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Schröfel Max-Günter, Cubic Spline Functions, Matrices and DERIVE
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Wiesenbauer Johann, Elliptic Curve Cryptography with DERIVE

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Galan Jose Luis a.o., Derive 6 as a Pedagogical CAS: Programming using Display Function

Herweyers Guido, Applied Linear Algebra

Hugelshofer René, Workshop with TI-Nspire

Koch Kerstin, Einsatz von CAS im Mathematikunterricht Klasse 8

Leinbach Carl, BioInformatics

Leinbach Pat, Mathematics in Action – Using Mathematical and Scientific Knowledge in Forensic Investigations

Lokar Matija & Kos Jasna, Selected Themes from Teaching Calculus with CAS

MacAogain Eoghan, Student Projects in Using DERIVE to help teach Topics in second level Mathematics

Moormann Marianne a.o., LeActive Workshop
Olive John, From arithmetic operations with real numbers to composition of functions using dynamic number lines

Pröpper Wolfgang, Die Kurvendiskussion ist tot – es lebe die Kurvendiskussion

Schmidt Karsten, Working with Random Variables and Random Numbers in Derive

Schröfel Max-Günther, Wie schwer ist die "Gloriosa", die größte Glocke des Frankfurter Doms?

Tonisson Eno a.o., Intelligent Problem Solving Environment T-Algebra

Townsley Lisa & Manmohan Kaur, DERIVE: A Mathematical Assistant

Wiesenbauer Johann, Some Derive Tools for Computations on Elliptic Curves and Their Applications
GRAPH_ALGORITHMS.MTH:
Graph Algorithms using Display Step in DERIVE 6

In this work we introduce GRAPH_ALGORITHMS.MTH. This file has been developed to be used in subjects that deal with discrete mathematics aimed to students or teachers. Specifically, it has been elaborated for teaching graph theory to computer science students.

This file implements most common graph algorithms such as Kruskal and Prim algorithms for minimum spanning trees, Dijkstra’s algorithm, Ford and Fulkerson Labelling algorithm and others.

The aim in the development of this work has been the implementation of these common algorithms in a way that eases the teaching and learning of Graph Theory. The use of "display step" facility has been shown as a powerful tool for a better comprehension of this matter and makes DERIVE 6 to become a white-box CAS.

Some examples of the algorithms are included with the corresponding comments on the advantages of using them for teaching algorithm like Ford and Fulkerson Labelling algorithm.

Graph Algorithms using Display Step in DERIVE 6 (pdf)

GRAPH_ALGORITHMS.dfw
Towards the European Higher Education Area: 
A Balanced Use of the CAS

The European Higher Education Area (EHEA) implies inevitably, at least in Spain, a change in the traditional teaching of mathematics in Engineering Schools. In some cases, the new teaching based on the acquisition of competences involves an authentic methodological revolution in which a tutorial activity and the use of new techniques, including Computer Algebra Systems (CAS), is going to play an important role.

In this paper we will show how to develop within this educational frame a teaching strategy that will permit to tackle in a global and multidisciplinary form the complete resolution of different Engineering problems that can be modelled in mathematical terms, using this context to get the students to acquire an effective command of the use of the mathematical method in the resolution of Engineering problems. To carry out this strategy implies for each problem we tackle, to establish concrete objectives, the exposition of the involved mathematical elements, the search for the resolution from the Engineering point of view, the choice of the adequate methods, the introduction of possible methods of solution, exact and approximate, the qualitative and quantitative study of the problem and so on.

Some reflections will be offered on the advantages and inconveniences of this strategy and on the other part, by means of different examples the opportunity and intensity of the use of the different CAS will by analysed. These examples show the advantages of using CAS in the learning process. Some examples related to electric and electronic circuits will be developed at length since its study covers a wide range of mathematical techniques such as systems of linear equations, solutions of ODEs, Laplace transforms, Fourier analysis, and control theory, etc.
Using Graphic Calculator in Teaching and Learning Mathematics: Effects on Students' Achievement and Metacognitive Skills

There has been a steady increase in interest in using hand-held technologies, in particular graphic calculators, by mathematics educators, curriculum developers and teachers. The choice of graphic calculators is motivated mainly by the potential for them to be available to essentially all students all of the time (Kissane, 2000). To date, there is a substantial body of research into the use of graphic calculators (Dunham and Dick, 1994; Ruthven, 1996; Penglese and Arnold, 1996; Heid, 1997; Dunham, 2000; Burrill et al., 2002).

Although handheld graphing technology has been available for nearly two decades, research on the use of technology is not robust (Burrill et al., 2002). Its use in secondary classrooms is not well understood, universally accepted, nor well-documented.

In Malaysia, research on the usage of graphics calculators is still in its infancy and therefore its use has to be explored. Thus, there is a need to further research in this area in the context of teaching mathematics at the Malaysian secondary school level.

This study employs a quasi-experimental design using two intact groups of form four secondary school students. The main objectives of the study is to investigate the effects of using graphics calculator on form four secondary school students' mathematics achievement and metacognitive skills in the learning area of relation and function. Students' views about their experiences using graphic calculators in learning of mathematics, the benefits of using graphic calculators in the learning of mathematics and the difficulties caused by using graphic calculators in practice were also sought. Preliminary findings of the study have provided pedagogical impact on the use of graphics calculator technology as a tool in teaching and learning of mathematics in Malaysia.
Der Einsatz von CBL/CBR im fächerübergreifenden naturwissenschaftlichen Unterricht

Vortrag


Experimente für den Einsatz von CBL/CBR
im fächerübergreifenden naturwissenschaftlichen Unterricht

Workshop


Ein Besuch des Pilotvortrags ist für den Besuch des Workshops nicht notwendig.

