Contents

Is Less More?: Calculator as CAS in the College Curriculum

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How does a graphic calculator, albeit one with approximately the same processor as my first Mac computer, stand up as an accessory to college math classes? My use of the TI-92 over the past year in the top end of the sophomore math curriculum give a very positive answer. In most ways my TI-92 is now my technology of choice for those courses – multivariate calculus, differential equations, and linear algebra. In this paper I'll sketch some arguments on behalf of the thesis that the TI-92 is enough technology for these courses and the corollary that the TI-92 even has some compelling advantages over familiar computer algebra systems (CAS).

Comparison Conclusions: TI-92 vs. Big-league CAS

Not long ago we did a comparative study of available Computer Algebra Systems to determine the next appropriate software addition for use in mathematics instruction at our large suburban community college. At the same time we were beginning to use the TI-92 in classroom activities. I soon found the programs for the TI-92 which are available from the DERIVE website[11], and I began to use the TI-92 where formerly I would only have used a larger machine and more sophisticated software. This set up a competitive face-off, almost a David and Goliath competition, between the CAS software for networked desktop computers and the much smaller, much less expensive, portable TI-92 graphic calculator. Here are some conclusions I made:

- *The learning curve is easier on the TI-92 than for any big-league CAS.*
  Syntax is not as demanding, and students (and faculty) are already familiar with operations format for the TI-92. "Training" on use of the hardware and software may be done with very little effort and may not even be necessary.

- *Little pedagogical advantage is sacrificed.*
  On the TI-92 I have found almost all computational features needed while teaching these three courses. I have found few textbook problems for which the TI-92 is unprepared. This includes problems in the technology supplements which were typically designed to be solved on Derive, Maple, Mathematica, or MATLAB.

- *Convenience and portability are superior.*
  For a single user the size advantage is obvious, but what I have in mind here is the ability to meet the needs of a class without traveling to a lab or scheduling special space or without scheduling bulky A/V equipment.

- *Ready access may increase frequency of use, improvement of skills, and interest.*

I find myself using my TI-92 while I'm watching television, much to the disgust of my family, who usually aren't as fascinated by the problem I'm hooked on. Some students also are as enthusiastic about using their TI-92s, and the ability to use the calculator virtually anywhere is really an advantage.

•*Customized programs are easy to develop and are inexpensive.*
I have downloaded, copied, adapted, and written many useful programs which have been useful to my specific course needs. So far, I haven't actually purchased *any* software for the TI-92.

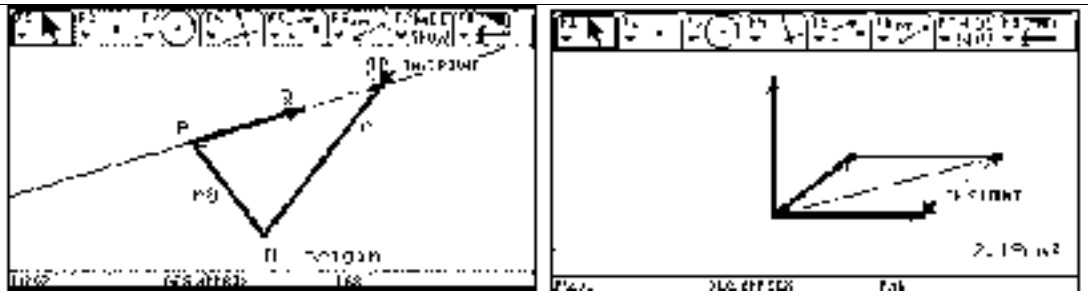
and the obvious observation,

•*Costs are less.*
The cost of the calculator itself often compares favorably with the cost of the student version of the CAS software. The cost of buying a calculator for every student in a small class is substantially less than the CAS site license for a comparable size lab. Add the hardware and there is a five to fifty-fold cost advantage.

