"What do we want them to know?"

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Introduction

"What do we want them to know?" is a question we don't ask ourselves frequently enough. When we teach a college algebra course (the subject under consideration in this paper), the students are learning something, but what are they learning? This paper investigates the different things we teach our students, which of these things is most important, the role that DERIVE can play in helping us teach the more important things (I have included personal notes on a radical experiment in teaching using DERIVE), and what obstacles still lie in our way of maximizing our effectiveness as teachers.

When we teach college algebra we conceivably could teach them many things:

1) symbolic manipulation: When we ask them to solve, factor, graph or simplify using traditional mathematical techniques, we basically want to see if they can move certain symbols around in the right order using the right rules to get the right answer.

2) conceptual knowledge and critical thinking about different mathematical ideas: The brave teacher will ask students questions where they have to write words that explain ideas. For example, "Give the definition of a function as well as an example of an equation which is not a function and explain why it doesn't conform to the definition." or "Can the graph of a rational function ever cross a vertical asymptote? Explain your answer." or "Without drawing a picture and without solving any equations, what is the most number and least number of times the graphs of $x^5 - 2x^3 - x + 7$ and $x^4 + 2x - 11$ will intersect? What is your reasoning?" Aside from having to wade through poor writing skills, some instructors don't want to have to assess the student's imperfect ability to explain concepts or illustrate ideas.

3) applications: We usually use this as a primary motivator for our students. "You better know how this models real world phenomena or you won't do well in your next class or at your next school or at your next job."

We normally don't ask students to prove things rigorously in a college algebra class, although that could be a different type of learning activity that we could add to the list.

What's Most Important?

Of these things that we teach them, which is most important? If we go by the amount of time we spend in class on the topic or by the percentage of the test that requires them to display knowledge about these issues, we would have to answer the question by saying, "symbolic manipulation". Few teachers compose tests where applications and conceptual questions comprise the majority of questions. Tests such as these are hard to make, hard to grade, and usually not well received by students.

While symbolic manipulation is important, gives good mental exercise and provides students an opportunity to see how the crank turns, is it the most important thing? If DERIVE can do the symbolic manipulations, doesn't this suggest that perhaps one ought to give more emphasis to conceptual thinking?

Examples of Poor Concept Formation

Most of us can come up with countless examples of frustration regarding poor student understanding of ideas even though symbolic manipulations may have been done reasonably well. My favorite example involves rational functions. Quite often a student will identify the asymptotes and intercepts correctly and then draw the graph of a rational function, but there will be no logical connection between the information they've gathered and the graph they've drawn! They may have identified an x-intercept to occur at say $x = 3$ and even have a "dot" on the x-axis at 3, but their graph does not come anywhere near this x-intercept and may never even cross the x-axis.

Once (after we had covered rational functions - or at least I thought we had!) I asked my students to locate the point(s) where a given rational function intersected its own horizontal asymptote. The responses troubled me. Some said a rational function could never intersect its own horizontal asymptote. Others did not know how to find the horizontal asymptote. Those that could, did not know how the point(s) of intersection could be found. One person knew they had to plug in the value of the asymptote for $f(x)$, but did not know how to proceed. Another knew they had to find the value of x that made the equation true, but didn't know what that mathematical process was called, or how to proceed after that. Yet many of them could graph that rational function with its graph crossing (oops! - how could that be?) its own horizontal asymptote!

Radical DERIVE Experiment

Suffice it to say that our students sometimes do symbolic manipulations correctly without understanding much of what their results mean, or why they did those manipulations anyway. DERIVE has afforded me the opportunity to perform a radical experiment in teaching with my college algebra students. In an effort to isolate this deficiency in conceptual understanding and critical thinking, I taught the first 5 weeks of a college algebra course without a textbook. We talked about terminology, definitions, concepts and critical thinking. We used DERIVE to do every symbolic manipulation. We graphed functions, solved equations, found inverses of functions, solved systems of equations, and performed many other symbolic manipulations with DERIVE, relating many different concepts in the process. During this first 5 weeks they took many vocabulary and concept quizzes and did many laboratory assignments using DERIVE.

