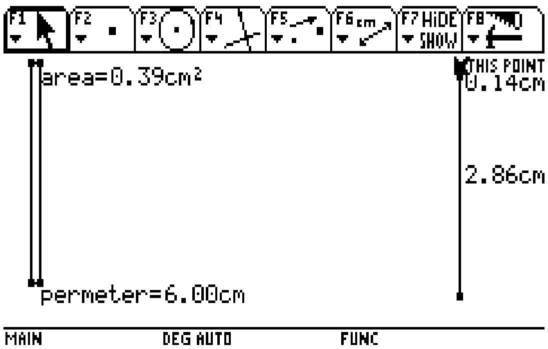
To find these derivatives using CAS on the TI-92, use the syntax ***d*****(f(x), x);**
the script *d* is in the **Calc** menu **[F3] 1:** ***d*****(differentiate** or **2nd 8**:

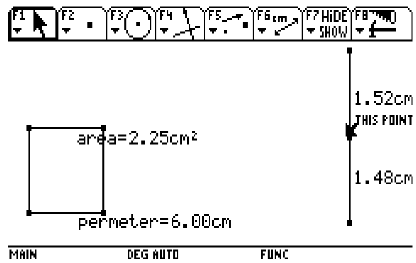
d**(x^3+2, x)** ***d*** **(sin (x), x)** ***d*** **(x^n, x)** to get

$3x^2$ **cos (x)** $\frac{n?x^n}{x}$.

Maximizing the Area of a Rectangle with a Fixed Perimeter

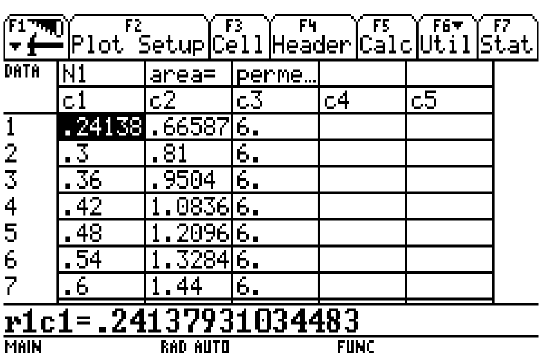
Using the Geometry Application on the TI-92, we can show students rectangles that have the same perimeter and different areas (I never feel students really understand what is happening when they do this problem). Then find the dimensions that maximize its area.





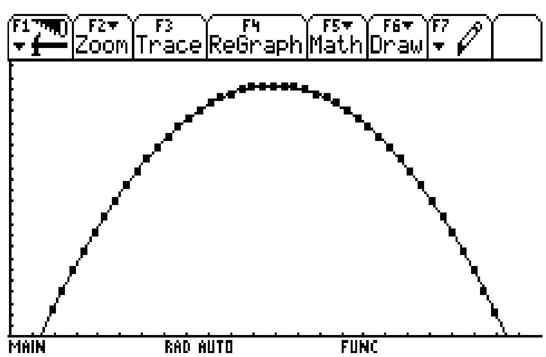
Create a rectangle of fixed perimeter 6.00 cm by using subsegments of the 3.00 cm segment at the right as its height and width. By moving the interior point on the 3.00 cm segment at the right of the screen, the area of the rectangle changes and approaches zero as its height approaches zero. By moving the point slowly when it is near the midpoint, we see that the maximum area of the rectangle is 2.25 cm^2. Detailed instructions for constructing the rectangle in the top figures are given on the last page of this article.

Although it is very easy to write the equation for the area [x (3 - x) = $-x^2 + 3x$], it is informative to animate the interior point moving on the segment and collect the height, area, and perimeter of the rectangle. Area can be plotted versus height to see that the data lies on a parabola. Power regression can be used to find its equation [Area = $-x^2 + 3x$]. Scrolling through the data shows that as the height increases, the area increases to about 2.25 and then decrease toward zero, and the perimeter remains constant.