Multivariable Calculus and Technology

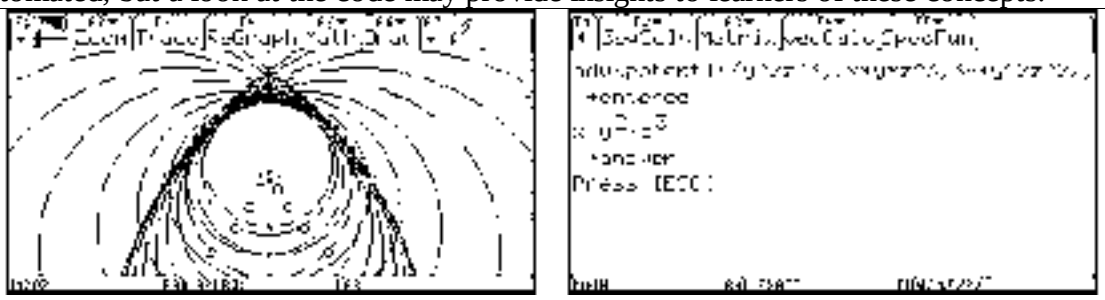
Specific examples of potential use of the TI-92 in Calculus III include vector manipulations, parametric equations, partial derivatives and multiple integrals, line integrals, and the basics of differential geometry. A CAS supports student experiences with all of these which foster conceptual understanding while freeing one from computational complexities. The act of entering functions and parameters into the calculator seems to reinforce the meaning of notation as well as concepts. While use of the calculator is frequently decried for allowing students to avoid mastery of some algebra skills, in this course we have seen a number of examples where an opposite effect occurred, namely better understanding was bolstered by the ability to display and experiment with advanced topics in the course. Here, using the TI-92, it has not been necessary to go to a lab nor to use specially prepared classrooms. Using a classroom set, students may use a unit either singly or in pairs during the class.

I have used the TI-92 to display dynamically the ideas of vector equations of lines and planes or the geometry of projections or cross products, sticking points for many students. I still use a hill-climbing extreme-value program developed by a student after he saw me use a script to demonstrate the gradient. One script may be used to calculate, step-by-step, all the differential geometry information of an arbitrary (well, almost) space curve, and the Frenet-Serret formulas may be verified, as well. This is material which I have rarely covered in such a course, but the concepts were easy to demo very quickly, while leaving students with a copy of the script and some confidence that these complicated ideas may be approached with the tools they have just studied. I wish I had that when I studied the subject as a graduate student!



Two Cabri displays of vector concepts. Dragging the designated point causes assciated vectors change to change lenge and position.

A program can be used to demonstrate concepts of curvature much more easily than with chalk or overheads and one may point at the locus of centers of curvature while it develops in plotting the circles. Finding the potential function of a vector field is easily automated, but a look at the code may provide insights to learners of these concepts.



Circles and centers of curvature

Finding a potential for a vector field

Our text for this course has been Anton's well-known and widely used book[2]. It includes about ten "Technology Exercises" at the end of each chapter, several of which are marked with a "green diamond" icon which is very similar to that on the TI-92 keyboard. That is supposed to label exercises "more appropriate for a CAS than a graphing calculator." *All* such problems in the chapters which cover vectors through Stokes' Theorem may be solved on the TI-92, which seems to say that this graphic calculator is sufficient for most of the technology needs of a multivariable calculus course. I suspect that all the technology exercises in the book likewise may be similarly solved.

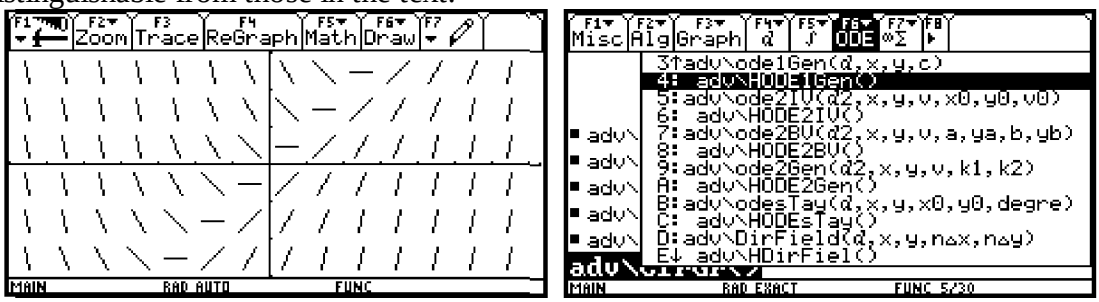
I'll send students to the lab for more support if they need it, as they will if I want them to see colorful or animated or detailed pictures of 3D vectors, implicit function surfaces, contour maps, and space curves, but can I do this without special instruction sessions for use on the software used? Even though the display capabilities are substantially improved for surfaces on the Plus, there is still room for improvement on the smaller "platform." Here's a place where Goliath still has it over David. One hopes for programs like AcroSpin or MPP3D on the TI-92 at some point in the future.

