THE CHALLENGE OF TEACHING WITH TECHNOLOGY
A Personal Reflection

Chris Barling
Swinburne University of Technology, Melbourne, Australia
cbarling@swin.edu.au

This is a paper based on some unusual personal experiences, and on the evaluation and reflection that has followed from it. It is not a typical piece of scientific education research, and carries no scholarly apparatus of references and cross-references; but I present it nevertheless as a work of scholarship in the traditional sense, as the fruits of a substantial period of reflection on my work as a teacher and of discussing it with valued colleagues. I think the points I make are well worth making, and want them to form part of a wider debate which I believe we all have to enter into as a matter of some urgency.

1. An Experience with CAS

My experience of teaching with a CAS was a rather unusual one. My School has always tried to place itself at the forefront of using technology in teaching mathematics, and in 1991 we became one of the first Australian universities to introduce the use of graphical calculators in tertiary mathematics. In the next year the University opened a second campus on the outer fringes of the city, and in order to meet the difficulties of its remote situation the University embarked on a program of what came to be called "multi-modal education", which envisaged students studying sometimes on the central campus but also often at home or at local study centres, linked to the University and each other electronically. A clear prerequisite was for each student to have modern computing facilities, so for the pilot years each student was equipped with a laptop computer to be used both at home and in class. The program selected for the pilot operation was the one I teach to students majoring in (applied) mathematics and computing.

The main program continued on our central campus, and one requirement of running the two programs in parallel was that the curriculum and assessment had to be as close as possible between the two campuses. On one we had a large group of students studying with the use of graphical calculators; on the other campus we had a small group of students required to study the same program but equipped with powerful laptop computers. Setting up the multimodal students with graphical calculators as well seemed ridiculous, so we decided to make the most of the opportunity and give them a package that would at least replace the graphical calculators, and probably do much else as well. In the end we obtained a licence to give each student a copy of Maple.

I have written elsewere [1] in greater detail about the program and its evaluation. In this paper I want to concentrate on the issues of curriculum that were highlighted by this unusual set of circumstances.

Even with the official requirement that the two streams be kept parallel it was clear that the program had to be thoroughly rethought. As a simple illustration, it is pointless to spend a whole class on partial fractions and another on doing integrals using them when every student has a machine in front of them that gives them the answer directly. If I tried that, and then showed the students the CAS commands to perform the task, the usual response was irritation (at least) that we had "wasted" all that time.

What I found myself doing instead was one of two things. Sometimes I would introduce a concept first through manual examples, develop a small amount of theory, and then look at how the CAS dealt with the concept. This I found to be particularly appropriate in problems where the nature of the problem is itself not immediately obvious, such as with differential equations or the inverse of a matrix. In other topics it was more interesting to use the CAS for an initial exploration and then gradually develop a theory to "explain" what was happening, with the CAS often also helping in developing the general theory.

In 1992 by far the most common approach to teaching with Maple was to give the students a number of worksheets that were to be completed in a laboratory situation, largely on the basis of copying or adapting given commands. In our situation, though, the students would be using Maple all the time, independently at home as well as under my supervision in class, so it was essential that they learn enough of the syntax to be able to manage on their own. This was a more difficult path, not least because the syntax is so strict and unforgiving.

Where I was not able to follow my instincts was in the selection of topics that form the curriculum: here I was obliged to work through the same list of things in the same order, and to assess them as far as possible with the same assignments and tests. This latter was indeed a problem, because there are clearly many tasks that are quite substantial for students using pencil-and-paper and calculators but almost trivial for students with a good CAS. My usual solution here was to use the same questions but give the CAS students fewer marks for getting the same results, and give them additional tasks or questions that required them to display their understanding or their knowledge of the syntax of the package.

The program ran for three years in all before being closed. I was able in that time to make both formal and informal comparisons between the two groups, and to survey the Maple students to some extent as to how they found the experiment.

2. Some observations and conclusions.

Every time a new technology is introduced we hear first from its advocates and proponents, eager to tell us about the wonderful things that can be achieved with it. We then hear from others who worry about the effects of the proposed change and are concerned at what will be lost as a consequence of its introduction. On the one hand we have an image of a world transformed; on the other one of loss and destruction. My experience has usually been that neither of these positions is realistic, and with this group of students in particular I certainly found that the outcome reflected neither position.