DATA	N1	area=	perme…		
	c1	c2	c3	c4	c5
1	.24138	.66587	6.		
2	.3	.81	6.		
3	.36	.9504	6.		
4	.42	1.0836	6.		
5	.48	1.2096	6.		
6	.54	1.3284	6.		
7	.6	1.44	6.		

r1c1=.24137931034483



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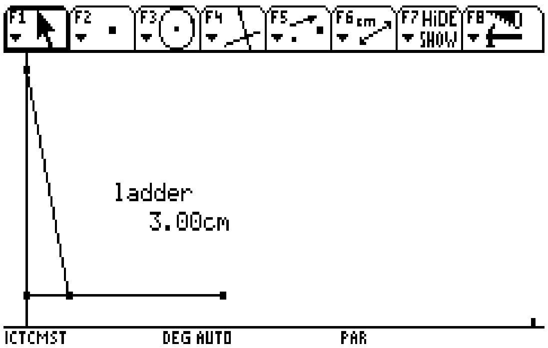
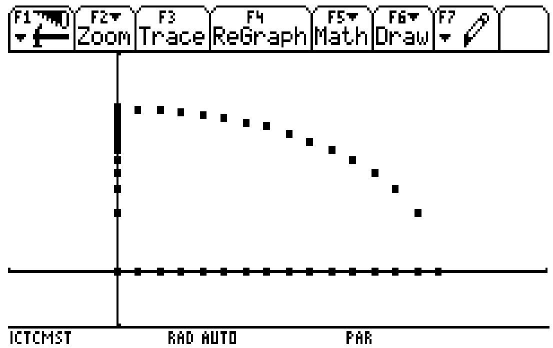
To use the derivative and CAS to find the maximum area, use **1:solve** in the **Algebra** menu **[F3]:**

solve (*d*(x (3-x), x)=0, x) **x = 3/2**
x (3-x)|x=3/2 ∪ **2.25** for the maximum area

Observing a Ladder Sliding along a Wall using Scripts

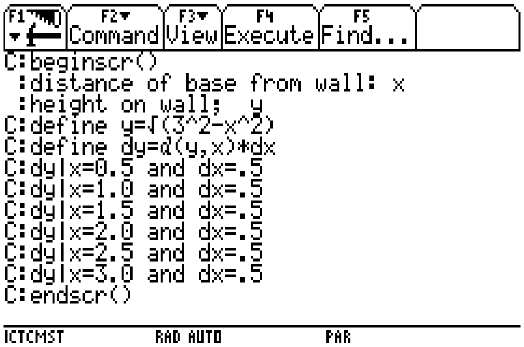
As another geometry application, draw a ladder whose base can be moved along the ground in equal increments and watch the rate at which the other end slides down the wall. The ladder's length, 3.00 cm, was chosen to fit comfortably on the screen; to be realistic, it could represent a 12-foot ladder. Observe the rate at which the ladder slides down the wall; is it sliding at a constant rate or does it appear to be sliding faster near the bottom?

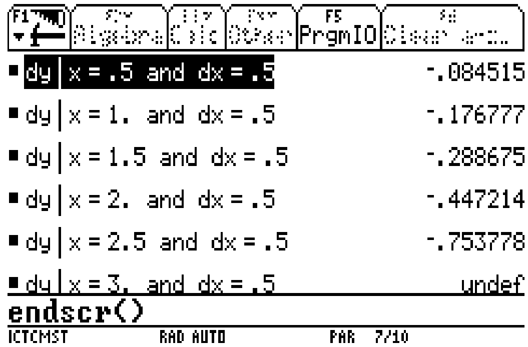




To see a different representation of this motion (based on a script written by Dr. Paul Beem, Indiana University at South Bend), let **t** represent the distance of the base of the ladder from the wall and **s(t)** = $\sqrt{(3^2 - t^2)}$ the height of the ladder on the wall. Graph the functions **xt1(t) =t, y1(t) = 0, x2(t) = 0, y2(t) = s(t), x3(t) = t, y3(t) = s(t)** in the parametric window **[0,3] x [-1, 4] x [-1, 4]** with **t-step = 0.2** as shown on the right above. The first (horizontal) function gives the positions of the base of the ladder along the ground (equal increments), the second (vertical) function the positions of the top of the ladder sliding along the wall, the third (2-d) function the height on the wall versus the distance along the ground. The second and third functions show that the changes in height are greater as the ladder moves away from the wall.

