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# Learning About Graphs Starting with 3D

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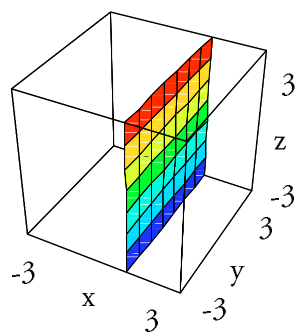
Traditionally, in the United States, people have learned about graphs by plotting points from ordered pairs, then a collection of such points that are associated with a particular function ($y = 2x$, $y = 2x - 1$, $y = x^2$, $y = \sin(x)$ and so on). Years later, if the students have survived classes in algebra 1, and algebra 2 and trigonometry and college algebra and two semesters of calculus then maybe some 3D graphs begin. As a result people who live every day in a 3D world have trouble with 3D graphs. In this paper I will suggest an alternate approach that I have tried and which I hope will be tried by some other teachers and students.

I will describe a sequence that I have used with students as young as 4th grade and with teachers of high school math. None of this could work unless I had help from proper technology which in this case is Cyclone 98, a WIN 95/NT program that combines power, speed, and is simple to use and which is programmed by David Parker. My procedure for presentation here will be a dialog between a student (I call Addison) and myself.

**Jerry**: I'd like you to make a graph of x = 1 in this program. I'll show you how to do this. Click on Restore, Double-click on defaults.mwc, go to Edit, erase the existing equation and type in x = 1 and click on Execute. Here is the picture you'll see




You can manipulate this picture by tapping on the arrow keys, Page Up or Page Down, and the Home key.

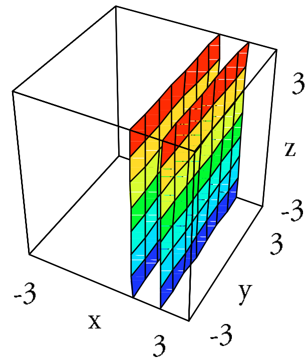
Now that you've seen the picture of x = 1 I'll ask you to guess what the graph x = 2 looks like.

**Addison**: How can I guess with so little experience?

**Jerry**: OK, enter the new equation and see.

**Addison**: It looks a lot like the old graph.

**Jerry**: Good. I'll show you how we can see both graphs at once. Enter both equations with a comma in between and parentheses around the two equations that tells the computer that this is a list.




**Addison**: Well, they do look the same ... one is moved over.

**Jerry**: Good, if you look at the numbers at the front of the graph it is indicating that x goes from -3 on the left to 3 on the right. Could you guess what the graph of x = 2.5 looks like?

**Addison**: Yes

Now we will skip the dialog and I will list the kind of sequence I often suggest. The individual equations separated by commas indicate graphs to explore one at a time. The equations enclosed with parentheses indicate a list of graphs which will be displayed at one time.

Sequence of equations to try and study and compare: x = -1, x = -2, y = 1, z = 1, x = sin(time/4), (x = 1, x = -1, y = 1, y = -1, z = 1, z = -1).

This next to last graph needs an explanation. Sin(x) goes from -1 to 1 and time is a built-in parameter which is proportional to the milliseconds since Windows was started so the graph of x = sin(time/4) is an animated picture of x = a number which varies from x = -1 to 1. Dividing by 4 slows down the motion. Now the floodgates are open. People can and should make up their own graphs and sometimes two people working together can help each other and share the excitement. After a while I like to check and see if this graphing experience allows the students to predict the next graph from its equation. Some can and some can't. More experience helps both groups especially if they are focused on developing the skill of predicting the results. I believe that if they can predict results with reasonable skill then they are learning. Otherwise, it may be a fancy video game with no math learned from it.

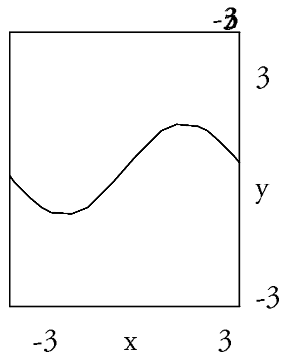
Many people create their own graphs with equations similar to the ones above. We all get imprinted quickly. So, like many teachers, I want my students to see the wider world. Back to the dialog.

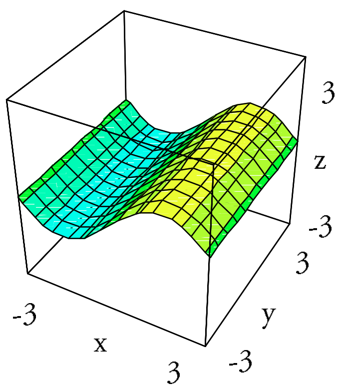
**Jerry**: Try y = $x^2$. I'll show you how to enter it. It is the same as y = x*x.

