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Do Computer Algebra Systems Change the Order in Which we Should Teach Mathematics?

Justo Cabezas (*) **Eugenio Roanes-Lozano (+)**
(*) I.E.S. Rodríguez-Moñino Badajoz &
Dept. Matemáticas, Fac. de Biblioteconomía, Univ. de Extremadura (Spain)
jcabezas@alcazaba.unex.es
(+) Dept. Algebra, Fac. Educación, Univ. Complutense de Madrid (Spain)
eroanes@eucmos.sim.ucm.es

Abstract

The contents of a *curriculum* can be ordered w.r.t. different criteria: epistemological, historical, psychological...We think that the development of CASs allows to change in a revolutionary way the ordering in which topics are introduced in the *curricula* of Mathematics. We are designing, implementing and testing a set of classroom experiences in order to try to prove that these changes of order are possible. How the content of the whole mathematical education (from Primary School to University) could be reordered is beyond the scope of this paper.

Introduction

For instance the computer (in particular a Computer Algebra System -CAS-) is a powerful tool that can make out for good the need of a deep knowledge of algorithms in the following cases.

a) In case repetitions of its practice doesn't add new capabilities. This way some time is set free and can be used to dive deeper into concepts and to dedicate more time to mathematical training.

There are some examples in the history of Mathematics teaching. Teaching the use of logarithm tables was a must until 30 years ago. Moreover, a long time was dedicated to master it. Since the spread of scientific calculators and computers, teaching it is out of the question. This proves that this skill has no value itself.

b) In case the concept together with all the mathematical processes needed are too hard to master for the student. The computer automates the mechanical part and let the student concentrate in the background idea. This way CASs allow to change the order in which Mathematics are taught. The CAS would be working here as an *elevator* to another level of the *House of Mathematics*.

For example the concept of area inside the circle is conceptually equivalent to the concept of “area under a more general curve”, that can not be introduced until much later (when the students master techniques of Integral Calculus, i.e., when they are more or less 18 year old). Therefore a CAS could be used as a black-box (Buchberger, 1992)(Heugl, 1997), i.e. an auxiliary tool if it is considered necessary.

c) In case the student doesn't know necessary topics (that are taught before).

c1) Because they are not implemented in all previous *curricula* that allow to study that particular subject.

For instance in Spain there are different itineraries to reach university. In some of them (access for adults, access for bachelors in another school...) not all the necessary topics are covered in real practice.

c2) Because the student didn't completely understand or has forgotten the concepts.

c3) Because the complexity of the knowledge needed to understand the algorithm doesn't allow its acquisition.

For example generating many classic curves (such as the catenary) can be understood, its shape can be recognised, but obtaining its equation is not easy in comparison.

This item c) is Bernhard Kutzler's well known *scaffolding principle* (Kutzler, 1996): a CAS can make possible for a student that hasn't completely developed all the skills in a level of the *House of Mathematics*, to begin at the next level without the constraints that this lack of knowledge would have imposed another way.

How All Started

The situation of the first author (teaching Mathematics to both Secondary School students and non-mathematicians undergraduates in University) is very interesting from the point of view of classroom research. It made him think about the possibility of introducing a first glint of some of the topics that are really hard to master for undergraduates from other schools different from the School of Mathematics even sooner (see 3.1).

The work has been produced in co-operation, and the experiences have followed the process below:

i) Determining the topic (taking into account its own interest and the rule of the CAS as a teaching aid).

ii) Development of a self-contained complete document (worksheet) for the students, (including examples, exercises...) and an examination plus a final "satisfaction test". The worksheets present the contents, detailing explicitly some of them, and direct the student in rediscovering the rest. Therefore, the teacher is liberated and acts as a supervisor and consultant. Except these differences they are similar to those in (García, 1994) and (Etchells, 1997).

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iii) Experimentation in the classroom (including the evaluation of the students by the teacher and of the teaching process by the students).

Experiences Already Carried Out

Up to now two experiences have been implemented, experimented and evaluated with the students of the first author. Another experiment is currently taking place.

3.1 A b-type Experience in a Secondary School: Negative Exponential Functions

In this experience (Cabezas, 1997) we try to prove that a CAS, DERIVE in this case, can make it possible to introduce topics that are usually in the university *curriculum* at secondary school level (*elevator principle*). Besides, some of these concepts are usually hard to understand by non-mathematicians**.**

This experience was carried out with 17 years old students. These functions appear in different fields: absorption and elimination of a compose of a medicament (Pharmacokinetics), survival of species (Ecology), radioactive desintegration (Physics), demand of scientific information (Bibliometrics)... The objectives of this experience were: knowing and detecting cases of this function, solving algebraic problems (given the equations), recognising Mathematics as a common language for problems in different contexts and showing how the Mathematics can be used as a tool for problem solving.