Some of these vocabulary and concept questions would be very simple such as, "What is the domain of the natural logarithm function?" or more complex such as "Can a non-linear system of equations have no solution? Either explain why it couldn't happen or give an example of such a system and explain why the example has no solution."

These quizzes were accompanied by laboratory assignments where the student would use DERIVE to perform some sort of action to arrive at an answer, but quite often they were asked to explain what the answer meant. For example one of the lab questions might be "Use DERIVE to graph $y = x^2 + x + 1$; then use DERIVE to solve the equation $x^2 + x + 1 = 0$; then explain how these two problems are related." Another example would be to give them a 4th degree polynomial with a non-obvious multiple zero. I ask them to write down how many zeroes the polynomial should have. (They all write down 4. At least I taught them something!) Then I ask them to use DERIVE to find the zeroes. Then I ask them to explain the apparent contradiction (Using DERIVE can help here also! If they don't know what to do, I encourage them to factor the polynomial.)

During the second 10 weeks we used the book and worked out all the symbolic manipulations with the traditional approach, with students doing most of the work on the board. I never lectured once in this second ten weeks. Armed with the concepts, I told them to read the book and work out the problems. Students were directed to ask the presenting students if they didn't understand something. During this 10 weeks they were still doing lab assignments using DERIVE, some of these lab assignments being reshuffled examples from the first 5 weeks. This repetition turned out to be valuable.

I enjoyed limited success with this approach. Many of the students latched onto ideas that they never would have before without the exposure to concepts that DERIVE affords. Some students never caught on and were intimidated by the technology as well as the fact that they were going to actually have to know and explain some mathematics.

In the future I would reduce the amount of time at the beginning devoted exclusively to concepts and DERIVE demonstrations. The students were quite anxious about not doing symbolic manipulations. I may consider integrating the DERIVE demonstrations on an as needed basis per chapter rather than all at once at the beginning.

How the Students Benefited

Aside from the intended result that students would acquire better concept formation with this “radical” approach, DERIVE was helpful for non-obvious reasons also. When my students saw that most everything that could be done during the semester could be done by DERIVE, I sensed that it became more obvious to them that they really needed to know answers to the "how" and "why" questions (in order to distinguish themselves from the software). DERIVE also seemed to evoke an attitude in the students of "I know all of this can be done because I've just done this on a lab assignment. If this machine can figure out how to do this problem, then so can I!" And perhaps most importantly the students developed a keener sense of ownership and responsibility in the class.

How the Teacher Benefited

Most math teachers think they are good lecturers. Some of them are correct in this assumption and some are not. I think I am a good lecturer, but if my students don’t think I am or if they are not learning, of what value is it? It was difficult for me to adopt this new approach because I was giving up a fair amount of control in the classroom and this made me uncomfortable. However I am so satisfied in seeing students learn in this non-traditional way, that I don’t ever want to lecture again – at least not using the traditional approach of presenting problems in their full glory using symbolic manipulation. Let DERIVE do the dirty work and they can learn the essential steps of the process from their book. If there’s something they don’t understand they can ask me.

Additionally I benefited because I was able to ask those conceptual questions that I had always wanted to ask but couldn’t because I didn’t have the time to teach the concepts and still get through the traditional material involving symbolic manipulations. DERIVE afforded me the opportunity to teach and test these concepts and still have the time required to finish the material proper.

Final Thoughts

While this approach was satisfying to me (and a culture shock for the students), I always wondered what would have happened if I could have spent the entire 15 weeks doing what I had done in the first 5 weeks. I could have done some neat, challenging topics. Of course the main reason we can't do this, is that we want them to be able to do the symbolic manipulations so they can do well in their next math class. Could we develop an experimental college algebra track that concentrates more on concepts than calculations? ... and eventually work it into the curriculum? Would this be wise?