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Dimensional Analysis with DERIVE

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Abstract

Computer Algebra Systems (CAS) are not only useful for relieving us of boring and/or difficult algebraic calculations (and numerical ones as well) but more and more they are used within mathematics education to gain more insight, to focus on the basic concepts, and to open many fields of applications for students at secondary school level. Besides all the standard items including Precalculus, Calculus and Linear Algebra I have found *DERIVE*™ [1] useful to cope with several other fields in maths education. In this lecture I will focus on a partial aspect of physics teaching which from my own experience might have caused and might still cause problems for generations of secondary school students: working correctly with physical quantities and units. The ideas presented can easily be transferred to other CAS available. As there is now a CA-able pocket calculator one of the examples will be shown using that revolutionary calculation tool.

Introduction

I was inspired to solve word problems using a CAS by a short chapter in "Mathematik mit *DERIVE*" [2]. The authors´ idea of working with "word variables" and using the various prepositions for a "hidden" multiplication by 1 was so attractive for me that I tried to extend this approach. As I had been using *DERIVE* as an educational tool for a long time it was clear to me also to use *DERIVE* for this [3]. Mauve and Moos worked with measuring systems and used the Dot as multiplication operator. As a very short introduction I will present one little example. (The settings in line #1 are important and they are the default mode for all examples. You might prefer setting `TimesOperator := Implicit`.)

Example 0

```
#1:  [TimesOperator := Dot, InputMode := Word, CaseMode := Sensitive]
#2:  [cm := 10·mm, dm := 10·cm, m := 10·dm, km := 1000·m]    Definitions
#3:  3·km + 576·m                                         What is the sum?
#4:  3576000·mm                                                  Simp(#3)
#5:  1·m + 3·dm + 10·cm + 6·mm = How_many·m                          User
#6:  How_many = 1.406                                            Solve(#5)
#7:  5·km + 367·m - How_many·m = 4·km + 863·m                 Answer in m
#8:  How_many = 504                                              Solve(#7)
#9:  5·km + 367·m - How_many·km = 4·km + 863·m               Answer in km
#10: How_many = 0.504                                            Solve(#9)
#11: "We can teach DERIVE working with areas"
     •          2                 •
#12: •a := 100·m , ha := 100·a•                               Definitions
#13: 34.4·ha = (1·km + 112·m)·length                      Find the length
```

Contents

```
#14: length = 309352.5·mm                                    Solve(#13)
#15: 34.4·ha = (1·km + 112·m)·length·m                    Length in m!!!
#16: length = 309.352                                        Solve(#15)
#17: "We convert English measuring units:"                        User
#18: [inch := 2.54·cm, ft := 12·inch, yd := 3·ft, mi := 5280·ft]
#19: how_many·ha = (1·mi + 35·yd)·136·yd                          User
#20: "Simplify #31 and explain the result!"                       User
                              2                     2
#21: 10000000000·how_many·mm  = 204115411123.2·mm           Simp(#19)
#22: how_many = 20.4115                                      Solve(#19)
```

I will not explain how to work with *DERIVE* in this lecture, I will present results. But it can be said that neither teacher nor the students need performing as *DERIVE* experts to have success with the problems presented. When you follow the annotations you will find explanatory comments.

There is a strong demand from the physics (-teachers) to represent a **physical quantity** as the product **measuring number** × **measuring unit**. In particular you have to pay attention to the correct value of the units, their dimensions and to the handling of these quantities when you are going to solve problems. So when I remember my physics lessons - long, long ago in secondary school - I always have in mind my teacher´s words: "Check the units and their dimensions!"

If a student has to find the distance x covered by a body in t = 20 minutes with a given basic velocity v = 30km/h and an acceleration of a = 0.25m sec^{-2} he might fail and use the formula

$$x = v \cdot t + 0.5\, a \cdot t\,.$$

We compare the units of each side of this equation, with (LU = length unit, TU = time unit)

$$[LU] = [LU \cdot TU^{-1}] \cdot [TU] + [LU \cdot TU^{-2}]\;.\;[TU] = [LU] + [LU \cdot TU^{-1}]$$

which is obviously false. So check the equation and you will find the mistake:

$$x = v \cdot t + 0.5\, a \cdot t^2\,.$$

Now dimensional analysis will give LU as a unit for the distance covered. Unfortunately most applications are not so simple.