The script below shows the height of the ladder as it is pulled away from the wall. The negative signs show that the ladder is falling; the increasing absolute values of dy show that it is falling faster as the ladder is pulled away from the wall.





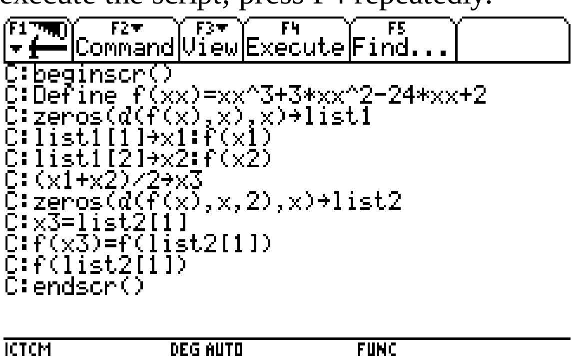
Investigating Properties of a Cubic using Scripts

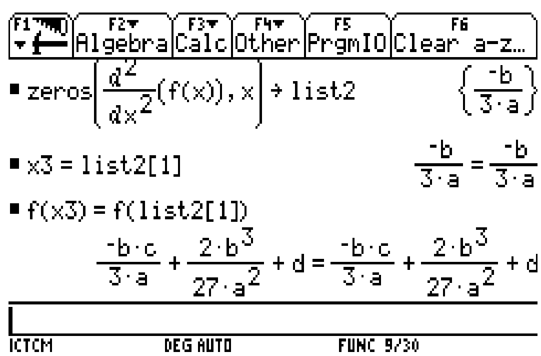
We can use the text editor to save work as a script using the command in the **Tools** menu **[F1] 2:Save Copy As** . . . To illustrate this we will use the **function** $\mathbf{f(x) = x^3 + 3x^2 - 24x + 2}$ **.** However, when defining the function use the variable **xx** to avoid the circular definition error. This function has a relative maximum at (-4, 82), a relative minimum at (2, -26), and an inflection point at (-1, 28) [the average of its relative extrema]. **Is the inflection point the average of the relative extrema for all cubics?** To explore this, first we'll find these values analytically for the function above.

Clear the home screen,

define the function	$\mathbf{f(xx) = xx^3 + 3xx^2 - 24xx + 2}$
save the zeroes of its derivative as list1	**{-4, 2}**
evaluate the function at each critical point	**f(-4) = 82; f(2) = -26**
find the average of the x-coordinates of the critical points	**-1**
save the zeroes of its second derivative as list2	**{-1}**
are the values from the previous two steps equal?	**true**
are the values of the function at these two values equal?	**true f(-1) = 28**

The screen on the left below shows this work saved as a script. The "C:" at the beginning of a line indicates that it is a command to be executed. The command "beginscr( )"calls a short program which divides the screen so that the work can be seen easily. The command "endscr( )" calls a short program which restores the screen to see the script only. To execute the script, press F4 repeatedly.