Experimente für den Einsatz von CBL/CBR (pdf)
Using DGS for Working on Realistic Billiard Tasks

So-called "Mathematical Billiard" using a variety of table geometries has been a topic of several didactical investigations, partially including the use of dynamic geometry systems (DGS). In our approach, a more realistic model called "Physical Billiard" is considered where balls are modelled as circles and loss of energy at collisions can be included. We investigate how a DGS (Cinderella) can be used for solving typical tasks in Carom Billiard. The constructions should be flexible such that the positions of balls can be changed using the dragging mode. Some of the tasks can be solved directly by constructing a suitable direction for hitting the cue ball, for others so-called "soft constructions" can be set up which can be used to find a solution in the dragging mode. The latter constructions can also be interpreted as blue prints for real mechanisms including rotational and translational joints, and such mechanisms can really be built. The "Billiard tasks" are particularly suitable for student projects.

Using DGS for working on realistic Billiard Tasks (pdf)
Posing and Solving Linear Algebra and Statistics Problems with Maple

When teaching applied mathematics to engineering graduate students emerge some specific topics that should focus with student's resources or maybe preferences. I analyze two problems that were posed and solved with Maple in a linear algebra course with Mexican graduate students in 2005. The main objective of the course was students' use of concepts and methods of linear algebra to formulate and solve engineering problems with software. The course was mostly oriented to student acquisition of linear algebra concepts and solving linear programming problems. That course was previous to others like optimization, operation research, and decision theory. At the beginning of the course, students where introduced to Maple basics commands of linear algebra and statistics, then they designed some research-class projects to apply linear algebra concepts and methods. The teaching approach was centered on student interest and the professor role was a promoter of conceptual change (Light, G. y Cox, R., 2001)

The first problem came from a structural analysis in civil engineering, and a solution was approximated with a Maple program that use the Jacobi's iterative method (see Balderas, 2005) The second one was thought to link linear algebra with some statistics concepts as multiple regression, by using least squares problem.

References
Michel Beaudin - Ecole de technologie superieure/Service des enseignements generaux, Montreal, Canada

Theory versus Technology: don't look for a competition, look for a collaboration

When time comes for a limitation—or even an elimination—of technology in learning and teaching mathematics, one of the major argument is the following: before using a symbolic machine, the student should be able to go through all the required calculations by hand, using pencil and paper techniques only. This may be surprising but many professors still think this way: after over 10 years of massive presence of technology at Ecole de technologie superieure (ETS: engineering school in Montreal, Canada), engineering teaching still follows a classic way and many teachers are satisfied with this. Using examples from differential equations, our lecture will try to show that a well balanced use of pencil and paper techniques, along with technology—ever available technology—yields a better learning environment. And you don't necessarily have to change (all) the curriculum!

Simply take a look at what you have to teach: with technologies like the ones built into Voyage 200 and Derive, we are convinced that basic concepts will be better assimilated by the students and that mathematics lessons will become more interesting. More precisely, we will illustrate the followings:

a) with the Voyage 200 symbolic calculator, problems involving Laplace transforms need to be solved using both known formulas and calculator built-in functions. This is a good illustration of what we have experienced at ETS since 1999.

b) using paper and pencil techniques only, students sometimes don't have any idea of the result they get. Derive, with it's ability to integrate piecewise continuous functions, is a good tool for understanding the Dirac delta "function ". Moreover, the slide bar can help in order to see what is going on.

c) sometimes, higher mathematics concepts can't be illustrated without a computer. And for live presentations, Derive allows the user to get to the point right away.

Theory versus Technology (pdf)
The TI symbolic calculator is now universally used as a teaching and learning tool at Ecole de technologie superieure (ETs). It was initially introduced in 1999 as mandatory in the introductory calculus course but is now used in all the mathematics and science courses at our university. From remedial mathematics to calculus and differential equations, our courses have seen a gradual change in paradigm since 1999. The most recent changes have affected the way, and the purpose for which, we use the graphing interface of the symbolic calculators.

The object of this talk is to share our experience in the use of the graphical interface of the symbolic calculator (together with the symbolic and numerical capabilities) in order to present how it improves both our teaching as well as our students' learning experience. More specifically, we will demonstrate how we use the different graphing modes available on the Voyage 200, to:

a) look more closely at theorem hypothesises and their importance in mathematical constructs;
b) bring forth a renewed appreciation of numerical methods;
c) make mathematics come alive by enabling students to develop modelling skills as early as possible by working with advanced concepts without having the usual algebraic dexterity traditionally acquired prior to the introduction of "real world" problems that are so important to the engineering student's instruction.

With this era of readily available technology, we have made some hard choices about what and how we teach. Detractors say that we are doing less than we used to, we know that we are doing so much more!

Confessions of a CAS User (pdf)
CAS-Calculators in a centralized national exam -
Report from a first experience in 2006

Luxembourg has a long tradition with centralized national exams. Until 2006, neither graphical nor CAS-based calculators were allowed in the mathematics-test in the final exam in secondary school. Since 2003 the CAS-based calculator Voyage 200 is a mandatory tool for teaching and learning in mathematics classes for grades 11, 12 and 13 and its purposeful use will be compulsory element for designing questions in the national final exam.

Since 4 years, the national education system is preparing this major change and the present paper will try to present answers to the following questions:

a) What were the reasons for this major change?

b) How was the project implemented?

c) What problems did the project encounter?

d) How did the assessment in the final exam change?

e) What lessons can drawn from a first experience?
Dynamic Geometry and Computer Algebra Systems are used more and more in mathematics education. A lot of excellent and effective software products is available for both technologies. There are pros and cons for all programs.

Unfortunately we can use these wonderful tools only separated from each other. So we do geometry using Cabri, Euklid, Sketchpad, Cinderella, GeoGebra to name some of the tools and then we proceed with a CAS to reach generalized results or to prove our conjectures or we go the reverse way.

The 2D- and 3D-slider bars which are implemented in DERIVE 6 make it possible to narrow this gap between the two technologies: we can now work parallel algebraically which is abstract and geometrically which is very concrete -and in this way connect two very important representation forms. They accomplish each other and mutually confirm their results. Further generalizations often lead to interesting and surprising conclusions.

In this workshop we will work out two not too expanded examples from secondary school level mathematics.

Slider Bars narrow the Gap (pdf)

Example 1 (Directrix) DERIVE file

Example 11 (Property of a Parallelogram) Derive file
Background Pictures as a Stimulating Means for Math Teaching

Derive 6 and the TI-92/V200 are offering the opportunity to place pictures (photographs, cliparts, scanned materials, etc.) in the background of the 2D- and 3D-Plot Window (Derive) and the Graphic Screen (TIs). This opens a wide field for classroom activities and gives the chance to include our environment into math teaching at all levels of math education.