I think that one could use the advantages of a CAS to make clearer the connections between the physical quantities and their connections to the basic units. Some examples shall demonstrate this non standard application of a CAS. (I will not explain the physical background in detail.)

Example 1:

A house is to be connected to a 220Volt line which is in a distance at 1km. At an intensity of 15A the loss of voltage should not exceed 10% . What is the minimum diameter of the copper wire? Check the dimension of the copper´s specific resistance! (Specific resistance of copper: $\rho_{Cu} = 0.018\Omega\cdot mm^2/m$)

(The physical quantities and the well known physical units are spelled with uppercase initials, the measuring numbers with lowercase ones. Exceptions are mm, cm, m, km, kg and sec)

```
                        Spec_Res·Length
#1:   Loss_of_voltage = ••••••••••••••••·Intensity            The Word Formula
                         Cross_Section

      •                                        3              3            2•
#2:   •VU := Volt, LU := mm, IU := Ampere, m := 10 ·LU, km := 10 ·m, AU := LU •

#3:   RU := Ohm                                                             User

                     Spec_Res·(length·LU)
#4:   voltage·VU = ••••••••••••••••••••••·(intensity·IU)            Subst in #1
                           area·AU

                     Volt·area·mm·voltage
#5:   Spec_Res = ••••••••••••••••••••••••                  Solve(#4) for Spec_Res
                   Ampere·intensity·length

#6:   "Dimension: mm*Volt/Ampere. We use the fact that Volt = Ohm * Ampere:"

#7:   Volt := Ohm·Ampere                                                    User

                   Ohm·area·mm·voltage
#8:   Spec_Res = •••••••••••••••••••••                                   Simp(#5)
                     intensity·length

#9:   "Dimension of Specific_Resistance = rho = •*mm"

                              2
                 0.018·Ohm·mm
                 •••••••••••••••·(2·km)
                         m
#10: 22·Volt = ••••••••••••••••••••••••·(15·Ampere)     Sub(#1) the given data
                            2
                           d
                         •••••·•
                            4

               0.545454·•330·mm
#11: d = •••••••••••••••••••                                      Solve(#10) for d
                        ••

                 0.545454·•330·mm
#12: d = - •••••••••••••••••••                           the second solution < 0
                          ••

#13: d = 5.59037·mm                                                   Approx(#11)

#14: Volt :=                                       unassign Volt and try once more!

           0.545454·•330·•Ampere·•Ohm·mm
#15: d = •••••••••••••••••••••••••••••••  Solve(#10) for d, the pos. solution!
                     •••·•Volt

#16: Volt := Ohm·Ampere                                assign Volt as Ohm * Ampere

           5.59037·mm·•(Ampere·Ohm)
#17: d = ••••••••••••••••••••••••••                                   Approx(#15)
                 •Ampere·•Ohm

#18: "Nothing happens, because we didn't declare Ampere and Ohm both >= 0"

#19: default :• Real [0, •)                                                  User

#20: d = 5.59037·mm                                     Approx(#17), and now it works!
```

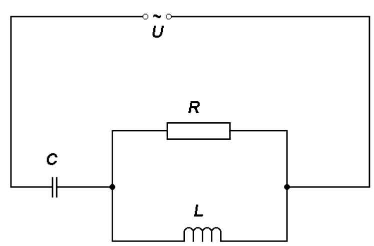
Expression #8 for the specific resistance shows two physical units Ohm·mm (= RU · LU) and that obviously matches with $\Omega \cdot mm^2/m$ (= $RU \cdot LU^2$ / LU).

It might happen that students without instruction would try the second approach, without declaring Volt := Ampere × Ohm. To reproduce this way, we "unassign" the variable Volt in line #14, solve the quadratic once more and then declare Volt := Ohm · Ampere. Surprisingly approximation does not cause cancellation in the fraction #17. It is necessary - totally correct, but usually totally ignored - to declare the variables non-negative (line #19) and then we obtain the result. One side-effect of working with a CAS is the fact that the students are forced to work correctly and to consider the domain of variables.

Example 2

Find the voltage, the current, the translation of the phase and the power of the following circuit.

