Contents

Assessment in the Age of Technology

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Three important issues that we are addressing this semester are:

1. What is assessment?
2. What is the role of writing in the teaching and learning of math?
3. How do we best integrate the use of technology into the math classroom?

In our efforts to address these questions we will review and give reports on various chapters of the N.C.T.M. (National Council of Teachers of Mathematics) yearbooks of the past decade (1988-1998) and also the journals: *The Mathematics Teacher*, *Mathematics Teaching in the Middle School*, etc.

First, we will review assessment procedures we are familiar with and then we will consider new modes of assessment. An effort will be made to integrate the assessment program into the learning situation and show that: "ASSESSMENT CAN BE MORE THAN TESTING."

We will design several classroom projects and show how working in teams (small groups), writing clear statements of the problem and final reports, and using technology can solve the required problem (when the need arises). These three related concepts will be demonstrated in several sample situations.

I. Finding Solutions to Non-standard Problems

The first example will give us an Algebra I concept review exercise. We wish to find ALL values of the real number x for which: $(x^2 - 5x + 5)^{(x\wedge 2 - 9x + 20)} = 1$. In the study of this problem students will need to refresh their memory of such related facts as:

1. $1^{(anything)} = 1$
2. $(-1)^{(even\ power)} = 1$
3. for $A \neq 0$, $A^0 = 1$

Having these, bases, exponents and powers firmly in hand, we are now ready to consider an experimental procedure to start the search for solutions.

In setting up this search, build a table of values with these headings:

X	$p(x) = x^2 - 5x + 5$	$q(x) = x^2 - 9x + 20$	$p(x)^{q(x)}$
0			
1			
2			
3			
4			
5			
6 cont			

Building tables or charts is a very useful procedure and most calculators and C.A.S. (computer algebra systems) have these readily available as a built-in feature.

In the above problem, finding the actual values of *x*, which satisfy the equation, is not the most important feature. More important are the questions:

1. How many such values exist?
2. How does each one arise?
3. How can we change the problem to cause the number of solutions to change?

Finally, the resolution of this problem is not complete until the students have written a detailed analysis of all of the questions raised.

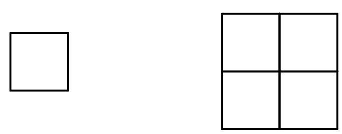
This example gives us a problem that required many of the N.C.T.M. standards to be addressed in the classroom situation.

This problem is based on a problem appearing in *Professional Standards for Teaching Mathematics* (NCTM March, 1991) p43.

We will develop several similar problems that require the use of all three concepts in our discussion.

II. Number Patterns from Geometric Constructions

Consider using toothpicks of equal length to form squares in the following manner:



Count the number of toothpicks in each square and in a 3×3, etc. Fill in the following table

Size	Number
0	0
1	4
2	
3	
4	
5	

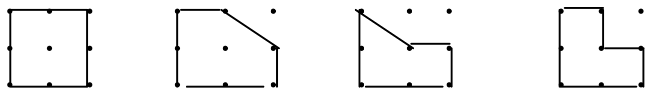
What pattern do you see in the second column? Can you prove your result?
HINT: Consider first and second differences

EXTENSION: Make a cubical lattice with gumdrops and toothpicks. Now what pattern do you see?

III. Peg Board Geometry

Place golf tees at each hole in a peg board. Place a rubber band around any number of the golf tees. This will form a simple connected region on the board.

Some of the golf tees will be inside the region, others will be on the border of the region, and still others will be outside the region. Assume that the area of the region can be expressed as an affine relationship of the number of pegs on the boarder and interior of the region. Find this relationship.

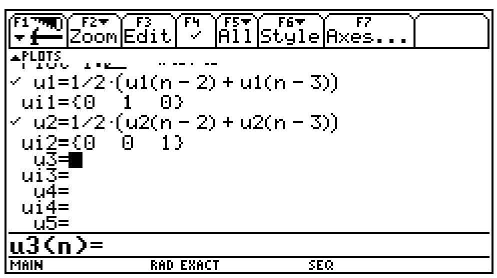


Assume that each square has area one and find the area of each of the above figures. Set up the linear equations for the number of pegs in each category. (see “Pick’s Theorem Extended and Generalized” by Christopher Polis (as an eighth grade student) in *The Mathematics Teacher*, May 1991 pp 399)

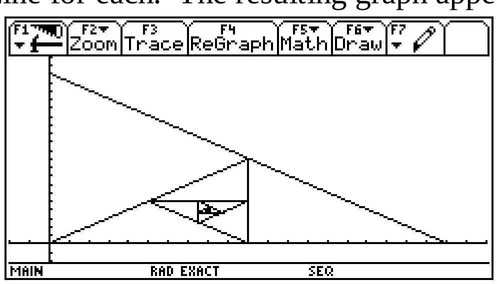
IV. A Geometric Limit

We begin by drawing a right triangle with vertices at (0, 0), (0, 1), and (1,0). Now we begin connecting (0, 0), the vertex at the right angle, to the midpoint of the hypotenuse, make a clockwise turn of 135^{o} and connect to the midpoint of the next side. This yields a new, smaller, right triangle. Repeat the process for this new triangle. Continue for several more times.

1. Show that the sequence of points that you generate as vertices of the triangles satisfy the equation $P(n)=\frac{1}{2}(P(n-2)+P(n-3))$
2. On your TI-92 use the sequence mode and enter the following recursive sequences.



3. Press $\boxed{\text{F7}}$ Axes , choose Custom. For the *x*-axis choose u1 and for the *y*-axis choose u2. For each of the recursive functions press $\boxed{\text{F6}}$ Style and choose Line for each. The resulting graph appears as:



It is obvious from the picture that the sequence of vertex points is converging to a limit. How do you know that this is true? Can you determine this limit?

V. A Solution Based on Graphical Observation

Consider the following situation: Two students go to a county fair and decide to play a game of pitch and catch with a baseball. They add to the difficulty of the game by having one of the students ride a Ferris Wheel. The Ferris Wheel has its center at (0,25) and is 40 feet in diameter. The wheel rotates at a rate of ½ radian per second. The first student is on the Ferris Wheel. The other student is standing at (28,0) with the ball being released at coordinate (28,5).

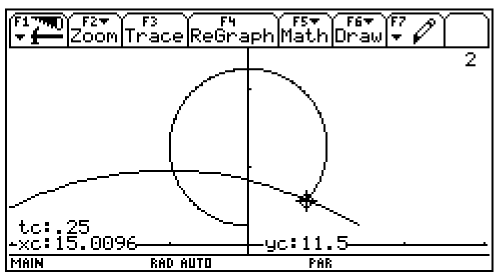
Here is the problem: The student on the Ferris Wheel is to catch the ball when located at a position making an angle of 45^o line of sight with the student pitching the ball and on the same side of the center of the wheel as that person. The student pitching the ball throws at a speed of 60 feet per second. Determine the angle, θ, at which the ball must be thrown in order to reach the student on the Ferris Wheel.

First task: Use Newton's Laws of Motion to compute the two component equations for the path of the ball.

$$x(t) = 28 - 60\cos(\theta) * t$$

$$y(t) = 5 + 60\sin(\theta) * t - 16t^2$$

Using this information we experiment to find the angle and time at which the ball should be released. We assume that we start the clock when the first student is at the point closest to the ground. Here is the result of one estimate for θ.



For this estimate we used $\theta = \dfrac{\pi}{6}$, which is close to the solution of the problem. Using the TI-92 we can test this solution and solve for the exact solution.