We can send out the students looking and picturing with their digital camera: "Hunting for Parabolas", "Curves, Curves, Curves, ...". "Logos of famous Companies reproduced", even ordinary traffic signs tempt hesitating students to do mathematics.

Background Pictures (ppt-presentation as pdf)
Hans-Joachim Brenner - Albert-Schweitzer-Gymnasium Erfurt, Germany

Darstellung der Entwicklung meines Mathematikunterrichts mit CAS-Rechnern - mein persönlicher Lernprozess


Darstellung der Entwicklung meines Mathematikunterrichts (pdf)
Jean-Jacques Dahan - IAM of Grenoble and IREM of Toulouse, France

Becoming quickly a skilled User of Cabri 3D Workshop

During this workshop we will learn how to become a good user of Cabri in realising some special files based on the introduction of conics as sections of cones and on manipulations of polyhedra and their nets. We will add architectural examples to give a ludic approach of Cabri 3D.

Visualizing solutions of differential equations of the second order with Cabri 2 Plus Workshop

I have shown in Montreal how to visualize solutions of differential equations and systems of the first order (using the first derivative functions). In Dresden I will show that it is also possible to obtain such a visualisation of solutions of differential equations and systems of the second order using techniques we have developed this year in my group of research of the IREM of Toulouse. Such techniques could be used for equations and systems of further orders but an experimentation needs to be done to test the power of Cabri for the calculations used in these techniques. These experimentations will be done soon and perhaps the results can be presented in Dresden. We will visualize the solutions most classical equations of the second order.

Many Cabri-files are in folder \\DES-contribs\\Dahan
Dynamic Calculus with GeoGebra

GeoGebra is an educational mathematics software that joins dynamic geometry, algebra and calculus in a new way in order to foster active, problem-oriented, experimental and discovery learning. One interesting application of GeoGebra is dynamic calculus which will be demonstrated using several examples.

Dynamic Calculus with GeoGebra (pdf)

GeoGebra files are in folder \DES_contribs\Dorfmayr

You can install GeoGebra from \DES_soft\.
"Learning Paths" in Classroom Teaching

The use of "Learning paths" in math teaching has strong influences on the organisation of classroom teaching. Various teaching skills are required to integrate "Learning paths" into classical teaching methods or to combine them with learning platforms.

Various forms how to organize the learning process are shown using concrete examples.

Learning Paths in Classroom Teaching (pdf)

14 Learning Paths (in German) (html)
Lernpfade zur Medienvielfalt im Mathematikunterricht

Workshop


14 Lernpfade (html)

You can install GeoGebra from \DES_soft\.
Click here.
Heronian Simplexes & Constructs

History documents a continuing search for formulations of the measure of the interior of constructs in terms of edges. A couple of thousand of years ago a remarkable formulation for the measure of a tri-lateral, Heron's formula, was documented. Almost two thousand years later another remarkable similar edge only formulation breakthrough occurred, Carnot's formula. With these two discoveries the formulations for the two and three-dimensional cubes and rectangular parallelepipeds can be derived. Tri-laterals and quadra-hedrons are two and three-dimensional simplexes. Some years ago Dr. Edgell composed a formulation for the measure of the interior of integral-dimensional simplexes as a function of edges only, and thereby the formulation for determining the measures of constructs in terms of edges. And, this formula is easily programmed onto TI-calculators. Thus students in-put edge data of the tessellating simplexes of a construct, using the vertices, and sum of the out-put data is the interior of the construct. For students there occurs an almost immediate correlation between the edges of tessellating simplexes and measure of a construct, a nice geometrical gestalt.

Educators, for a multitude of rationales, tend to be interested in acquiring a collection of simplexes/constructs with “nice” facets to use with the sharing/teaching of ideas of geometry. Teachers tend to think of “nice” as being integral. Simplexes/constructs with integral facets are referred to as Heronian. Consequently the focus of this paper/presentations is upon applying the simplex/edge formula to determining the measure of the interior of Heronian constructs. Further, the quest for fourth-dimensional Heronian simplexes/constructs and the role of the simplex/edge formula will be discussed.

together with

Teaching: Simplexes – Technology Connection

n-D simplexes tessellate constructs at the vertices. Thus the measure of the region bounded by the facets of a construct is the sum of the measures of the interiors of the tessellating simplexes. Unfortunately in most public schools in the United States students wade through a limited maze of specific constructs with the expectation that students will hopefully select the “correct” formula from a corresponding myriad of formulas and then not make a mistake in calculating. Dr. Edgell, while researching with classes of middle school students on aspects of this issue developed a formula for calculating the interior of any n-D simplex in terms of edges. All too often seemingly tedious mathematical procedures in combination with mistakes unnecessarily prevent students from understanding ideas. Friendly pre-programmed calculators can allow students to focus upon the figure, the edge attributes, and virtually make an immediate
association with the figure and the size of the interior, which is important. Dr. Edgell is a “constructionist” in sharing mathematical ideas and with respect to the importance of this pre-mentioned association, figure/edges/interior, and some other important issues, he incorporates technology. Dr. Edgell continues to research simplexes and constructs as well as this simplex-calculator connection with classes of students at all levels. He has similarly prepared/taught thousands of pre-service mathematics teachers at all levels, and likewise shares such with in-service professionals in workshops across the nation. Further, the simplex formula - technology connection is contributing to determining and understanding Heronian Simplexes, a source of “nice” simplexes which tessellate “nice” constructs. “Nice” is in reference to the comfort of teachers and students in working with figures having integral results. Dr. Edgell will feature Heronian simplexes in sharing the simplex formula, programming techniques for the TI-82 calculator, examples/practice, and experimental/teaching experiences.