They did not burst from their shells as super-students, able to investigate hard problems and perform complex tasks without assistance. On the other hand they did not lose such understanding as they had by using the technology, a frequently expressed fear. I designed short tests to measure comprehension of key ideas without reference to the use of particular technologies, and the two groups — those trained with graphical calculators and those who had learned Maple — showed about the same level of understanding.

My own observations are that for the bulk of students, the "ordinary" student, using a CAS made very little difference to the bottom line of how reliably they can solve a simple and routine problem. It did make a number of differences, sometimes major differences, along the edges: the better students were able at least some of the time to do interesting things, and the weakest students, who without help would have achieved little or nothing, were able at least to get something done, and often to produce right answers. There were no blank pages.

Given that I wanted the students to learn the package and its syntax rather than just copying model commands, the hardest stage of all was right at the beginning. For students who have grown up with standard electronic calculators there is a serious conceptual difficulty when they first meet an algebraic processor, when the machine for instance tells them that • 2 is • 2 rather than the expected 1.414213562 .... We had to begin with an exploration of the difference between exact and numerical work, and why the machine prefers the former. Then we had to spend a lot of class time on the details of the syntax, of the complications of entering expressions and functions, the ways of browsing through and editing a session, and a great many other things that amount to learning the package and its language rather than learning mathematics. This is unavoidable: a broad observation is that the sort of student that has to work hard to learn the standard techniques and still has difficulties becomes the sort of student that has to work hard to learn a complex language and still makes many errors with the syntax.

3. The challenge of curriculum change.

It will be clear by now that I am an advocate of revision and change in the traditional curriculum, but I have not yet spelled out exactly why I take this position.

First, and most obvious in the context of this gathering, is the imperative force of new technologies. After several centuries of little perceptible change in the physical way mathematics is done, we have in the second half of this century witnessed a total revolution in practical everyday computation, both in everyday hand-held devices and in computers and the packages written for them. These have changed forever the sorts of knowledge and behaviour that will in future be required of mathematicians and of users of mathematics, from shop assistants to astrophysicists, and it as a process with no foreseeable end or even limiting behaviour. We have to imagine and deal with a future of continual and accelerating change.

Secondly, demographic and social changes have brought into senior mathematics classes a large body of students who in past decades would never have dreamed of such achievements, and in many cases are poorly prepared or motivated for the traditional study of mathematics. For most of them the standard curriculum is both unrealistic and irrelevant. We can't pretend that the world has not changed, and are obliged to provide a curriculum that suits the real needs of all of our students, not just those suited to the old ways.

"Curriculum", of course, covers more than just the list of topics we teach. In its broadest sense it includes all the things we want our students to learn, to know, to understand, to be able to do; it describes the circumstances in which and with which they are to do them, and the ways we are going to measure or assess those skills and understandings. Broadly, I think, we have got the list of "big" topics more or less right, but perhaps not the order, and certainly not all the details. The first set of questions for debate are to determine what topics are "in" or "out", and the emphasis each topic is to receive in the overall picture.

The circumstances of learning, of course, include the technological medium that the students (and teachers) are to use in demonstrating the skill or the knowledge they have acquired. "Technology" can include any of the things we use to do mathematics, from pencil and paper (or chalk and blackboard etc) to the various machines we compute on, and all the other tools we use, including tables of various sorts. Doing a calculation mentally is quite a different process from doing the same calculation on paper, and different again from doing it on a calculator or computer. Furthermore we have to consider that different means might be used on different occasions for the same type of problem. The "normal" way we want our students to use is going to be the most important in their understanding, and that will be affected by issues of portability, availability and expense.

The third big parcel of issues concerns the students themselves. Quite apart from the fact that there are more of them than aver before, we have to recognize their different levels of need for mathematics, and different levels of motivation, from "real" mathematicians to technicians who will use some mathematics, technicians who won't use much at all, and the vast bulk of students who for whatever reason have ended up in our classes but will in later life have little "use" for what we are teaching them. Independently of this, to a large extent, are their different levels of ability and of intelligence, and their different levels of interest in and curiosity about mathematics. Probably only a minority have a "natural interest" in the subject, with the bulk choosing mathematics for less-than-ideal reasons — as a prerequisite for further studies, to maximize options, for financial reasons. Even those with a real interest in the subject and a real talent for it will often find they cannot devote to it the time we would wish, because of legitimate competing interests.