U = 40V
R = 100Ω
L = 0.4 H(enry)
$C = 8 \cdot 10^{-5}$ F(arad)
$\omega = 250\ sec^{-1}$



In this example we will use complex numbers. One answer to the many questions of our students in secondary schools: "What are complex numbers good for?"

Resistance of two resistors in parallel connection: 1/R = 1/R1 + 1/R2
Resistance of two resistors in serial connection: R = R1 +R2
Capacitive Resistance Cap_Res = Capacitance: $X_C = 1/(\omega\, C)$
Inductive Resistance Ind_Res: $X_L = \omega\, L$

For working with complex numbers we set $X_C = 1/(i\, \omega\, C)$ and $X_L = i\, \omega\, L$.

The coil L and the resistor R build a partial system (parallel connected) with resistance Res_part which is serial connected with the capacitor C. Using Word formulas and all the physical units make the session self explanatory to a high degree. Lines #5 - #11 calculate the resistance of the parallel connected subsystem resistor R and coil L. Ohm´s resistance is represented by the real part of the complex number.

```
                                   -1                            -1
#1:   Henry := Volt·sec·Ampere  , Farad := Ampere·sec·Volt             User

#2:   Ind_resistance := î·Frequency·Inductance                          User

                                      1
#3:   Cap_resistance := ──────────────────────                          User
                          î·Frequency·Capacity
```

The next lines calculate the resistance of the parallel connected subsystem resistor R and coil L. Ohm´s resistance is represented by the real part of the complex result (= 50 Ω)

Contents

$$\#4:\quad \frac{1}{\text{Res\_part}} = \frac{1}{\text{Resistance}} + \frac{1}{\text{Ind\_resistance}} \qquad \text{User}$$

$$\#5:\quad \frac{1}{\text{Res\_part}} = \frac{1}{\text{Resistance}} - \frac{\hat{\imath}}{\text{Frequency}\cdot\text{Inductance}} \qquad \text{Simp(\#4)}$$

$$\#6:\quad \frac{1}{\text{Res\_part}} = \frac{1}{100\cdot\text{Ohm}} - \frac{\hat{\imath}}{(250\cdot\text{sec}^{-1})\cdot(0.4\cdot\text{Henry})} \qquad \text{User}$$

$$\#7:\quad \text{Res\_part} = \frac{100\cdot\text{Ohm}\cdot\text{Volt}^2}{\text{Ampere}^2\cdot\text{Ohm}^2 + \text{Volt}^2} + \frac{100\cdot\hat{\imath}\cdot\text{Ampere}\cdot\text{Ohm}^2\cdot\text{Volt}}{\text{Ampere}^2\cdot\text{Ohm}^2 + \text{Volt}^2}$$

$$\#8:\quad \text{Volt} := \text{Ohm}\cdot\text{Ampere} \qquad \text{User}$$

$$\#9:\quad \text{Res\_part} = 50\cdot\text{Ohm} + 50\cdot\hat{\imath}\cdot\text{Ohm} \qquad \text{Simp(\#7)}$$

$$\#10:\quad \text{Res\_part} := 50\cdot\text{Ohm} + 50\cdot\hat{\imath}\cdot\text{Ohm} \qquad \text{User}$$

$$\#11:\quad \text{Res\_total} = \text{Res\_part} + \text{Cap\_resistance} \qquad \text{User}$$

$$\#12:\quad \text{Res\_total} = 50\cdot\text{Ohm} + \frac{\hat{\imath}\cdot(50\cdot\text{Capacity}\cdot\text{Frequency}\cdot\text{Ohm} - 1)}{\text{Capacity}\cdot\text{Frequency}} \qquad \text{Simp(\#11)}$$

$$\#13:\quad \text{Res\_total} = 50\cdot\text{Ohm} + \frac{\hat{\imath}\cdot(50\cdot(8\cdot10^{-5}\cdot\text{Farad})\cdot(250\cdot\text{sec}^{-1})\cdot\text{Ohm} - 1)}{(8\cdot10^{-5}\cdot\text{Farad})\cdot(250\cdot\text{sec}^{-1})}$$

$$\#14:\quad \text{Res\_total} = 50\cdot\text{Ohm} \qquad \text{Simp(\#13)}$$

That is the resistance of the whole system. Phase translation is described by ARCTAN of the quotient of imaginary and real part of the resistance.