Heronian Simplexes (pdf)
Franz Embacher – Universität Wien/Inst. für Theoretische Physik, Vienna, Austria

The Didactical Significance of Interactive Animations

A picture says more than thousand words. A dynamical animation says more than thousand pictures. By using interactive animations in mathematics education new patterns of perception may be activated and presumably lead to new forms of the inner representation of mathematical issues. This conjecture is illustrated in terms of some.

The Didactical Significance of Interactive Animations (pdf)
A New Toolkit for Simplifying Trigonometric Expressions

Simplification of trigonometric expressions is an important problem that hasn’t been completely solved by current computer algebra systems. This paper presents a new method for simplifying trigonometric expressions. The method is based on a number of unique prescriptions for the ordering of some trigonometric rules, which have been derived by observing how human experts follow their intuitive rules, and it can conveniently simplify trigonometric expressions automatically or interactively. We have implemented the procedure in LISP because of its suitability for formula manipulations and rule-based reasoning systems. Consequently, it can achieve much better results than Maple and Mathematica in the simplification of many trigonometric expressions, and it can generate readable proofs that can be verified step by step. Therefore, the toolkit is helpful in K12 education.

readme.pdf

A New Toolkit for Simplifying Trigonometric Expressions (pdf)
Launch Program sjhs  (then select sjhs.img in folder \des_contribs\fu)
In this paper we present the file MULTIPLE-INTEGRALS.MTH, created to be used in subjects that deal with double and triple integrals, aimed at Engineering students. Such file contains a series of macros which permit to solve multiple integrals problems.

The macros contained in the file can be grouped within the following blocks:

♦ Double integration (in Cartesian, polar and other coordinates).
♦ Triple integration (in Cartesian, cylindrical, spherical and other coordinates).
♦ Green-Riemann's theorem.
♦ Applications of multiple integrals.

We also show in the paper some examples of applications that have been carried out with our students of Telecommunication Engineering. The macros have been developed using the Display function in order to be used as didactical tools with explications of what the macros do step by step (using DERIVE 6 as a PeCAS or as a white-box CAS).

Finally, we include the conclusions obtained after using this file with our students and also some future work on this subject.
DERIVE 6 as a Pedagogical CAS: Programming Using Display Function Workshop

In this workshop we will use some examples of programming with DERIVE 6 using the Display function in order to use this software as a PeCAS. We have developed this kind of workshop with our students of Technical Telecommunication Engineering. After the results obtained we think that programming should be part of the teaching of Mathematics at the undergraduate level.

The main innovative aspect of this way of teaching is that students have an active role. Specifically, they have to elaborate by themselves utility files to solve the typical problems for the different subjects. In our case, this fact implies that students need to deal with programming in DERIVE 6, understand the subject and know how to solve typical problems.

The didactical method we will use in the workshop can be resumed in the following two points:

1. During the first part of the workshop, the conductor will show the participant how to create different macros to solve typical problems of Engineering, including some graphical utilities to help in the process of resolution.

2. The second part will be dedicated to help the participants in building additional macros related with this subject.

We will provide some material with the theoretical aspects needed in order to make the workshop easier to follow.

- Auto-check exercise
In this paper we present the file LINE-INTEGRALS.MTH, created to be used in subjects that deal with line integrals, aimed at Engineering students. Such file contains a series of macros which permit to solve line integrals problems.

The macros contained in the file can be grouped within the following blocks:

♦ Parameterization of curves.
♦ Exact differential form.
♦ Potential function.
♦ Line integral.
♦ Line integral with respect to arc length.
♦ Applications of line integrals.

We also show in the paper some examples of applications that have been carried out with our students of Telecommunication Engineering. The macros have been developed using the Display function in order to be used as didactical tools with explications of what the macros do step by step (using DERIVE 6 as a PeCAS or as a white-box CAS).

Finally, we include the conclusions obtained after using this file with our students and also some future work on this subject.
Using DERIVE 6 to find the Equation and to visualize a Locus of Points in the 3D Space

In the last year of secondary school, a classical 2D analytic geometry exercise is to determine the locus of points whose sum of the distances to two given intersecting lines is a positive constant. Once a suitable system of coordinates has been chosen, the only difficult part of that exercise for the students is to realize what the locus precisely looks like. With DERIVE 6, this part becomes trivial but it remains interesting to ask the students to explain why the locus looks like it does.

Let us generalize the problem to the 3D space. We look for the locus of points of the space having the property that the sum of their distances to two given intersecting lines is a positive constant. In this case, the students encounter several difficulties. The first one is to find a formula for the distance of a point in the space to a given line, formula that the students generally do not know. The second one is to obtain a suitable expression for the equation of the locus and the third is clearly to visualize the locus for several values of the parameters contained in the equation. Using DERIVE 6 the second and third difficulties disappear though it may prepare us some surprises ...

An interesting extension is then to try to understand better the nature of the locus by determining and visualizing its intersection with a variety of planes. Here again, DERIVE 6 can play a decisive role.

Using DERIVE 6 to find … (pdf)

Derive-files are in folder \DES-contribs\Gossez
Using Computer Technology to Enhance the Teaching & Learning of Engineering Mathematics

The HELM (Helping Engineers Learn Mathematics) project was a major three-year curriculum development project undertaken by a consortium of five UK universities and sponsored by UK government funding from October 2002 -September 2005. It used the expertise from consortium partners and computer technology to enhance the mathematical education of engineering undergraduates through the development of a range of flexible learning resources in the form of Workbooks and web delivered interactive courseware elements together with an integrated web-delivered CM implementation.

This paper first describes the HELM learning resources. These consist of Workbooks, Computer-Aided Learning (CAL) courseware and Computer-Aided Assessments (CM). As well covering the mathematics essential for engineering undergraduates in the first two years of their degrees, the 50 Workbooks include engineering examples and case studies, a students' guide and a tutors' guide. The CAL courseware, consisting of on-line interactive lessons to aid understanding, is web-delivered and based on many of the first 20 Workbooks.

An extensive CM regime, which facilitates the regular testing of large numbers of students, is used to drive student learning. It takes two forms, either an integrated web-delivered version or an alternative stand-alone CD-based version. Its implementation and its use for both formative and summative assessment of engineering students learning mathematics are outlined. The CAA regime powerfully encourages students to engage more in their own learning and has been essential to the success of the project.

