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On Some Aspects of the Mathematics Teacher Training

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What should be done to teach schoolchildren how to think? This is the key question for many educators. It means that thinking should be developed us to such a level that is characterised by plausible reasoning mastering, search for acquiring knowledge and the ability to use the acquired knowledge in strange situations.

We agree with those educators who believe that the process of acquiring knowledge is more important than its result, i.e. the way in which mathematical material is acquired is more significant for thinking development than its subject matter acquired by pupils.

It goes without saying that these ideas have been thoroughly developed since inculcation of new technologies like DERIVE and Cabri-geometre in teaching mathematics. Nowadays it is necessary to modify programs and standards of the mathematics teachers training that concern using of technologies in their future pedagogical activity since modern software can promote modification of view on the essence of a pedagogical activity. In expectation of outcomes of long-term pedagogical investigations the inaction is short-sighted and inefficient as well as refusal of preparation of the future teachers for forthcoming reforms.

The purpose of this report is to represent the review of special courses that offered for the undergraduate students of a “mathematics - computer science” speciality. A general problem, that integrated these courses is modern tendentions of development of pedagogical and methodical ideas, approaches in teachinging and learning mathematics using systems like DERIVE and Cabri-geometre:

1) The Use of DERIVE for Explorations in Algebra and Mathematics Analysis;
2) Computer Explorations in Plane Geometry;
3) The Use DERIVE for Solving Problems with Economic Contents.

Our methods of teaching, as well as traditional school methods, are based on the laws of empirical generalisations. The present report is devoted to the survey of teacher students’ research directions in course and diploma papers. At first the foundations of existing methods of teaching mathematics are taught with using technologies like DERIVE and Cabri-geometre during special courses. Then the main directions of future independent research are discussed. Only for some of them we plan to direct their efforts towards the development of theoretical thinking (creativity) because this is more complicated process.

Among main directions of teacher students' research with using technologies are as follows:

- pupils' mental development;
- the change of mathematical curricula.

It should be noted that psychological and pedagogical laws of mental activities formation form the foundation of all research.

The students investigate themselves new themes, new methodical ideas with using DERIVE, for example:

1) guiding computer explorations;
2) problem solving of real world;
3) open-end problems;
4) methods of solving trigonometric equations;
5) complex numbers and their geometrical applications;
6) geometry of curves;
7) elements of linear programming etc.

Background

It is evident that we take into consideration dialectical character of relations between knowledge and thinking. Namely, as the true acquisition of knowledge is impossible without an adequate level of thinking, the subject matter of knowledge determines the character and direction of thinking development.

Thus, while evaluating schoolchildren's mathematical abilities, two basically different ways of acquiring mathematical material have been discovered: "Along side with the way of gradual generalisation of mathematical material based on varying some particular cases (the way characteristic of most pupils), there is also another way: bright pupils neither compare "similar" cases nor make special exercises, nor perform teachers' directions; they generalise mathematical objects independently, "all of a sudden" having analysed only one phenomenon in a set of similar phenomena" (Krutetzkii 1968).

According to generalisation types, thinking is referred to either as empirical or theoretical. Empirical thinking is characterised by the reflection of external relations without penetrating into their essence. According to Krutetzkii's conclusions, it characterised the average level of schoolchildren's mathematical abilities.

Theoretical thinking reflects internal relations of objects and the laws of their development used in scientific research. Empirical thinking is characterised mostly by the inductive way of cognition, while theoretical thinking — by the deductive one, therefore the latter is also called a scientific way. The way of empirical thinking is ascending from the concrete up to the abstract, while theoretical thinking descends from the abstract down to the concrete. There are algorithmic, geometric and logical components of creativity in both types of thinking.

Taken into account the modern development similar ideas (paradigm of the "open-end" approach method (Nohda, Pehkonen)) the main task of teacher students' research is the development of thinking according to individual preferences in mental structure, namely predicative versus functional cognitive structures (Schwank 1986, 1993). The ideas of these research group (Cohors-Fresenborg 1993, Cohors-Fresenborg & Kaune & Griep 1991, 1992, 1993) are especially important for investigating the possibilities and limits of the use of technologies in representing intuitive ideas.

The next component of our approach background is a theory on visual thinking and its development. The most important stage of visual thinking (Arnchame, Luria) is a stage of mental framing hypothesis about the methods of solving, possible ways of solving with analysing and forecasting the outcome that can be received.

Research on guiding computer explorations

Studying methods of teaching teacher students have to remember that computer explorations performance presupposes the following sequence of a pupil's mental acts: analysis, comparison, abstraction and generalisation. Computer explorations are very important for the process of studying as well as computing experiment for the process of scientific cognition. In the first case, pupils study subjectively unknown facts, in the second — scientists' research objectively unknown facts of nature. Teacher students have to guide this process which consists of some steps:

- posing of problems;
- having pupils' explorations based on computing experiments;
- framing hypothesis about the ways of problem solving;
- proving the hypothesis or creating the counterexample for it.