$$\#15:\quad \text{Res\_total} := 50\cdot\text{Ohm} \qquad \text{User}$$

$$\#16:\quad \text{SOLVE}\left(\text{TAN}(\bullet) = \frac{\text{IM(Res\_total)}}{\text{RE(Res\_total)}}, \bullet\right) = [\bullet = 0, \bullet = \bullet, \bullet = -\bullet]$$

$$\#17:\quad \text{Intensity} = \frac{\text{Voltage}}{\text{Resistance}} \qquad \text{User}$$

$$\#18:\quad \text{Intensity\_tot} = \frac{40\cdot\text{Volt}}{\text{Res\_total}} \qquad \text{Sub(\#17)}$$

$$\#19:\quad \text{Intensity\_tot} = 0.8\cdot\text{Ampere} \qquad \text{Approx(\#18)}$$

$$\#20:\quad \text{Power} = \text{Voltage}\cdot\text{Intensity}\cdot\text{COS(Phase)} \qquad \text{User}$$

$$\#21:\quad \text{Power\_total} = (40\cdot\text{Volt})\cdot(0.8\cdot\text{Ampere})\cdot\text{COS}(0) \qquad \text{Sub(\#20)}$$

$$\#22:\quad \text{Power\_total} = 32\cdot\text{Ampere}^2\cdot\text{Ohm} \qquad \text{Simp(\#21)}$$

$$\#23:\quad \left[\text{Volt} :=, \text{Ohm} := \frac{\text{Volt}}{\text{Ampere}}, \text{Ampere} := \frac{\text{Watt}}{\text{Volt}}\right] \qquad \text{User}$$

$$\#24:\quad \text{Power\_total} = 32\cdot\text{Watt} \qquad \text{Simp(\#22)}$$

$$\#25:\quad \text{Cap\_volt} = \bullet\text{Cap\_resistance}\bullet\cdot\text{Intensity} \qquad \text{User}$$

$$\#26:\quad \text{Cap\_volt} = \frac{\text{Intensity}}{\text{Capacity}\cdot\text{Frequency}} \qquad \text{Simp(\#25)}$$

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$$\#27:\quad \text{Cap\_volt} = \frac{0.8\cdot\text{Ampere}}{(8\cdot 10^{-5}\cdot\text{Farad})\cdot(250\cdot\text{sec}^{-1})} \qquad \text{Sub(\#26)}$$

$$\#28:\quad \text{Cap\_volt} = 40\cdot\text{Volt} \qquad \text{Simp(\#27)}$$

$$\#29:\quad \text{Volt\_part} = \bullet\text{Res\_part}\bullet\cdot\text{Intensity} \qquad \text{User}$$

$$\#30:\quad \text{Volt\_part} = \frac{50\cdot\bullet 2\cdot\text{Intensity}\cdot\text{Volt}^2}{\text{Watt}} \qquad \text{Simp(\#29)}$$

$$\#31:\quad \text{Volt\_part} = \frac{50\cdot\bullet 2\cdot(0.8\cdot\text{Ampere})\cdot\text{Volt}^2}{\text{Watt}} \qquad \text{Sub(\#30)}$$

$$\#32:\quad \text{Volt\_part} = 56.5685\cdot\text{Volt} \qquad \text{Approx(\#31)}$$

The voltage on the capacitor is 40 Volt and on the coil 56.59 Volt.
It is very informative to combine the concept of using complex numbers with the consistent use of physical units to obtain correct answers. That might help to develop a feeling for the consistence of mathematical notation and to illustrate how each idea fits like a stone in a huge mosaic.

Example 3

What is the ratio of the electrostatic force and the gravitation force acting between two protons.