Finally the viability of adopting the HELM learning resources and implementing the CAA regime at other institutions is examined. Use of these resources can have a positive impact on the learning experience of students.
Geometry with Derive

Derive 6 possesses some qualities and tools which make it useful in teaching. In the paper we focus our attention on teaching geometry using these features. We particularly take advantage of qualities such as a uniquely simple way from an algebraic representation of a formula to its plot, possibility of dynamic way of exploration using the slider bar in graphical windows, transparent notation corresponding with the usual representation of geometrical figures and utilization of a simple programming language to tailor functions of the program to solutions of particular problems.

The way of incorporation of Derive into the teaching of geometry at high schools and in introductory courses at the university is demonstrated in the paper. Several examples of growing complexity concern with problems of intersections of geometrical figures and problems of loci of points. Although the problems are solved analytically the visual presentations of solutions are stressed. A package of several simple procedures that are used in solutions of problems is also introduced in the paper.
Mathematics with Toads, Cockchafers, CAS and much more

The use of new tools for mathematics at school wins increasingly importance. It follows from this that they are consequences as well as on aims and contents of mathematics at school us like on methods in the lesson. The use of new technologies opens a variety of new possibilities. The lecture should showed general aims of mathematic education at the example of the new teaching curricula in Saxon. With motivating examples shall be represented the contribution of graphic computers and CAS for the realization of didactic strategies how:

♦ experiment,
♦ visualize of mathematics,
♦ motivate,
♦ discovering study,
♦ use the mathematics,
♦ processing's open tasks.

The task examples refer to the different levels of school mathematics.
Mathematik mit Kröten, Maikäfern und CAS


Mathematik mit Kröten, … (pdf)

TI-Programm im Verzeichnis \des-contribs\Heinrich
Sara Hershkovitz - Center for Educational Technology/Mathematics, Tel-Aviv, Israel
Beba Shterenberg - Center for Educational Technology/Mathematics, Tel-Aviv, Israel

Presenting Non-Standard Math Word Problems for Elementary School Students via the Internet

The project described in this work, is a national internet activity dealing with non-standards problems. The main goal of the project was to encourage written communication among students about their solutions and strategies, and to develop new kind of intervention and evaluation by teachers.

We have concentrated on non-standard problems of the following nature: problems that have many solutions; problems that include contradiction and therefore have no solution; and problems that call for non-routine heuristics.

In order to work in community and enable the students to communicate with each other, we established an open discussion group. The group has received four problems each month. The mentor did not react to each student individually; still he conserved his position as a mentor in highlighting new and interesting solutions and strategies that were presented by the members of the group. At that year, 25 schools from all over the country participated in the project. Yet, we have observed that the reaction of the mentor to specific solutions hindered the flaw of the work of the group.

In the following year, on the basis of previous experience (under schools pressure to participate) we have tried a new mode: Again, each month four problems were presented. After a month the mentor summarized the solutions strategies, and typical errors in the discussion group. At that time the number of participants grew to more than 40 schools and the students were divided into three separated discussion groups.

The teachers were also involved. They have learned the new types of problems, the way of communicating via the Internet, and they encouraged their students to be active. The students worked on the problems, mostly in small groups.

In the workshop we will present examples and discuss the students' performance, discussion and teachers ways of evaluation.

Presenting Non-Standard Math Word Problems (pdf)
Guido Herweyers – KHBO/Industrial Sciences and Technology, Oostende, Belgium

Applied Linear Algebra Workshop

In this workshop some fundamental concepts of applied linear algebra will be explored through exercises with the symbolic calculator 11-89 Titanium. Keywords are: matrix algebra, systems of linear equations, vector spaces, linear dependence and independence, eigenvalues and eigenvectors, discrete dynamical systems, least-squares problems.

The exercises are based on "Linear Algebra and Its Applications, third edition update" by David C. Lay.
Didactical Principles of Mathematics Education supported by Electronic Media

The lecture deals with some didactical principles which are significant for the interaction of several electronic media like interactive animations, GeoGebra and Computer Algebra Systems (CAS). One focus is on the role of CAS in the learning paths of our project "Variety of electronic media in mathematics education".

Didactical Principles (pdf)
Dynamic Inspirations with TI-Nspire

TI-Nspire is an innovative new learning system for PC/Mac and TI-calculators. I will present new possibilities with challenging examples from algebra and geometry and a systematic method that provides you with a new access to examples focussing on parameters and the dynamic given by them. Even examples that were written for calculus can be solved easier by this method with elementary tools on a lower school level.

For example, the following well-known problem can easily be solved in the 9th school year: A cubic polynomial has three zeros a, b, c. Show that the tangent line to the cubic through (a; 0) has the osculating point at x = (b + c)/2.

Dynamic Inspirations (pdf)

Workshop with TI-Nspire

Participants are working with the PC version of TI-Nspire and can also test the handheld version. The new possibilities of integrated applications in TI-Inspire and the linking of parameters are used for new examples from algebra and geometry focusing on parameters and the dynamic given by them (compare presentation #31 "Dynamic Inspirations with TI-Nspire").

Workshop with Nspire (pdf)

Several Nspire-files are in Folder \DES_contribs\Hugelshofer
Comparison of Student Errors made during Linear Equation Solving on Paper and in Interactive Learning Environment

T-algebra is an interactive learning environment for step-by-step solving of algebra problems in four areas of mathematics, including linear equations. To make the diagnosis of mistakes more complete and the program more intelligent we designed our own rule dialogue. Each solution step in T-algebra consists of three stages: selection of the transformation rule, marking the parts of expression, entering the result of the operation. The three-stage dialogue of T-algebra gives several advantages for precise diagnosis of student mistakes. We can diagnose separately incorrect choice of operation, wrong selection of operands for the chosen operation and erroneous application of the selected operation.

