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The Pantograph Linkage on the TI-92

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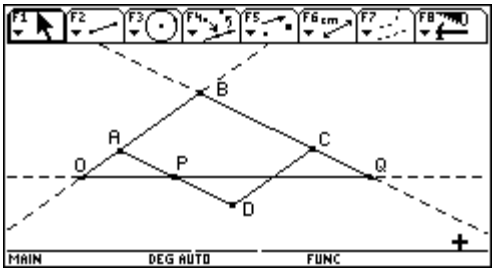
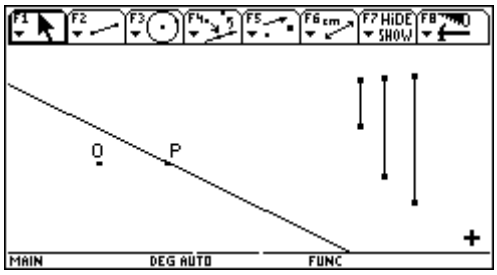
In 1630, Christoph Scheiner invented the Pantograph Linkage, a device used to produce an enlarged or reduced copy of any given plane geometric figure. In this talk, we will model this device on the TI-92 calculator.

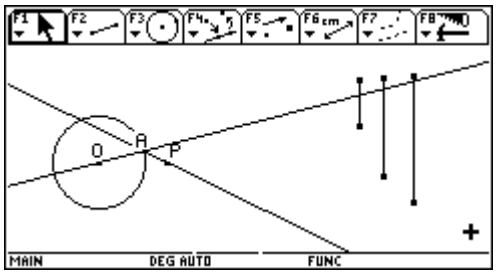
The Pantograph Linkage is a rigid framework formed by four rods $\overline{AB}, \overline{BC}, \overline{CD}$ and $\overline{AD}$, constructed as the sides of a parallelogram with hinges at the corners, so that the shape of the parallelogram can vary but not the length of its sides. The device is anchored at point *O* and as the point *P* is moved, the points *A* and *B* slide along a rod, as do *P* and *Q*, forcing *O*, *A* and *B* to be collinear as well as *O*, *P* and *Q*. As *P* is moved along any path, the point *Q* traces out a similar curve.



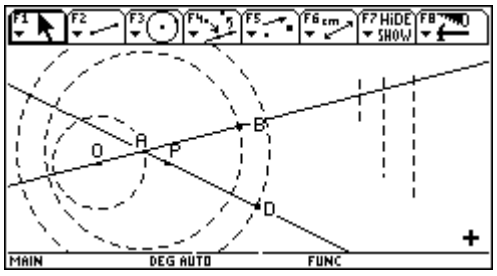
Start by opening a new geometry application. Construct points *O* and *P* and three segments of different lengths. Position the segments out of the way of the diagram. The shortest segment will represent the length of $\overline{OA}$ and the other two will represent the sides of the parallelogram. Finally, construct a line through *P*. This will become $\overleftrightarrow{OA}$.



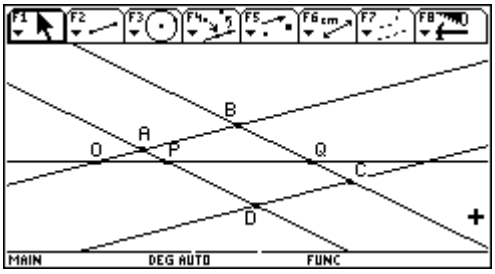
Now locate the point *A* on the line by using the Compass Tool and the shortest segment. *A* should be constructed as the point of intersection of the circle resulting from the Compass Tool and the line. If these curves don't meet, move the line until they do.

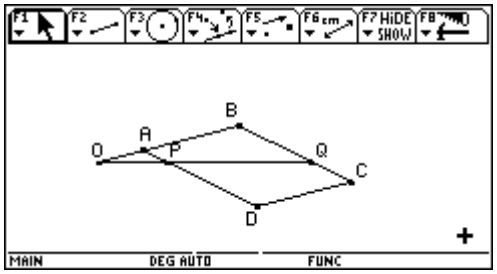


Use the Compass Tool and the remaining two segments to locate the points *B* and *D* on $\overrightarrow{OA}$ and $\overrightarrow{AP}$ respectively. Then hide the three original segments and the circles resulting from the Compass Tool. (In the figure, we have just dotted these elements, rather than hidden them.)

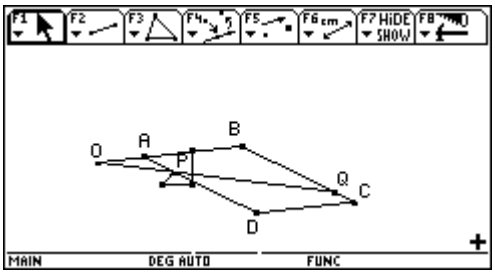


Now complete the parallelogram by locating C as the intersection of the parallel to $\overleftrightarrow{AD}$ through *B* and the parallel to $\overleftrightarrow{AB}$ through *D*. Construct $\overleftrightarrow{OP}$ and locate *Q* as the intersection of $\overleftrightarrow{OP}$ and $\overleftrightarrow{BC}$. This completes the construction. You should test things by grabbing the point *P* and moving it around. When you are satisfied everything works as it should, hide the lines and draw in the segments, as they would appear in the physical linkage.

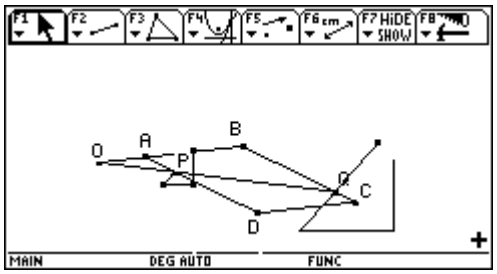


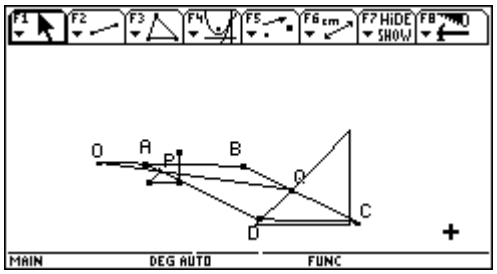


We will test the Pantograph using the Locus Tool. Start by drawing a small right triangle near the point *P*. We are going to attach *P* to this triangle. Select the Redefine Point Tool and then select *P*. The calculator responds by giving you four options. Select the second option: Point On Object. This tells the TI-92 that you want to redefine *P* to lie on some object. Respond to the calculator's prompt by selecting the right triangle you just created. The point *P* will jump to the triangle at the point you select, so don't pick it too far away from *P*.



Now use the Locus Tool to construct the locus for *Q*. You should get a triangle similar to the right triangle that *P* is now attached to. If you don't get a complete triangle as the locus, or if the figure disappears entirely (!) try moving the test triangle. If that doesn't work, you might try deleting the triangle and redrawing it. If you do this, be sure to detach *P* from the triangle first. Otherwise it and the rest of the figure will also be deleted!





Now detach *P* from the test triangle by using the Redefine Tool. This time select the first option after selecting *P* as the point to redefine. This will establish *P* as an independent point. Delete the test triangle after making sure that *P* is not still attached to it.

Now repeat the previous steps, this time using a circle near *P* rather than a triangle. (You might find that the locus for *Q* is drawn automatically after you attach *P* to the circle, without your having to reuse the Locus Tool.) The locus for *Q* ought to be a circle also. You can continue by using the Pantograph to copy other figures or by using the Animation Tool to mimic the actual action of the physical linkage.