And there is a fourth set of issues that cannot be ignored, which go under different labels but are basically economic, in that we are all under pressure to deliver our "products" in more efficient and (they hope) cheaper ways. In my country there is a general trend to unify curricula between states and within institutions which is being pushed from at least three different directions: an economic imperative to cut costs and eliminate duplication, a vaguer social perception that differentiation leads sooner or later to inequity and is therefore better avoided, and a commonsense desire to promote co-operation and both local and national unity by avoiding differences where none are necessary. Not unrelated to these pressures are the additional forces pushing us into delivering our courses with fewer resources and in less class time: where educational reasons are given for such changes they are usually implausible.

All of these trends and pressures are working together to present us with a single enormous challenge: to design a useful, interesting, and mathematically sound curriculum that will satisfy the varying needs of students of different abilities and interests, will stimulate the "good" students without distressing the "weak" students, will provide opportunities for any student that wants to learn, when they choose, and will make optimal use of the best available technology to help bring this about!

I want to emphasize one further general point. When I talk about developing a new curriculum I am not imagining a five-year process leading to a document or a set of class materials that will solve all our problems. It is obvious from looking at just the last ten years that we now have to accustom ourselves to continuous, endless change in technology and hence in the details of what we deliver and how we deliver it. Anyone who has recently tried to write prescriptive materials related to one model or one genre of technical equipment, or one version of a computer package, will know how rapidly such things now become obsolete. If you take my point about how intimately curriculum depends on the technology available, it follows inexorably that curriculum itself must now submit itself to continuous and perpetual change and revision. If we used to think of curriculum in terms of a document or a book, we now have to think of it as a process, a continual cycle of reflection and review, where each year builds on the experiences of the previous year and develops them in the light of the latest technological advances.

4. Curriculum fossils: three examples.

(1) The example most often quoted of an algorithm once taught but now no longer necessary is the manual method of extracting square roots, which "looks like long division but more complicated". I agree with the diagnosis, but suspect we should go further, and look at long division itself. In my country it is dying of benign neglect: while no-one seems to be advocating its removal from the primary syllabus, it has been de-emphasized to the extent that many teachers in effect ignore it, and their students seem not to notice any deficiency. For any division more complex than dividing by a single integer (or perhaps 11 or 12) they will reach for a calculator. So would I.

Tertiary mathematicians usually respond to this by saying that students "need" to study long division in primary school because otherwise they will not be able to divide polynomials in upper secondary school. To some extent I agree, for now, but TI-92 users will see the obvious next step: when portable algebraic processors are as commonplace as electronic calculators, that argument has no further validity: one divides polynomials, as one divides numbers, mentally or by hand when it is easy and by machine when it isn't.