While designing the program, we have taken into account the results of a study on student mistakes made with pencil and paper. The study was carried out in winter of 2005 in Tartu. We have attempted to leave an opportunity for the student to make the same mistakes in T-algebra, but also to provide the program with information about the intentions of the student for exact error diagnosis. This gives T-algebra the ability to collect data for adequate research on student mistakes in expression manipulation. In winter of 2006 we conducted a study on errors made by the students who solved linear equations in the T-algebra environment. This article thoroughly describes the errors made in the solution of linear equations on paper and the corresponding errors made in the T-algebra environment. It compares them and presents interesting findings arising from this comparison.
The Mathematics and Industrial Applications associated with the Singing Wineglass using CAS

When a disturbance such as an earthquake, hurricane or tornado moves down the edge of a bay, harbour or lake, resonant waves can be generated. Many linear and non-linear mathematical models have been developed to describe this (and similar phenomena) over the years.

The qualitative effect of such a disturbance can be observed in a classroom, with minimal apparatus, on a small scale by anyone who causes a partially filled wineglass to "sing" (resonate) using a wet finger. If one carefully examines the surface of the wine it is possible to see that a pulse (travelling wave) has developed, concentrated dose to the finger, following the finger around the rim. Three other similar pulses (spaced about the rim) can also be observed. The surface of the liquid also exhibits other, smaller patterns (called edge waves in the literature) which we do not attempt to model here. The vibration pattern of the glass is called a "quadrupole vibration" in the literature and the effect of this vibration has undoubtedly been know ever since mankind discovered the delights of drinking liquids from fine glass containers. Indeed the effect of a singing wineglass is noted in the literature by the ever inquisitive Faraday!

We discuss a simulation of the pulses (not the edge waves) using mathematics easy enough for senior undergraduates to understand as well as an animation of this effect using the CAS MATHEMATICA.

We also describe an industrial application of "Bryan's effect (1890)", used in advanced navigation systems. Bryan's effect can be observed when the wineglass is placed on a turntable in order to observe the vibrations from a "fixed position".

This paper thus focuses on some non-trivial mathematics at an intermediate level, the use of a CAS to analyze and calculate some of the equations involved as well as some real-life applications of the effects observed using simple classroom apparatus.
Coordinating the Process and Object Features of Mathematical Knowledge by CAS

Being able to consider mathematical entities both as processes and objects is a key competence crucial to success in learning mathematics, which frequently cannot be demonstrated by many students. Although CAS seems to be a suitable tool to help us improve the matters, its use without explicit learning requirements directed toward coordinating the process and object features of mathematical knowledge may even make things worse. Why may this be so? A task whose standard solution requires a process approach may in a CAS environment be treated by an object approach. Also, a task whose natural solution requires an object approach may be attempted by a process approach. How to coordinate the two views of mathematical knowledge? Or, in other words, how to promote proceptual thinking? To achieve this end, two requirements are to be fulfilled. The first requirement is general: When using mathematics, don't forget available tool(s); when utilizing tool, don't forget the underlying mathematics.

If, for example, one deals with the sine or cosine function, he/she would, if appropriate, directly use simple yet useful facts such as \(-1 \leq \sin x \leq 1\) instead of sketching the graph of the trigonometric function in question and searching for its extreme values.

The second requirement is more specific: To solve the assigned task, use, whenever possible, a process approach as well as an object approach, working in multiple-registers (algebraic and graphical, for example). If, for example, an object approach to the limit in question results in the status "undefined", a process approach would suggest the value of that limit, whereas an object approach would verify it. By using concrete examples from the upper secondary mathematics education, this presentation will illustrate how these requirements may be fulfilled for the benefit of the learner.
Simple Programming in CAS to Create Online Mathematics Quizzes

Frequent quizzes form an essential component of mathematics courses. Through quizzes, we not only assess student learning before it is too late, but also make sure that the student revisits important concepts multiple times. We have seen that in a course with frequent quizzes, students score better overall. Most available online mathematics assessment tools are not suitable to most small liberal arts schools that only support Windows platform. We will explain how we use CAS to create a database of mathematics problems in WebCT in an economical way, what difficulties came our way while administering online quizzes, and how we countered them.

Simple Programming in CAS (pdf)

CAS for 'Discovery' in Multivariable Calculus

In order to get undergraduates interested in mathematics, it is essential to involve them in its 'discovery'. CAS can do the repetitive and tedious tasks, thus making experimentation easy, and allowing the student to focus on the fundamental principles.

In this talk, the speaker will explain how DERIVE and knowledge of lower dimensional calculus can be used to help students develop intuition leading to their discovering the first derivative rule in multivariable calculus.

CAS for Discovery (pdf)
Patterns in Numbers

Key identities and fundamental notions of mathematics such as symmetry appear in an exposition linking three relationships groups of equations involving consecutive integers.

\[ 1 + 2 = 3 \]

\[ 4 + 5 + 6 = 7 + 8 \]

\[ 9 + 10 + 11 + 12 = 13 + 14 + 15 \]

\[ (t - a) + (t - a + 1) + ... + t = (t + 1) + ... + (t + a) \]

Patterns in Numbers (pdf)
Beispiele für den Einsatz von CAS im Lernbereich
Funktionen und lineare Gleichungssysteme
Workshop

Übungsaufgaben in Anlehnung an das neue Arbeitsheft von TI zum Einsatz von CAS in
Klasse 8 am Gymnasium in Sachsen

Beispiele für den Einsatz von CAS (pdf)
Motivierender Mathematikunterricht mit CAS, ein Erfahrungsbericht

How can English final exam make mathematics lessons more interesting or Mathematical aspect of the book “The Curious Incident” by Mark Haddon

In my presentation I will introduce the mathematical content of the book: "The Curious Incident of the Dog in the Night-time" by Mark Haddon. At the end of the high school, Slovene students have to pass final exams. For English they have to read this book containing many mathematical tasks, models, and remarks. The mystery novel is told in the first person by the protagonist Christopher Boone. He is fifteen years old and suffers from Asperger's Syndrome, a form of high functioning autism. Christopher has the mathematical abilities.

One of the themes in the book is prime numbers. I have prepared tasks, which the students solve with the use of the internet. The worksheet is arranged for different students' abilities and interests. We mention the use of the primes in cryptography. I introduce the RSA-algorithm, in the class we check the algorithm by using Derive. Perhaps the most amusing mathematical interlude of the novel is the discussion the Monty Hall Problem. In the class the students use an interactive applet to simulate individual trials of the situation from the Monty Hall Problem and then they simulate a large number of trials.