Computer explorations become more effective by DERIVE and Cabri-geometre facilitating accurate counting, transformations that produce equivalent expressions and equations solving. The development of the pupils' ability on formulation new hypotheses is an important problem of the modern teacher training.

A fruitful approach, which provides a success in developing mathematical creativity, is a system of problems arranged in series so that the experience of solving the previous problem helps to pose the next one and to form a hypothesis. For example, exploration on the connection between a function and its first derivative helps in posing problems (and forming hypothesis) on the connection between a function and its second derivative. Then last exploration (on connection between a function and its second derivative) helps in posing problems on the Mean Value Theorem and on the Number ***e***.

The essence of the exploration on the Number e is in the comparison and analyzing the positions of tangent lines of curves a^x (with different base value a) in the points $(0,1)$. Besides it is in comparison and analyzing their angles of slope and to find the value of base a that slope angle of tangent line is equal to 1. The pupils' conclusion: the value of base $a = e$.

In plane geometry we propose some series of arranging problems which help in posing out problem (and forming hypothesis), for example:

- theorems on the equality (or congruence conditions) of the triangles and theorems on the similarity of the triangles (as for any triangles as for right triangles), properties of sides and angles in triangle;
- theorems on the equality (or congruence conditions) of the angles formed by two parallel lines and intersecting line (internal and external crosswise lying angles, internal and external corresponding angles etc.), theorems on the sum of the angles of triangle; value of external angle of

triangle;
- area of right triangle, area of any triangle, Pythagorean theorem, properties of perpendicular, inclined line and projection.

We propose to organise the explorations in three levels as they are described below (in general they are not depending from the information technology support). At the first level a teacher demonstrates how to explore information models and how to compare and analyse the results of graphic, numerical or symbolic information. Pupils repeat the teacher steps in exploration at their computers and try to formulate a hypothesis under the teacher's control.

At the second level pupils explore their problems by themselves. By changing parameter values pupils try (by themselves or under the teacher's control) to find their mutual connections. If necessary the teacher helps the pupils in their work. At the third level pupils work themselves (without supervision). This structure helps to develop students' discovery competencies for solving any problem.

Consider the pupils' exploration for Mean Value Theorem. We remind you of this theorem:
If a function $f(x)$ is continuous on $[a,b]$ and has derivative $f'(x)$ in (a,b) then there exist at least one point c in (a,b) such that $f'(c) = (f(b) - f(a)) / (b - a)$.

To interpret the mean value theorem geometrically we propose to consider a continuous curve $y = f(x)$ such that there is a tangent at every point between A with the abscissa $x = a$ and B with the abscissa $x = b$ on the screen (the plot window of DERIVE). It is easy to notice that the ratio (right side of expression) is the slope of the intersecting line through points A and B and $f'(c)$ is the slope of the tangent line to a curve $y = f(x)$ at the point C with the abscissa $x = c$.

Pupils have to analyse the positions of tangent lines in the different points of curve and the straight intersecting line AB, their angles of slope and to find one point C with the abscissa $x = c$ at the curve between a and b at which the tangent to the curve $y = f(x)$ is parallel to the straight intersecting line AB. For this purpose they repeat the explorations some times for different points at the curve with the abscissa x between a and b , fill the table of angles, compare and generalise.

One of the most important task of the teacher in change of study research is to lead his/her pupils to the idea of logical reasoning significance, power and beauty, i.e. the proof of the "evident". The realisation of this fact puts his pupils up to a higher stage of learning. Proving theorems with well-based hypothesis is closely connected with constructing counterexamples, which are capable of refuting wrong hypothesis. If the theorem was successfully proved, a computer exploration allows us to use it for solving applied problems and theory systematisation.

Research on study of the trigonometric equation solving

The possibilities of a system DERIVE on management of trigonometric transformations are unique for teaching traditional school methods of trigonometric equations solving (factorization, simplifying to a quadratic equation of one functions or homogeneous etc.) as well as more complicated methods which isn't study in school.

A teacher's important task is to teach his pupil to see the thing that is instilled in images, i.e. to analyze visual information. It is the discovery of certain fragments and the identification of similar ones (either in form on meaning) that take place first. But the working out of problem solving plan is the most important stage. DERIVE is very useful in this process as well as in generalization methods of equation solving (discovery of algorithms).

For researching of teacher students in this direction we propose to reconstruct a pupil's possible way of thinking while analyzing the following problem. To solve the equation $3\sin^2 x - 2\sin 2x + 5\cos^2 x = 2$:

The equation includes trigonometric functions that is why it is a trigonometric equation.

1. The equation includes different trigonometric functions with different arguments.
2. All the summands of the both sides can be represented as a function of one argument.
3. All the summands have a similar degree and we can divide by $\cos^2 x$ for obtaining an equation of one trigonometric function.
4. We know two types of equations simplest $f(x) = a$ and more complicated $af^2(x) + bf(x) + c = 0$.
5. We obtain quadric equation of one trigonometric function.