What in fact is the value inherent in the long division algorithm? Here is a simple example:

```
        1.802.....
137   | 246.92
        137
        1099
        1096
         320
         274
           460
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You can see quite a few things by examining the process. First, it assumes complete competence at subtraction and at multiplication at least by a single digit. Second, it also requires intelligent guessing and sometimes making a trial: in this example it is not at all clear until we do it whether the second digit of the quotient will be a 7 or an 8.

But given the combination of intelligence, the willingness to explore and guess, and total competence at multiplication, what is wrong with a simple trial-and-error hunt like this:

$137 \times 1 = 137$	$137 \times 2 = 274$	
$137 \times 1.8 = 246.6$	$137 \times 1.9 = 260.3$	
	$137 \times 1.81 = 247.97$	
$137 \times 1.802 = 246.874$	$137 \times 1.803 = 247.011$	?

The trial-and-error process is more transparent and easier to follow once one has mastered the key idea of what division is — what we are trying to do, in fact. A calculator can do the multiplications, or can tabulate the successive possibilities for each new subdivision of the proposed quotient, so that we can "easily" do a division without using the calculator's division key.

What, then, is the value of the long division algorithm? I think it has two values. One is that it is a brute-force method that does not require any understanding of the aim of the exercise: one just does it. Generations of students have been bludgeoned into learning the process without the faintest idea of what they are trying to do or of why this arcane process achieves it. I find that difficult to label as an advantage. The other value of the long-division method is that, compared with the simpler trial-and-error process, it is more efficient in computation. Once we know, for instance, that the quotient (in the example) is between 1.80 and 1.81, it is wasteful to work out the products with both 1.802 and 1.803, when we only need to know what integer times 137 is under but nearest to 320. "Wasteful", that is, in terms of multiplying more places than we need, which is an important issue in hand computation but not really a problem for a machine (unless we are overloading its capacity). It seems to me that long division is a time-honoured process for making the successive guesses at places of the quotient as efficiently as possible, and for removing from consideration the net effect of what we already know, before moving on to consider the next place. It is state-of-the-art technology for hand computation on paper. But I question its value for students equipped with modern calculators. The square root algorithm is an almost identical case: the algorithm is a process for removing the net effect of the places already determined before examining the choices for the next place. Both algorithms are important for programmers, and for those wanting to push a given machine past its normal limits of accuracy, but do not seem to me to have any positive value for ordinary users, only the negative one of obscuring the process and to some extent making it available to those who do not understand its purpose.

(2) If I present you with a cubic polynomial like $f(x) = x^3 - 6x^2 - 13x + 42$ or $g(x) = x^3 - 6x^2 - 12x + 42$ and ask about its factors, what is your instinct? What would your students do? Those with a TI-92 would probably just enter the polynomial into a "factor" command. What would you do if you only had a graphical calculator, or nothing?

Most of my colleagues would start to hunt for integer solutions, like ±1, ±2, ... It turns out that f(x) has a root at x = 2, and if we then divide it by (x – 2) we get $f(x) = (x - 2)(x^2 - 4x - 21) = (x - 2)(x - 7)(x + 3)$. g(x), on the other hand, yields no apparent roots. Most of my students who did not baulk at the problem would tackle it by similar processes.

In the absence of a symbolic processor but with a graphical calculator I would suggest that the most intelligent first step is to examine a graph. The graph of f(x) "clearly" has three intercepts at x = –3, 2 and 7, and those who are (properly) sceptical can check that these apparent roots are genuine, by a substitution. So the factorization is done almost at once. The graph of g(x) equally clearly has three non-integer intercepts, and some elementary knowledge about integer coefficient polynomials tells us that they will be irrational, so there is no point in looking for exact rational roots.

So what is the value of the traditional hunt-for-integer-solutions-first method? In carefully selected examples it yields roots and factors; in others it fails entirely, and there is no way of telling at first which examples will work and which won't. Examining a graph is more reliable and promotes more understanding of the overall picture; using a CAS just produces the answer without comment. Students think the special method is the principal one, because for most of their lives they only see carefully constructed examples where it works. I think it has value as an instructional tool for beginners in the field, but I think we also need to move on and become more realistic a lot sooner than we do in most curricula — a step that is really only possible with technology.

This imbalance become even clearer in the world of post-secondary mathematics where I work, where the typical problem is not to factorize the cubic as such but perhaps to antidifferentiate a rational polynomial like

$$(2x^3 – 14x^2 – 53x + 121) / (x^3 – 6x^2 – 13x + 42)$$

To "solve" such a problem requires:

- dividing by the denominator to get 2 + a proper fraction
- factorizing the denominator
- splitting the result into its partial fractions
- integrating the resulting terms

and possibly then combining the result into a single logarithmic expression. The fact that the partial fractions work out to be

$$2 + 1/(x – 2) + 2/(x + 3) – 5/(x – 7)$$