Christopher says that sometimes a mystery is not a mystery and as example he takes the logistic equation to prove it. The logistic curve is the topic we do not have in our syllabus but it should be part of the repertoire of functions we teach in high school. It describes the behaviour of data in science and business and using CAS we have the opportunity to study the logistic function in all variations, including chaotic variation. In my presentation I will introduce the project I have done in my class. With this project I have related the mathematics to the realistic problems in life, which is a significant part of the curriculum.

Mark Haddon brings the mathematics in to the novel in a natural way. On the same way by using CAS I discuss new mathematical problems from his book in the class.
Josef Lechner. Dr. -Gymnasium Amstetten, Anzengruberstrasse 6, A-3322 Amstetten, Austria

See it Complex – Make it Easy!

There are some isolated topics in school maths that are not very popular and therefore sometimes neglected. Different vector products, plane curves and complex numbers respectively, are among them.

There exist two vector multiplications in school maths: dot product (with dimension 1) and cross product (with dimension 3) A product with dimension 2 is missing. An reasonable operation this kind exists in form of multiplication of complex numbers, but this multiplication is rarely seen as a vector product. This is a pity, because it would be a unique opportunity to help pupils give more meaning to complex numbers.

Moreover such a vector-oriented view of complex multiplication and complex numbers as a whole enables us to develop different visualization methods for complex numbers.

By using such visualization techniques concerning complex numbers the lecture is finally intended to show surprising an amazing connections among different mathematical branches.

See it Complex – Make it Easy … (pdf)

DERIVE files

Geogebra files

Geogebra can be installed from the CD (just click on the link)

Having installed GeoGebra update for Version 2.7.1 (just click on the link)
Mathematik animieren


The Power of Modules

Several software systems, especially CAS, are including modules functions with parameters. The lecture will show you the power of teaching math with modules and their parameters in the classroom. Use system-modules, define them by yourself, solve problems with them and analyse modules. Use them to visualize and animate terms and work with them experimentally!
Using DERIVE to Teach Bioinformatics Algorithms Workshop

In this workshop the participants will explore the algorithms used in basic bioinformatics investigations. The first series of exercises will be concerned with DNA sequence analysis and the alignment of two DNA samples. Included in this portion of the workshop will be a brief introduction to local alignments and BLAST. The second portion will involve the use of Markov Chains to determine phylogenetic distances and the construction of phylogenetic trees.

Bioinformatics (pdf)
DNAGenerator.dfw
SequenceAlignment.dfw
Blast.html
Neighbor Joining.html
Patricia Leinbach - Adams County, Pennsylvania/Coroner, retired, Gettysburg, Pennsylvania, USA;
Carl Leinbach - Gettysburg College/Computer Science and Mathematics, Gettysburg, Pennsylvania, USA

Mathematics in Action - Using Mathematical and Scientific Knowledge in Forensic Investigations Workshop

Presenter: Patricia Leinbach
Assistant: Carl Leinbach

Popular television series have created a great interest in "Forensic Science" amongst several sectors of the population. In particular, several Secondary Schools have instituted courses dealing with Forensics or including exercises in their courses. Forensics, used in this context, means the application of science for examining evidence suitable for presentation in a Court of Law. In this workshop, four examples will be given for the application of mathematics to the gathering of Forensic evidence. The topics of the investigation are: Determining Height From Stride Length; Calculating Vehicle Speed From the Length of Skid Marks; Analyzing Blood Spatters; and Estimating Time Since Death.

Forensic Workshop (pdf)
Solving polynomial transformation problems is one of the important skills taught at school. It is also one of the four topics of school mathematics implemented in the intelligent learning environment T-algebra. In T-algebra problems are solved step-by-step using transformation rules taught at school. When making solution steps in T-algebra the students make all the decisions themselves: select which rule to apply, select the operands of the rule and enter the result of application of the rule. A special three-stage solution step dialogue was designed for that purpose.

Prior to designing the solution process and available rules in T-algebra, we conducted research on how the students apply different rules and what errors they make when solving transformation problems using pencil and paper. We implemented all transformation rules that are taught in school textbooks, added some other simplification rules and some more rules that we collected from student papers. When designing different rules we attempted to leave a possibility for the students to make most of the common mistakes and to implement diagnostic procedures for them.

In this paper we give an overview of all the rules available in T-algebra and designed for the polynomial transformation problems. We describe the necessary student input for the application of those rules: what parts should students select as objects and what parts (and in what form) of the resulting expressions should be entered. We also give some examples of applying the rules and present some possibilities of how the students can make errors in applying these rules.

Finally, we describe the solution algorithms that are used by T-algebra (how does it solve problems using the implemented set of rules) and can be used by the students for solving polynomial transformation problems.
Signal Processing using a CAS

The purpose is to show that a Computer Algebra System is also very suitable in a lot of subjects in Digital Signal Processing like Aliasing, Discrete Fourier Transform and the Sampling Theorem.

1. Introduction.

It is impossible to overestimate the importance of signals to modern life, if we think of signals as information carriers, we might imagine TV, mobile phones, radar, sonar, X-ray, music, video, voicemail etc. Clearly, Analog and Digital Signal Processing is a major topic in Electrical Engineering. However, Signal Processing demands a lot of mathematical skills which even to better than average students proved to be a formidable obstacle.

The students involved should be completely familiar with a wide variety of techniques but even than they would be slowed down by lengthy calculating exercises, leaving insufficient time to acquire a thorough understanding. Many of the concepts and algorithm, needing for analyzing and exploring Signal Processing, are incorporated in software, such as MATLAB, but in my view it is essential to understand what is computed and how it should be interpreted. The use of a CAS like Derive, on a PC or on the Ti-89, is an excellent way to receive that necessary background.

2. Signals.

The paper will show, starting with the most simple but very important class of signals, sinusoids, which are so important because they are de basis for making more complicated signals by so-called Fourier series, so that we can make a simple step to the Discrete Fourier Transform

3. Discrete Signals.

As mentioned the paper will show how we derive the Discrete Fourier Transform (DFT) from the Fourier coefficients. It will show also, in a few examples, how we use the OFT to analyse signals, one of the most important issues in Signal Processing these days.