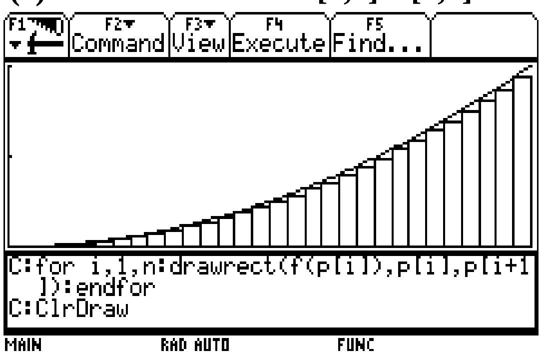


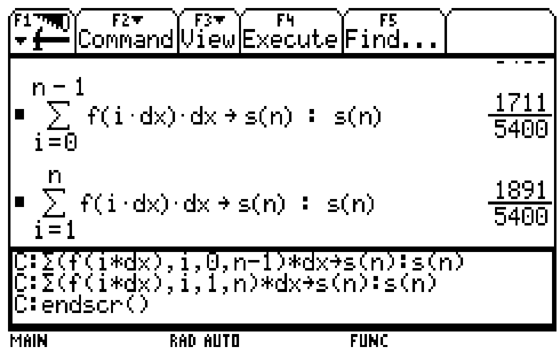
To repeat this sequence of steps for a different function, simply change the function in the second line. Executing the script for this new function shows that the property holds again. To see if it holds for **any** cubic, define f(x) as $ax^3 + bx^2 + cx + d$. Executing the script again, we see in the screen on the right above that for any cubic, its inflection point is the average of its two critical points.

The more I use scripts the more I realize their power. Why use scripts instead of writing a program? First, scripts use regular TI-92 commands, not special programming commands and logic. Secondly, scripts allow you to see the result of each command not just a final result emerging from a "black box". Thus, they are much more accessible for students to use without needing to learn a new language. Instructors can write scripts and link them to students to use as worksheets. Students can save their work as lab reports and print them out to turn in by using TI's GraphLink software.

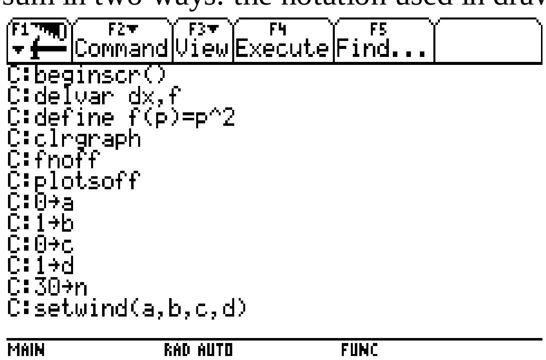
Riemann Sums

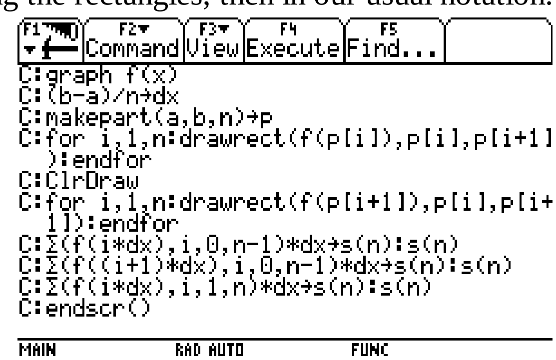
We can use a script to draw 30 lower rectangles for Riemann sums under the curve **f(x) = x²** in the window **[0,1] x [0,1]** and evaluate these sums.



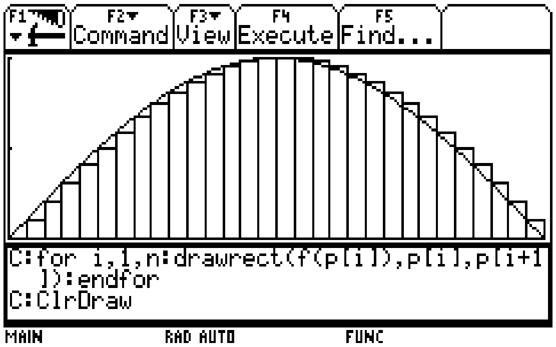


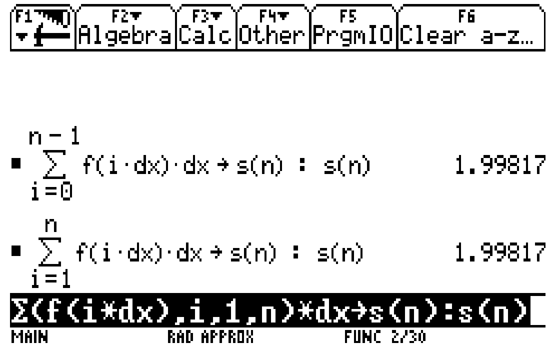
The script below defines the function, sets the window, graphs the function, draws the upper and lower rectangles, and calculates the upper and lower sums. It uses programs beginscr, setwind, makepart, and drawrect. Because the partition created by makepart is saved as a list, the left endpoint is p[1] and the right endpoint is p[n+1] instead of the f(0) and f(n) that we usually use. The next to last two lines of the script calculate the upper sum in two ways: the notation used in drawing the rectangles, then in our usual notation.