The values for the elementary charge, the proton´s mass and the electric constant $\varepsilon_{0'}$ (permittivity) can be found in each physical almanac. We do not need any value for the distance because of cancellation.

$$\#1:\quad \text{F\_e} := \frac{1}{4\cdot\bullet\cdot\text{Permittivity}}\cdot\frac{\text{Elem\_charge}^2}{\text{Distance}^2} \qquad \text{Formula for elect.force}$$

$$\#2:\quad \text{F\_grav} := \frac{\text{Grav\_const}\cdot\text{Mass\_Proton}^2}{\text{Distance}^2} \qquad \text{Formula for grav.force}$$

$$\#3:\quad \text{ratio} = \frac{\text{F\_e}}{\text{F\_grav}} \qquad \text{User}$$

$$\#4:\quad \text{ratio} = \frac{\text{Elem\_charge}^2}{4\cdot\bullet\cdot\text{Grav\_const}\cdot\text{Mass\_Proton}^2\cdot\text{Permittivity}} \qquad \text{Simp(\#3)}$$

Substitute in expression #4:

ratio =

#5:

$$\frac{(1.6\cdot 10^{-19}\cdot \mathrm{Coulomb})^{2}}{4\cdot\dfrac{6.67\cdot 10^{-11}\cdot \mathrm{Newton}\cdot \mathrm{m}^{2}}{\mathrm{kg}^{2}}\cdot(1.67\cdot 10^{-27}\cdot \mathrm{kg})^{2}\cdot\dfrac{8.854\cdot 10^{-12}\cdot \mathrm{Coulomb}^{2}}{\mathrm{Joule}\cdot \mathrm{m}}}$$

#6: $\mathrm{ratio} = \dfrac{1.23689\cdot 10^{36}\cdot \mathrm{Joule}}{\mathrm{Newton}\cdot \mathrm{m}}$ Approx(#5)

#7: Joule := Newton·m User

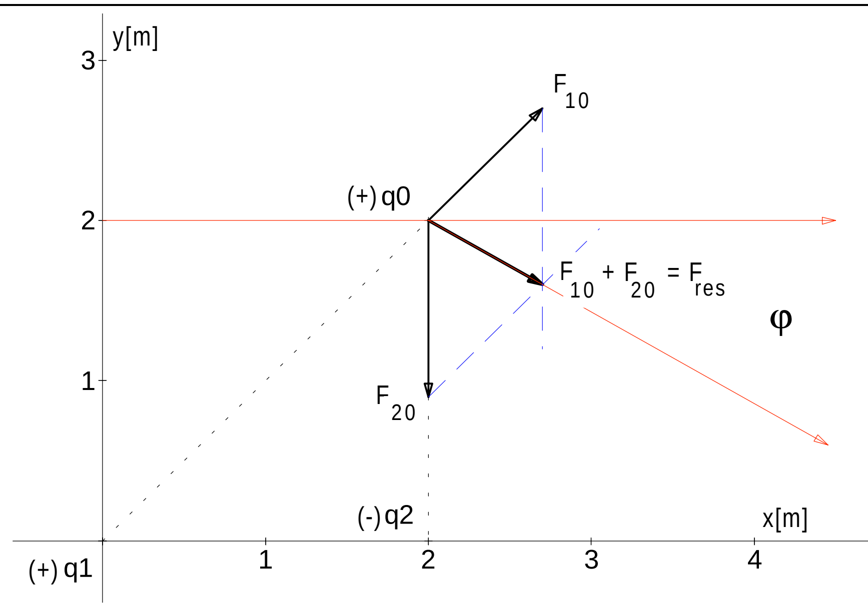
#8: $\mathrm{ratio} = 1.23689\cdot 10^{36}$ Approx(#6)

Please note the bulky expression #5, containing all units. If there is any mistake in the input it will not match with the claim for the fact that a ratio should appear as a scalar without any unit.

One more example from the field of electricity, which in addition provides an opportunity to use vectors.

Example 4:

Charge q_1 = +25nC is located in the origin, charge q_2 = –15nC on the x-axis with x = 2m and charge q_0 = +20nC can be found in point (x = 2m, y = 2m). Which force acts on q_0?



In general Force_AB is the force between the charges qA and qB, which can be decomposed into its two components Force_AB_x and Force_AB_y. In our example we have to look for the resulting force composed of F_10 and F_20. The basic formula for Force_AB is given in line #2.