Ordinary Differential Equations and Technology

In differential equations there are a number of packages and programs which allow closed or numeric solution of most problems in a first course. Much of the "reform" in the ODE

course involves addressing the subject qualitatively as well as in developing skills to determine closed form solutions. I've taught this course every semester for three years, and have only occasionally found problems which drove me to the lab to consult Derive or some other software – either for analytic solutions or for conveying geometric solutions. Of course, I have bolstered my TI-92 with many programs, some found in conference notes, some found on the internet, and some even written by me. In some instances the computational speed of the calculator is a disadvantage, but in general the TI-92 is enough for most desired applications. The TI-92 Plus adds enough capabilities for this course that you may not have to add programs to meet your needs. See the cited review of the TI-92 Plus by Jean-Michel Ferrard[3].

I used the technology oriented version of the Edwards and Penney[4] text this year. Again it had special sections for technology projects. Again, the TI-92 was almost always adequate to the task. The slope field pictures which I showed in class on my TI-92 were indistinguishable from those in the text.



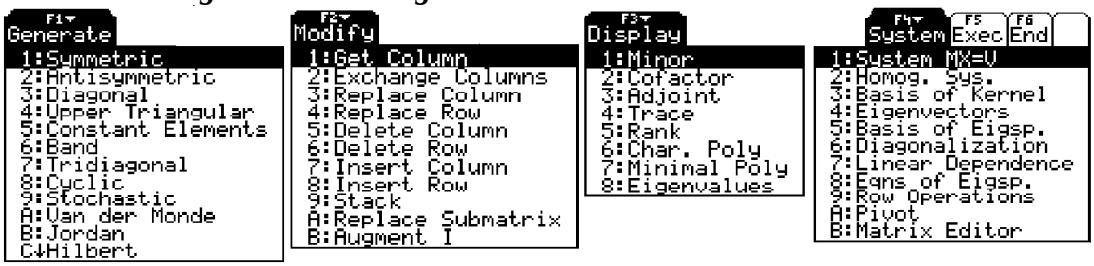
Screens from the DERIVE TI-92 package

The numerical solutions from Runge-Kutta and other numerical methods were easily implemented on the calculator. (Formatting the output for such a program is a little problematical.) Closed form solutions of all the second-order IVPs could be found using one of several programs I found, and a phase plane plot could be demonstrated adequately, but a little slowly. (It made me wonder if my favorite program of this type – written for the Mac – might soon be available on the TI-92.) A nice graphical demo of Fourier series was at hand. While there are some Laplace transform programs available, I avoided them and depended instead on the substantial algebra skills of the TI-92 to do partial fraction decomposition necessary for Laplace inversion. Graphing the solutions of problems with step-function or Dirac delta inputs was easily accomplished and displayed in class. Again, I was most satisfied.

Linear Algebra and Technology

The linear algebra course provides a setting where all of the content and most of the standard problems may be nicely addressed on the TI-92. Computational problems involving matrices and vectors of "small" dimension, many of the demonstrations which provide insight into vector spaces and linear transformations, and more sophisticated applications such as the standard matrix factorizations or even use of affine transformations to generate fractals yield to this little portable computing laboratory.

The built-in function rref() and other primitives of the language on the TI-92 provide all that is necessary to support technology needs in a course using matrices with about six or fewer rows or columns, no restriction on getting the basic ideas across. I used a new text this year, one which has a number of exercises labeled as "do by hand" or "use technology," and there were no cases when I yearned for more technology than that provided by my TI-92. Indeed, in this case I took calculators for each student to class for use in virtually every class session. The illustration shows a partial list of some functions which were available from a program, MATRIX()[3], which I loaded for each student machine, and I am sure that, while the power of the calculator was adequate, the advantages of use in class were superior to what we could have done using other technologies.