This is possible to experiment thanks to the flexible Secondary School Educational System of Spain, where a teacher can propose the content of a certain non-compulsory subject (the content of the compulsory ones is fixed by the authorities). In this case the experiment took place with three groups of students of the subject *Informática* (Computer Science). Two hours a week during two consecutive years are dedicated to this subject. There are 20 students in each group and classes take place in the computer classroom, where every two of them share a computer.

The environment is very positive. First, the level of these students is slightly higher than the average, because most of the students want to choose this subject, and those who have better marks have priority. Moreover, the general cultural level of the families is good. Second, they had already been using the computers for one year.

Only the DERIVE commands needed were introduced (Cabezas, 1994) with a worksheet that was prepared by the teacher and given to the students. They practised with it for 1 hour. As it has been said before, in these classes the teacher usually acts as a supervisor and consultant.

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Another worksheet introducing the topics mentioned above was prepared. Two hours were dedicated to it. Despite the fact that they had practised calculating derivatives, they had severe difficulties in interpreting geometrically the first and second derivative at a point.

Curiously, it was detected that some of them were surprised when adapting the model to real data (i.e. by the result of the process: *mathematical model* → *real world*). For instance some couldn't understand in the beginning how a non integer number could be obtained for a number of specimens.

We had already taught Mathematics with computers for several years. We knew of the increase in the students' interest w.r.t. traditional classes (one of the well known advantages of the "learning through discovery" approach in teaching Mathematics). Nevertheless, we have been really surprised by the extremely warm welcome given to this unit. Observations like "Why don't you always teach Maths this way?" or "Why don't we study interesting things like this in the Maths class?" were beyond our expectations. There was no complaining about unsuitability or unintelligibility.

The design, objectives and materials (hardware, software and courseware) received a very positive evaluation by the students and teachers. Particularly, the acquisition of contents was measured giving the students some paragraphs from undergraduate textbooks (elimination of an inert gas from the lungs, magnetic fields...). A good ratio of the students was able to read these paragraphs and to solve simple exercises derived from them. Moreover, there was a correlation between the results of these tests and the marks in the Mathematics subject.

3.2 A c1-type Experience a School of Librarians: Density Functions

This experience has taken place with 4th year students of the School of Librarians of the Extremadura University (Cabezas and Roanes, 1998). Almost all of them didn't really know or had forgotten the concept of integral. The experience was offered as an optative activity during the usual schedule. They all did choose to participate.

As in the previous experience a DERIVE introductory worksheet was given to the students, who practised with it for two hours. They learned how to plot a function and a family of functions and how to calculate integrals.

In the following worksheets, density functions of continuous distribution were introduced using DERIVE (thanks to its graphical capabilities and its excellent integrator). The definition and some of the properties of the density functions of the Normal, ji-2, t-Student, F-Fischer-Snedecor distributions were studied.

There were no major difficulties in working with the worksheets. Nevertheless, timing had to be adapted while the experience was taking place because of the big differences in the times the students needed. These differences were a consequence of their very different previous experience with computers.

A month later a very simple test was presented to the students in order to check if the lack of knowledge had disappeared. W.r.t. the objective, a reasonable level of success was obtained. The opinion about the design and implementation of he experience was very positive.

4 Conclusions

The experiences already carried out confirm our thesis in the experimented cases. In both experiences topics have been allocated in places different then usual. Our intention is to follow this line, deepening and improving the methodology, experimenting more examples of the same types, and of those that haven't been experimented yet.

5 Biographical Note

Justo Cabezas is a secondary school professor and he also teaches in the School of Librarians of the Extremadura University. He holds a degree in Mathematics. He has worked for several years in the use of computers in education (implementing experiences in Basic, Logo and DERIVE). Now he is mainly interested in the use of CASs in the classroom (the topic of his Ph.D. Thesis in Education, now under development).

Eugenio Roanes-Lozano is interested in Applications of Computer Algebra (to Artificial Intelligence, Automatic theorem Proving, Education...). He holds a Ph.D. in Mathematics and a second Ph.D. in Computer Science. He is a “DERIVE Consultant” and “beta tester” and has also developed packages now included in the regular versions of Maple V and Macsyma. He presently teaches in the School of Education and the School of Mathematics of the Complutense University of Madrid.

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