Contents

```
        •              -12        2       -1  -1                   •
#1:    •Permittivity := 8.854·10   ·Coulomb ·Joule  ·m  , Joule := Newton·m•

                                   1
                        •••••••••••••••••••·(Charge_A·Charge_B)
                         4·•·Permittivity
#2:   Force_AB = ••••••••••••••••••••••••••••••••••••••••                 User
                                                  2
                                      Distance_AB

Sub(#2)

                          1                -9                 -9
                •••••••••••••••••·((- 15·10  ·Coulomb)·(20·10  ·Coulomb))
                 4·•·Permittivity
#3:  Force_20 = ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••
                                                2
                                          (2·m)

                                 -7
#4:   Force_20 = - 6.74080·10   ·Newton                              Approx(#3)

        •                                           -7           •
#5:    •Force_20_x := 0, Force_20_y := - 6.7408·10   ·Newton•           User
```

Corresponding to the fact that F20 shows in y-direction the components of the vector Force_20 are defined in line #5; the components of vector F10 are obviously equal, hence Distance_AB is substituted by $2\sqrt{2}$. Fortunately we find another reason to use complex numbers.

```
Sub(#2)

                          1              -9                 -9
                •••••••••••••••••·((25·10  ·Coulomb)·(20·10  ·Coulomb))
                 4·•·Permittivity
#6:  Force_10 = ••••••••••••••••••••••••••••••••••••••••••••••••••••••••••••
                                                   2
                                          (2·•2·m)

                             -7
#7:   Force_10 = 5.61734·10   ·Newton                                Approx(#6)

                              -7
#8:   Force_10 := 5.61734·10   ·Newton                                   User

        •                Force_10                       Force_10 •
#9:    •Force_10_x := ••••••••••, Force_10_y := •••••••••••              User
        •                  •2                             •2     •

#10: Force_res_x := Force_10_x + Force_20_x                              User
#11: Force_res_y := Force_10_y + Force_20_y                              User
#12: Force_res := Force_res_x + î·Force_res_y                            User

                                  -7
#13: •Force_res• = 4.84181·10   ·Newton                      the acting force

      PHASE(Force_res)·180
#14: •••••••••••••••••••••• = -34.8785                  direction in degrees
                •
```

The next problems deals with mechanics, usually the students´ first contact with physics.

Example 5

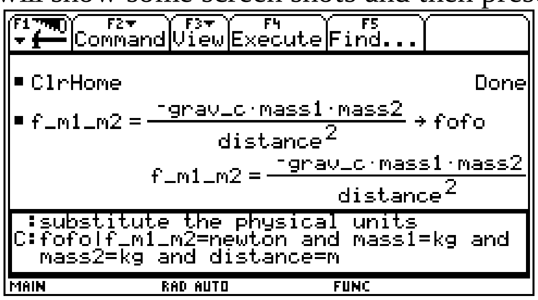
We want to know the escape velocity on the surface of planet Mercury.
Check first the gravity constant´s unit and value. ($G = 6{,}67 \cdot 10^{-11} N \cdot m^2 \cdot kg^{-2}$)
(Mass of Mercury = $3.31 \cdot 10^{23}$ kg, Radius of Mercury = $2.44 \cdot 10^6$ m)

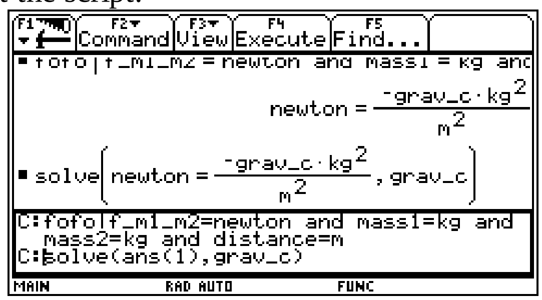
Despite the fact that this is a *DERIVE* lecture I´d like to demonstrate that you easily can realize the basic idea of this lecture on a *TI*-92, too. So prepare a "script" and run it in class explaining it step by step, or transfer it to your students´ *TI*s and let them study, discuss and describe the script. One method is to split the screen in Home Screen and Text Editor. Each line beginning with `C:` in the script is an executable line. Pressing F4 executes the command in the Home screen. Lines without a `C:` are used as comments.

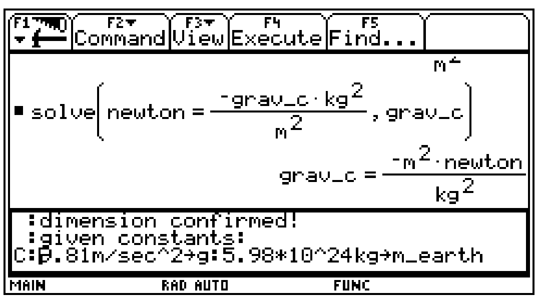
The new *TI*-92+ shows a new feature: F2 `5:Execute to EOF` executes the whole script. Switching to the Home screen you can follow the process in the history area.