The function in the script can be changed to f(p) = sin (p) and the right endpoint, b, to π.



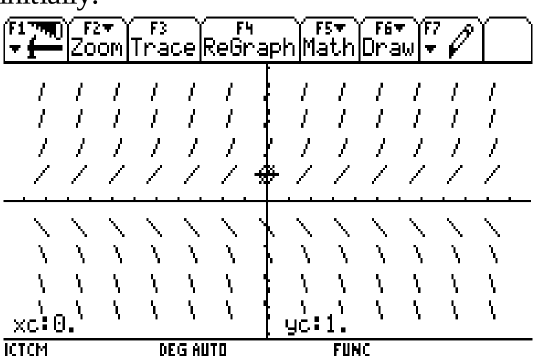


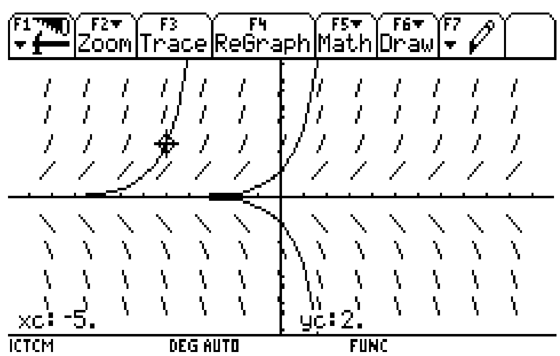
To compute definite integrals using CAS on the TI-92, use the **CALC** menu **[F3], 2: (integrate** or **[2nd 7]**: **(x^2, x, 0, 1)** or **(sin (x), x, c)** to get

1/3 or **- cos (x) + c**.

Differential Equations

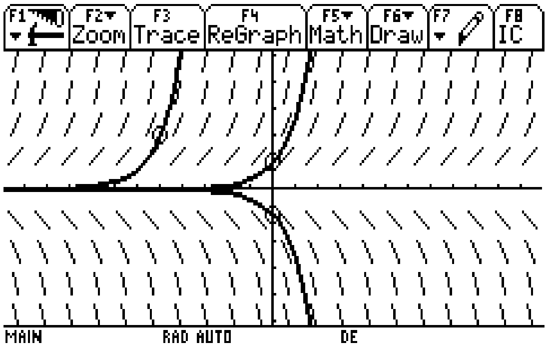
Slopefields are a good way to visualize a differential equation. The program **slope2** can be used to plot short segments of tangent lines at grid points on the screen. To solve the differential equation $\frac{dy}{dx} = y$, turn all functions and plots off; use the **ZoomDec** window; enter **slope2("y")** on the home screen. Notice that all the slopes along a horizontal line are equal since the slope of each point is equal to its y-coordinate. To draw a solution through the point (0,1), move the cursor to (0, 1); press **ENTER**. The solution points are drawn to the left of the cursor, then to its right. Solutions have also been drawn through (-5, 2) and (0, -.1). To check the functions graphed on the slopefields, graph their solutions **y1 = e^x**, **y2 = $2e^{x+5}$**, and **y3 = $-e^x$**. These functions will graph first when you enter slope2("y") on the home screen, then the slope field will appear. When you move the cursor to each initial point , the curve drawn overlays the "solution" drawn initially.