**Addison**: OK but let me finish these two I'm doing now.

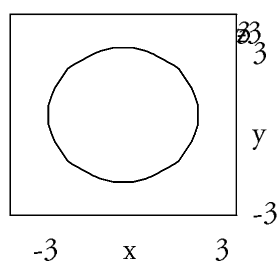
**Jerry**: OK

Everything I want to do in 2D is now available in 3D. If I want to do trig graphs I do y = sin(x) with perspective turned to false and view set to top. Of course I can also do z = sin(x) or $z^2$ = sin(y)



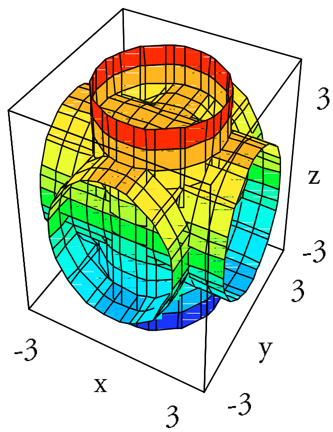




If I want to study circles I look at $x^2 + y^2 = 4$ from the top with perspective false.

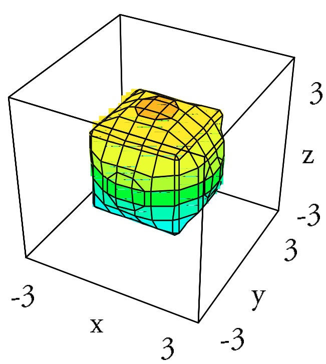



If a viewer of this graph in Cyclone 98 taps an arrow key they will find that their circle is a cylinder that we have been looking at end-on. Why is this? Well, we said $x^2 + y^2 = 4$ and said nothing about z so it could take on any values. What would the graph of $y^2 + z^2 = 4$ look like? What about

$(x^2 + y^2 = 4, y^2 + z^2 = 4, z^2 + x^2 = 4)$ )?

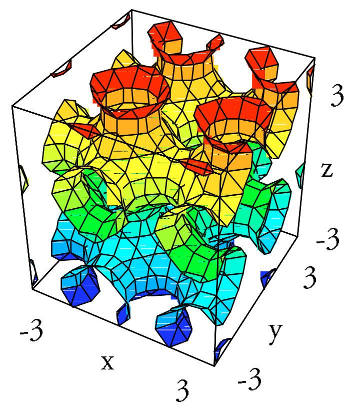



What about the intersection of these three graphs?

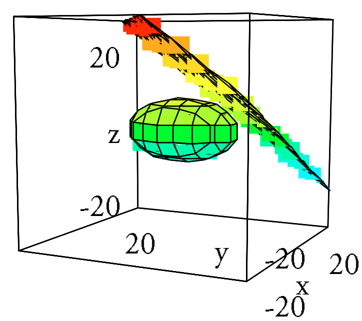



We also have four parameters available so the student can have some direct control. What is the graph of ax + b = c? How does the graph depend on a and b and c? Each can be controlled independently with a slider and the result seen in real-time. The same is true of $x^2 + y^2 + d = z^2$ , as d changes from -1 to 0 to 1 we see the hyperboloid of one sheet change to a cone and to a hyperboloid of two sheets.

We can also do graphs I never thought to try until a few months ago. Graphs whose equations are so simple that the only explanation for my not trying it was a lack of experience in 3D. The graph I tried was sin(2x) + sin(2y) = sin(2z). I was shocked by the result. What does the reader predict for the graph?

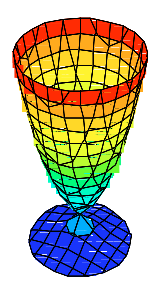



We may have a chance to rethink some traditional topics in the math curriculum if we have available appropriate 3D technology. A typical application comes from Larson, Hostetler, and Edwards Calculus 5th edition on page 918: Find the minimum value of f(x,y,z) $= 2x^2 + y^2 + 3z^2$ subject to the constraint 2x - 3y + 4z = 49. The solution uses Lagrange Multipliers and three variables. In our 3D language we are looking for the point of tangency of these two surfaces for a particular value of f(x,y,z). We can manipulate the viewpoint in real-time, and by setting f(x,y,z) = d and then changing the value of d with a slider we can make a decent answer by eye.




As with most suggestions for alternate approaches the best solution is often to do both approaches rather than one or the other.

I'd like to end with an aesthetic contribution of a Swedish wineglass by David Sjostrand:



The equation is: $x^2 + y^2 = (\ln(z + 3.2))^2$ .

*Demo copies of Cyclone 98 are availble for downloading on the internet at (www.mathware.com)*