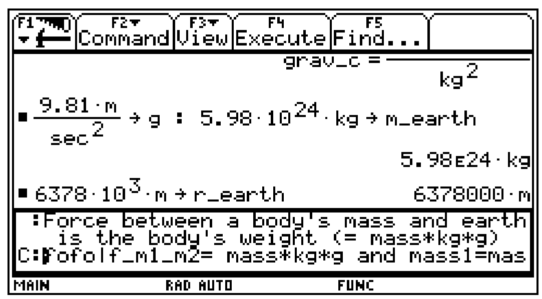
Due to Newton´s Law of Gravity the force acting between two masses M1 and M2 is proportional to both masses and reciprocally proportional to their squared distance.

I will show some screen shots and then present the script.

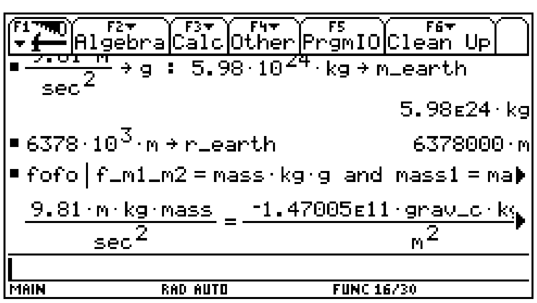


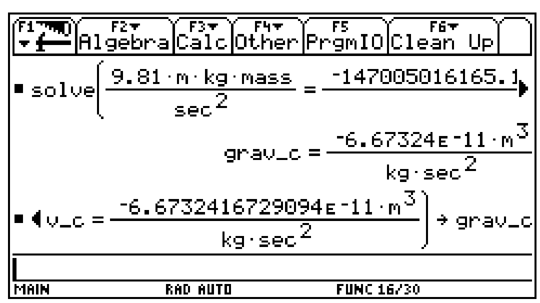






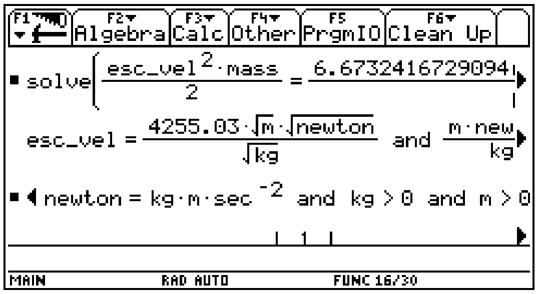
Some screen shots from the "full executed" script:

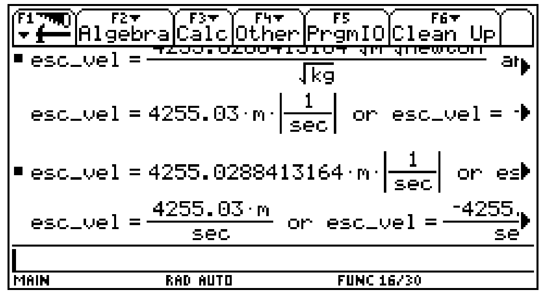




.... the end of the script:

Contents





The escape velocity on Mercury is 4255.04 m sec^{-1}.

The script - including explanatory comments will follow on the next page. (Variable names are restricted on 8 characters and unlike to *DERIVE* you are not allowed to use uppercase letters!)

Script Example5.92t

```
C:clrhome
 :
 :Law of Gravity
C:f_m1_m2=ªgrav_c*mass1*mass2/distance^2»fofo
 :
 :substitute the physical units
 :C:fofo|f_m1_m2=newton and mass1=kg and mass2=kg and distance=m
C:solve(ans(1),grav_c)
 :
 :dimension confirmed!
 :given constants:
C:9.81m/sec^2»g:5.98*10^24kg»m_earth
C:6378*10^3m»r_earth
 :
 :Force between a body's mass and earth is the body's weight (= mass*kg*g)
C:fofo|f_m1_m2= mass*kg*g and mass1=mass*kg and mass2=m_earth and
distance=r_earth
C:solve(ans(1),grav_c)
C:right(ans(1))»grav_c
 :substitute for sec
C:solve(newton=kg m sec^ª2,sec)
C:ªgrav_c|sec=§(m)*§(kg)/§(newton)»grav_c
 :
```

:that is ok!