Menus for the MATRIX() program translated from Ferrard

Comments on Logistics of Using the TI-92

A terse comparison of the logistics of the TI-92 portable lab approach with more "powerful" technologies is: more portable, more easily expandable, less bureaucratic, equally demanding of planning details, and greater sense of control. Appropriate classroom support matters including program sharing and distribution, machine sharing and distribution, printing, and access to printed or internet sources are addressed with less effort than for computer lab use, especially in settings with limited technical support personnel available.

Many, if not most institutions, still face severe resource problems in providing sufficient technology support for labs, general student access, and faculty development. Placing the technology in the hands of students and under their control in the form of a TI-92 calculator almost eliminates space demands and student access problems, while costs are drastically reduced in comparison to the use of workstations. Also, putting control of the computing environment in the hands of the users is often a good way of assuring that it may meet intended needs.

Some will argue that students must be provided with "professional" tools so that they are prepared when they leave our institutions. I agree entirely, even though, unlike computer aided design or spreadsheet use, it is not as clear as to what this might mean. But that may be done at the upper undergraduate level. The CAS systems which the TI-92 can almost displace early in one's college career were designed with different goals. They are general purpose "professional" math tools in terms of the intended scope of problems and applications to be solved using them. Some of the better known CASs have to be bent in

awkward ways to the needs of beginning students, often because so many options have been provided.

Take the promotional display piece of any CAS system, mark the examples which you may want students to be able to do in connection with your favorite course, and ask whether you or they could do the same on a TI-92. I have done that for Maple, Mathematica, and MATLAB, and the results were in each case: the same result could be obtained. Of course, I need to rule out examples which depend on the amount of memory (100X100 matrices) or super graphics capabilities to make this claim. But the ability to deal with the basic math ideas one encounters in the first two years of collegiate math for a math major or an engineering student are definitely there. And the transition to the professional tools will be easily made when necessary.

Let's teach students to use tools which are appropriate to the task at hand. When the primary task is learning, a minimalist philosophy of technology use seems appropriate. Such a philosophy is also less demanding of scarce resources such as space, capital expense for equipment and software, technical support, faculty expertise required, and student time. It may even find more support from our less-technology oriented colleagues.

A final note on a "professional" use of the TI-92, as a tool for one's own education, self-improvement, or "faculty development." A colleague recently asked me a question about the relationship between bases of two vector spaces and a basis of the intersection. I answered the question in terms of my recent familiarity with computations through my TI-92. Subsequently, I saw a short note published on just this problem. I was able to implement an algorithm described in the paper as a script. I doubt that I could have responded so "professionally" without my TI-92 and the interest in doing math with technology which it has nuttured.

So I now have a new hobby, one which is based on what is now a frequent experience which goes like this: I see an article in a journal such as American Mathematical Monthly, College Math Journal, AMATYC Journal, or Mathematics Magazine which includes some code for one of the more expensive Computer Algebra Systems or which describes an algorithm. Will it work on the TI-92? If so, it will usually be slower or the output will not be as slick, at least in the hands of this user, but, more often than not, the answer is *Yes!* And I will have accomplished the task more immediately and with a sense of satisfaction and reinforcement which probably is greater than that obtained from submitting the problem to a more powerful computer! And certainly cheaper! Here as in other examples given above, it is often the case that *Less is More*.

[1] [1] See www.derive.com and www.imaxx.net/~gdorner/TI92_paper. The latter is a review of the TI-92 software available from the former.

[2] [2] Anton, Howard, in collaboration with Albert Herr, Multivariable Calculus, 5th ed., John Wiley & Sons, Inc., New York (1995)

[3] See www.imaxx.net/~gdorner/TI92_plus for a review of the TI-92 Plus upgrade..

[4] Edwards, C.H., Penney,D.E., Differential Equations and Boundary Value Problems, Computing and Modeling, Prentice-Hall, Inc., Upper Saddle River, NJ, (1996)

[3] [5] Ferrard, Jean-Michel, TI-92: les programmes!,
Dunod/Texas Instruments France (1996)
(An English translation of the book and the programs will be available. Inquire at gdorner@imaxx.net.)