Research on acquisition of methods for solving real world problems

Traditionally physical and geometrical interpretations are used for teaching and learning mathematics. Solving of real world problems is a good way for introducing mathematical concepts to pupils or moreover for forming ones. But we have chosen mathematical problems with microeconomics contents because it's actual especially for our country.

This is a list of topics, which used in solving economic problems:

- Equations of Second Order Curves
- Linear Equations
- Systems of Linear Inequalities
- Matrices
- Limit and Continuity of Function
- Derivative of Function
- Functions of Several Variables and its Partial Derivatives
- Definite Integrals
- Linear Partial Differential Equations

It goes without saying that elements of linear programming with using DERIVE is better for schoolchildren understanding with the average level of mathematical abilities.

DERIVE is very useful not only for calculating and graphing activity during investigating information models but also for correcting ones and making models more exact. It is very important for understanding connection between the mathematical concepts and their economical interpretations. But we have to say that at first stage the students consider economical concepts in close connection with their mathematical ones. Only on the second stage they solve economical problems operating only economical images of mathematical concepts.

As we can see economic problems which use the notion of a derivative are very important for understanding that derivative characterise the rapidity of changes of economical processes. In the practice of economic researches the wide application has obtained a production functions used for definition of dependencies, for example, production output of costs resources, manufacturing costs of total products, return of goods for sale etc. On the supposition of differentiation of a production functions their differential characteristics connected with the notion of derivative acquire great importance because they describe of production functions.

Besides DERIVE successfully helps in acquisition of methods for solving real life problems. It is of great interest especially in applied

Math courses, in particular, in economics. It requires four steps, as usual:

- choosing a suitable model;
- translating the real world problem into the mathematical problem;
- calculating the model solution by applying known methods;
- interpreting the model solution into a real world solution.

For researching of teacher students in this direction we propose to use the method of constructing and reconstructing of problems, to compare the obtaining results from different ways of problem solving, to generalise a solution method for this type of problem etc. For example one economical problem which uses equations of second order curves and two reconstructing questions.

The distance between two enterprises is equal to 100 km. The price of the same products manufactured by each enterprise is equal to P. Transport cost (travelling expenses) for 1 km from enterprise A to a consumer is equal to 1 unit of money (financial resources), from enterprise B to a consumer is equal to 2 units of money. Try to find the consumers' expenses for the purchase of 1 unit of production of enterprises A and B are equal? Analyse locations of different consumers.

1. Find the solution of the previous problem when the transport cost for 1 km from both enterprises A and B to the consumer are equal to 1 unit of money. The price of 1 unit of production of the enterprises A and B are equal to accordingly 200 and 225 units of money.

2. Find the solution of the previous problem when the transport cost for 1 km from the enterprise A is 2 times less than B.

All problems are useful material for learning different mathematical material. We hope the vast majority of pupils who interested in economics can more easier assimilate mathematical problems in connection with reality of our life situation.

Research on study of the geometric applications of complex numbers

This direction is a good example of changing of mathematical curricula. Unfortunately the geometric applications of complex numbers are forgot in a school curricular. Though this method allows solving plane geometry problems by the elementary calculations using the known formulas which immediately follow from the problem condition. Our interest to the method of complex numbers with using DERIVE is connected to the greater simplicity of its application in comparison with traditional coordinate and vector methods, which demand considerable quickness of pupil thinking. Even few simplest statements (that follow from the geometric interpretation of complex numbers) allow to solve rather useful problems on the proof of properties of triangles and tetragons. Besides they allow to prove the known classical theorems of elementary geometry.

For example: Points *M* and *N* are middle of diagonals *AC* and *BD* of tetragon *ABCD*. Proving $|AB|^2+|BC|^2+|CD|^2+|DA|^2=|AC|^2+|BD|^2+4|MN|^2$ is easy if we suppose that complex number *a, b, c, d, m, n* corespondent points *A, B, C, D, M, N* and $m=\frac{1}{2}(a+c)$, $n=\frac{1}{2}(b+d)$.

This method is very useful for proving classical theorems such as Gauss' theorem, Pascal's theorem and Monger's theorem. For example Gauss' theorem: if a straight line intersects the straight lines, which contain sides *BC, CA, AB* of triangle *ABC* correspondingly in points A_1, B_1, C_1, then the middle of segments AA_1, BB_1, CC_1 lie on one straight line.

Conclusion

The realizations of these special courses shows that the use DERIVE and Cabri-geometre helps the teacher students in didactical research and designing teaching strategies on the development of pupils' thinking (abstraction, generalization, specialization etc.) and formation creative (scientific) style of activity. The main purpose of theacher students research is investigating pupils' mental development with using technology and investigating technology possibilities in changing of mathematical curricula.

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