tells us that this is no random example: it has in fact been carefully constructed, starting with a "friendly" answer and working backwards.

In the traditional curriculum this would be considered a difficult example by most students, simply because it will take a couple of pages of manual work, and a considerable amount of time; and for most students "long" and "hard" overlap in meaning. Of course, students with a TI–92 would just type in the problem and get the answer in a few seconds.

One of the real difficulties we face is that a lot of us who have become math teachers by choice actually enjoyed working at problems like this when we were students. There are certain sorts of mathematics that appeal to certain sorts of personalities, just like some people like crosswords or jigsaw puzzles or shiny engines, and others don't. And in many classrooms we have enthusiasts trying to cajole reluctant students into doing things for which they simply don't share our interest.

And we don't help the situation if they realize that the tricks we use only work in special situations. If I change some of the coefficients in the top line the problem is still doable, but the partial fractions now have nasty complicated fractions rather than friendly whole numbers. And if I change the "13" in the denominator to a "12", changing from f(x) to g(x), the whole nature of the problem changes: the TI–92 won't co-operate, and to get an antiderivative I will have to get numerical values for each of the roots and do an approximate numerical synthetic division. Suddenly even the enthusiasts aren't enjoying themselves any more.

(3) My third example of a curriculum fossil is much simpler: what is the sine of • /4 radians?

Without exception, every one of my colleagues and students to whom I put the question answered (when persuaded it was not a trick), 1 / • 2 . Is there a problem?

The TI–92, and Maple, and Mathematica, all give the response • 2 / 2. Why?

I believe that what we have here is a genuine mathematical fossil, surviving in the form of a fetish. When I was a student we were carefully trained to rationalize all surd expressions, so that • 2 / 2 was preferred to 1 / • 2 . What reason might there be for this? Algebraists have suggested that it is useful to make the transformation, because it demonstrates that the number in question is in fact part of the field of rationals extended by the square root of 2, but I doubt that that is of any serious concern to students of elementary trigonometry. I think rather that the reason lies in the history of computation.

Working from my trusty six-figure tables, I find that the square root of 2 is 1.414214 ; to then find its reciprocal I need to do a linear interpolation between the nearest two values in my table of reciprocals to get a best estimate of $1/\sqrt{2}$. It is much simpler, and much more accurate, if I realize that the number I want is just the square root of 2 divided by 2, which gives me 0.707107 directly. And the contrast is true in general: an expression with a surd denominator is numerically more complicated than one with an integer or rational denominator, *if I am doing the calculations by hand.* It makes no difference to an electronic calculator, or a TI–92, or a computer (except possibly in fine rounding errors, which we can adjust if important). Most of us would find it easier to get a quick fix on the value of $1/\sqrt{2}$ than of $\sqrt{2}/2$, I think. So why do all of the more sophisticated devices give us the latter?

5. Some further issues.

Which technology are we going to adopt? Some of us are in a position to prescribe a single machine for our whole class, which makes for quite pleasant teaching: you can be quite specific about how particular things are done, and get all your students working together when appropriate. Others of us have classrooms in which there is already a wide variety of brands and models, or of different types of machine, which makes it much harder to give useful instruction in notations or keystrokes or syntax, and implies a certain amount of independent learning, and very flexible teaching! In laboratory work students' experiences are more likely to be uniform, but I have never found that laboratory work is a powerful influence on students' understanding or abilities. The thing that drives their understanding is their "normal" way of doing things, and that means the portable technology that they carry with them all the time and reach for when in doubt. It is difficult to change.

If you are able to prescribe a piece of technology, it is not just a matter of going for the best available, especially if by that you mean the most sophisticated. It is possible to make a choice that is inappropriately complicated or works at too high a level for a particular group of students at a particular stage of their lives. With a really deep package like Maple or Mathematica, or even with a TI-92, there is a tendency for the bits our simpler students need to be swamped by the vastly more sophisticated sea that surrounds them, and a constant tendency for simple things to suddenly develop unwanted complications, like complex roots to polynomials or screens full of incomprehensible "exact" expressions. There were times when I was teaching with Maple when I had doubts about its ultimate suitability at first year post-secondary: its syntax is so complex, its notation so fussy, its operation so unforgiving, and there is so much to explain, so many hurdles to get over before you can do useful work, that I sometimes thought it beyond many of my first-year tertiary students.

Teaching good use of a piece of technology is often as hard as the alternative of teaching manually the mathematical techniques it performs, and to some extent you usually have to do both.