:body's kinetic energy = its weight

C:1/2 (mass*esc_vel^2)=(grav_c*mass*mass_pl)/r_planet

C:solve(ans(1),esc_vel)|mass_pl=3.31*10^23kg and r_planet=2.44*10^6m

C:ans(1)|newton=kg m sec^ª2 and kg>0 and m>0

:

:positive root is the escape velocity

C:ans(1)|sec>0

:

:housekeeping

C:delvar fofo,grav_c,r_earth,m_earth,g

Example 6

A mass of 500kg is hung on a steel rope with cross section 0.15cm^2 and length 3m. How much does the rope stretch out?

The stretching or tensile stress is defined by the ratio of the tensile force acting on an elastic body and its cross section - Formula(1). Formula(2) doesn´t need any explanation, and in Formula(3) we define a natural constant - Young´s modulo - which is characteristic for various materials. I found its value for steel with 200 GN/m^2 in a table. This model is valid within the range of validity of Hook´s Law.

#1: $\text{Tensile\_stress} = \dfrac{\text{Force}}{\text{Cross\_section}}$ Formula(1)

#2: $\text{Rel\_change\_of\_length} = \dfrac{\text{Abs\_change\_of\_length}}{\text{Length}}$ Formula(2)

#3: $\text{Youngs\_mod} = \dfrac{\text{Tensile\_stress}}{\text{Rel\_change\_of\_length}}$ Formula(3)

#4: $\left[\text{Cross\_section} := 0.15 \cdot \text{cm}^2,\ \text{cm} := 10^{-2} \cdot \text{m},\ \text{Length} := 3 \cdot \text{m}\right]$ User

#5: $\left[\text{Force} := 500 \cdot \text{kg} \cdot 9.81 \cdot \text{m} \cdot \text{sec}^{-2},\ \text{Youngs\_mod} := 200 \cdot 10^{9} \cdot \text{Newton} \cdot \text{m}^{-2}\right]$

#6: $\text{Tensile\_stress} := \dfrac{3.27 \cdot 10^{8} \cdot \text{kg}}{\text{m} \cdot \text{sec}^{2}}$ Assignation not an equation!

#7: $\text{Rel\_change\_of\_length} = \dfrac{\text{Abs\_change\_of\_length}}{3 \cdot \text{m}}$ Simp(#2)

#8: $\dfrac{2 \cdot 10^{11} \cdot \text{Newton}}{\text{m}^{2}} = \dfrac{\text{Tensile\_stress}}{\text{Rel\_change\_of\_length}}$ Simp(#3)

We multiply expression #8 by the variable **Rel_change_of_length** and then obtain together with equation #7 a system of two linear equations with two unknown - the changes of length, absolute and relative.

$$\#9:\quad \left[Rel\_change\_of\_length = \frac{Abs\_change\_of\_length}{3\cdot m},\ \frac{2\cdot 10^{11}\cdot Newton\cdot Rel\_change\_of\_length}{m^{2}} = \frac{3.27\cdot 10^{8}\cdot kg}{m\cdot sec^{2}}\right]$$

Simp(User)

$$\#10:\quad \left[Abs\_change\_of\_length = \frac{0.004905\cdot kg\cdot m^{2}}{Newton\cdot sec^{2}},\ Rel\_change\_of\_length = \frac{0.001635\cdot kg\cdot m}{Newton\cdot sec^{2}}\right]$$

Solve(#9); give a better form!

$$\#11:\quad sec := \sqrt{\frac{kg\cdot m}{Newton}}$$

Newton = kg m/sec^2; User

Simp(#10)

#12: [Abs_change_of_length=0.004905·m, Rel_change_of_length=0.001635]

The absolute change of length is 4.9mm, the relative change is approximately 1.6%.

Conclusion

Using a CAS to work with "word formulae" and with the physical units, and connections between them, could help to close the gap between verbally given problems and their mathematical model. The meaningful use of dimensional analysis could be demonstrated and it might become an important support for the students in other fields. I have to admit that I benefitted a lot when I used my ideas for solving word problems using a CAS the first time in a physical environment.

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