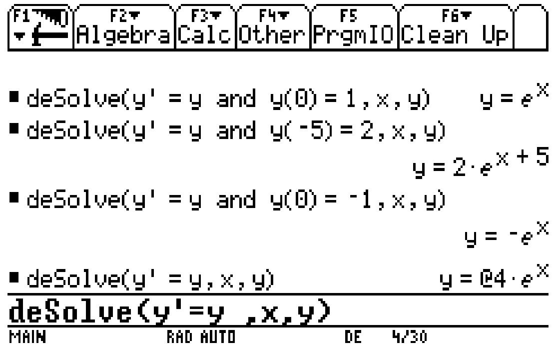




The **Advanced Mathematics Software** on the new **TI-92 Plus Module** contains a differential equation graph mode. Thus it graphs slopefields without needing a program. Set the graph mode to **DIFF EQUATIONS**. Use the **ZoomDec**imal window with **t 0 = 0, t max = 10, t step=.1, t plot=0**. In the **Y =** menu, enter **t 0 = 0, y1' = y1, yi1 = {1, -1}** to graph the functions on the right above. To see the graph on the left, press **F8 IC** (initial conditions) **-5 ENTER 2 ENTER**.

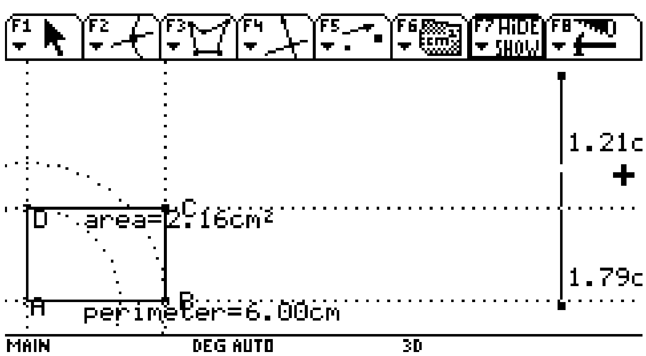
The **TI-92 Plus Module** also contains a new command in the **F3 Calc** menu **C:deSolve(** for solving differential equations. To use it, enter the differential equation **and** the initial condition followed by the independent, then dependent variable, separated by commas. To type the "prime" in **y**', press **2nd b**. **@4** in the last line of the screen below indicates the arbitrary constant.





To Create The Construction For A Rectangle With Fixed Perimeter

Open the geometry application [**APPS, 8, 3**] and name your construction **area**. To do this, construct a vertical line segment [**F2, 5**] near the right edge of the screen. Measure its length [**F6, 1**]. Adjust the segment until its length is exactly 3.00 cm. Place a point anywhere on the segment [**F2, 2**]. The two subsegments formed will be used as the dimensions of a rectangle with perimeter 6.00 cm. To use these subsegments separately, hide the segment [**F7, 1**] and construct the two subsegments [**F2, 5**] from the interior point to each endpoint. Delete the length 3.00 **cm [F1, 1, DEL**] and measure the length of each subsegment [**F6, 1**]. Now to begin constructing the rectangle, draw a horizontal line [**F2, 4**] through a point in the lower left of the screen and a line perpendicular to it [**F4, 1**] through the point. Label [**F7, 4**] the point A. Use the compass tool [**F4, 8**] to construct a circle centered at A with its radius the length of the lower vertical subsegment. Locate the point of intersection of the circle and the horizontal line [**F2, 3**] and label [**F4, 8**] it B. Repeat to construct a point D on the vertical line through A so that the segment AD has the length of the upper vertical subsegment. Hide [**F7, 1**] both circles.



Construct lines [**F4, 1**] through D perpendicular to the vertical side of the rectangle and through B perpendicular to the horizontal side of the rectangle. Locate the point of intersection of these lines [**F2, 3**] and label [**F4, 8**] it C. Hide [**F7, 1**] the four horizontal and vertical lines through A, B, C, and D. Draw the polygon [**F3,4**] ABCD. Measure the perimeter of the polygon [**F6,1**]; when 6.00 cm shows, type "**perimeter=**". Measure the area of the polygon [**F6,1**]; when the number shows, type "**area=**". Now move the point on the vertical line at the right of the **screen** [**F1, 1, "hand"**]. The length, width, and area of the rectangle should change while the perimeter remains 6.00 cm. Locate the maximum area. Unfortunately, or maybe fortunately for forcing students to think about it, the length of the upper subsegment jumps from 1.48 to 1.52 cm. In both cases the area is 2.25 cm^2.