If you decide to adopt a particular piece or class of technology, I think you have to be whole-hearted about it. Too often we see old text books supposedly adapted to embrace new methods, but in reality just the old editions with added problems in the exercises. I think we have rarely followed through the implications of new technologies in terms of what skills they replace, or what new skills are required. Even the now ubiquitous pocket calculator, for instance, is used very badly by most students in my experience: they will write down partial answers and re-enter them (rounded); they will "simplify" expressions using flawed algebra before entering them, and they have little awareness of or feel for

errors and their likely magnitude. Many competent students prefer to work manually, at some cost of time and accuracy. Many students have been trained to feel slightly guilty at using a calculator, as if it is somehow "cheating". All of these are only reinforced as we move into graphical calculators an symbolic processors.
I would like to conclude with a sort of diagram that I find helpful in picturing where we find ourselves in all of this. Here is a model of how in a traditional curriculum we might explain the structure of a typical real-world problem:

TRADITIONAL APPROACH TO MATHEMATICS

1. Formulation of the problem.
2. Modelling of the problem in mathematical forms.
3. Analysis of the model to produce mathematical solutions.
4. Interpretation of the solution(s) in the light of the original problem.

Section 3 is where we do the routine manipulation of formulas and apply our knowledge of solving various types of abstract problems. It is where we put most of our effort as teachers, especially for weaker students. It is where the students put most of their effort, trying to master abstract processes they barely understand. At the same time it is probably the easiest part of the chart for most students, who find the chores of turning words into formulas or of interpreting their work extraordinarily difficult; and it is probably the easiest part to teach, because it is well mapped out and "routine". We might lament the lack of time to concentrate on the other parts of the cycle, but in practice we generally spend most of our time and effort in the manipulation of abstract problems.
The irony is that this is precisely the part of our curriculum that is simplified or replaced by technology. And because that is traditionally where we have spent our time, and where we feel most competent as teachers, that new technologies provide such a threat and such a challenge to us. Taking up that challenge is never going to be easy. But in the next century (and next millennium) we have unprecedented opportunities to take on wholly new directions in mathematics teaching, using technology to liberate us and our students from the unrewarding rigours of hack work and allowing really confident exploration of real and interesting problems. The enthusiasts have always said this would happen, but there has been little guidance or example as to how it might happen; and it cannot happen to any real extent without a major rethink to our curriculum process.
On the final page is my alternative model of this brave new world:

ALTERNATIVE APPROACH TO MATHEMATICS

1. Formulation of the problem.
2. Modelling of the problem in mathematical forms.
3. Use of appropriate technology to produce mathematical solutions.
4. Interpretation of the solution(s) in the light of the original problem.

To which we can add two ancillary boxes:

4a. Use of technology to check solution(s) obtained.
[always needed, but so difficult to get students to take seriously!]

and, for those who want it:

3a. How does it work?
Theory Details of the analysis
Possible technical complications Limitations of the process

In my ideal curriculum very little of the traditional material is discarded altogether, but much of it is relegated to one side. There will always be some students who want to know how something is done, and we should encourage their curiosity; good use of technology will let them explore it without much of the pain of the past. Some methods apply only to selected problems: they are still interesting, but in the wider world of real problems they will become something of a curiosity. We will find that from time to time some of the better students will want to fast track for all sorts of reasons, and also that some of the less obviously talented students will find something that grips their imagination; and with flexible learning opportunities and friendly technology we can help them learn some genuine mathematics without frightening them away through difficult manipulations. I think we might be surprised at some of the learning that will take place.

To equip our students for the future we have to provide them with a number of things. They will need a fair proportion of the traditional mathematical toolbox, mostly mediated through appropriate technology. They will need experience and some skill to choose from that toolbox an appropriate tool for each problem they encounter, and the confidence to make a guess and "have a go" when the choice is not clear. Students are not always comfortable with choices, and one effect of new technologies is to expand the range of choices available. We need to train them to deal with this, discussing availability, flexibility, portability, efficiency. Often there are several plausible and valid choices, and the decision may vary from occasion to occasion and certainly from person to person.

The most important gift I can give my students is confidence, which comes from the experience of success. In vocational education the aim is not academic perfection but a functional, competent professional. Beyond their basic skills and vocabulary, what our students need is the ability to learn a new topic, to know they can solve problems, to know what to do in an unfamiliar situation. Giving them maximum access to professional-standard technology surely has an vital role in achieving this.

Reference

[1] Barling, C. R. (1995) Incorporating Computer Algebra into College Level Mathematics, in *Innovative Use of Technology for Teaching & Research in Mathematics*, Proceedings of the First Asian Technology Conference in Mathematics, Association of Mathematics Educators, Singapore, 85–88.