The paper will also present a first step in understanding the concept of Aliasing.

4. Concluding remarks
Selected Themes from Teaching Calculus with CAS Workshop

The ways in which basic themes of calculus like limit, differentiation and integration are introduced differ. In the workshop we would present several worksheets taken from the all aspect of the subject - from the concept of the limit to the usage of integrals. We want to exploit various possibilities of technology. With the aid of computers students learn new mathematical facts by themselves. Of course such an approach should be supported by systematically prepared lectures.

We intend to show such a way of teaching calculus with several examples. In the workshop we intend to present different ways where Derive and other programs combined with internet resources can be used. Our worksheets are prepared as a foundation for self-discovering work. as a reinforcement of knowledge of certain topics. as a main material for a guided hour etc. Let us briefly describe some of them:

Students often have problems connecting the geometrical meaning of the derivative and the tangent line of the graph to the function. The worksheet ‘Geometric Meaning of the Derivative’, prepared as a guided hour, tries to circumvent this. With stepwise solving, self-discovery can be often carried out more efficiently. We exploit this in the worksheet where we try to lead the students to discover derivative rules by themselves. In the worksheet Volume of revolution we exploit the power of Derive to show students how tools are just helpers we still have to make our mathematical model, select suitable function and use the proper methods. This worksheet is an example that in spite of all progress in technology, sound mathematical knowledge is still needed.

The main intention of the workshop is to discuss prepared materials. We would briefly make an overview of worksheets with an emphasis on technical aspects. In the second part participants would use the worksheets in order to facilitate the discussion, where ideas about materials and approaches to teaching calculus will be discussed.
Student Projects in using DERIVE to help Teach Topics in second-level Mathematics Workshop

Conference participants can look through a selection of second year students' projects on the theme: "Take a particular topic in second-level mathematics and see how DERIVE could be used to help teach it".

Each project typically consists of (i) a DERIVE session of about 100 expressions and (ii) a report on the project of about words. These projects exemplify students' ideas on how to use CAS in the classroom and as such may be of interest.

Students Projects using DERIVE (pdf)
Interactive Work with Computer Algebra Systems when Solving Problems on Physics

It is an obvious fact that using Computer Algebra Systems (CAS) for solving physics problem is not always straightforward. Very often an interactive human contribution is necessary, not only on conceptual level but also on the level of technical side of the calculus. In this presentation we show a few examples of such problems and the action that leads to the solution. One should note however, that even if the human interference is sometimes necessary one can not overestimate the help provided by CAS.

As the first example it was shown how to obtain the general expressions for the radial and transversal components of velocity and acceleration. In the second example the general analysis of two bodies collision in 1D has been done, in which the perfectly elastic and perfectly inelastic collisions appear as two extreme cases. The analysis has then been generalized on 2D case of the Compton Effect. For the analysis the DERIVE 6.1 and the MATHEMATICA 5.0 were used.

\[
\frac{m_1v_1^2}{2} + \frac{m_2v_2^2}{2} = \frac{m_1u_1^2}{2} + \frac{m_2u_2^2}{2}
\]
Giora Mann, Beit Chanan, Israel, 
Nurit Zehavi - Weizmann Institute of Science/Science Teaching, Rehovot, Israel

Changing the viewing angle on a conic section: 
Exploring the interplay between reflection and execution

For a given hyperbola, what are the points in the plane through which pass tangents to the hyperbola? Can we identify regions from which the hyperbola is viewed in an acute/obtuse angle? What is the role of the asymptotes and the "director circle" of the hyperbola? What is the contribution of methods in dynamic geometry to our exploration with computer algebra software? What is the evolving role of Operative Knowledge as mediator between Reflection and Execution?

Using CAS, we will demonstrate that the outside of every hyperbola is divided into two disjoint regions by the asymptotes and the director circle - one region of vertices of acute viewing angles; the other of vertices of obtuse viewing angles. The angle between the asymptotes determines if there is a real director circle, and hence, if each of the two regions is composed of 4 disjoint sub-regions or only two. Using slider bars enabled us to explore dynamically the unfamiliar results.

In the research and development process of a resource e-book for teaching Analytic Geometry with CAS we explored the interplay between reflection and execution. We realized that involving CAS in the Execution of the tasks monitored by Reflection acts on our Operative Knowledge that must adapt itself to its evolving role. In return, the evolution of Operative Knowledge has obvious influences on the ways of Reflecting.
A Refined Algorithm for Solving Polynomial Equations

This paper outlines an improved algorithm for the solution of polynomial equations by the use of Descartes' Rule of Signs, The Rational Root Theorem, and a lesser known theorem which have termed "The Coefficient Sum Theorem".

A Refined Algorithm (pdf)
LeActiveMath Workshop

This workshop extends Prof. Dr. Reiss’ keynote and offers the possibility to work with a prototype version of the e-learning environment LeActiveMath that was developed in close collaboration between experts in math education and computer science. The LeActiveMath system is an innovative third generation e-learning system for school and university level learning as well as for self-study. It offers new ways to learn mathematics by adapting itself to the goals of the individual learner, to her/his competencies and motivation(s). Interactive tools serve to deepen the mathematical knowledge. The system is based on modern pedagogical approaches.

In the workshop, we will present an overview on the LeActiveMath system, its components and its pedagogical foundations. Participants will get to know the basic features of the software and "learn" with it on their own. Special emphasis is put on the adaptivity of the system, especially the dynamic course generation.

The workshop will be truly interactive and we look forward to lively discussions.
Potential Uses of Technology in Mathematical Problem-Solving

The explosive development of digital instruments makes it necessary to reflect on the extent to which their use influences students' ways of understanding and solving mathematical problems. Understanding to what extent the use of dynamic mathematics software becomes relevant to explore mathematical relationships is part of a framework that helps students reflect on mathematical practice. This includes posing questions, using distinct representations, making conjectures, and communicating results. It becomes important to identify what type of mathematical reasoning students develop as a result of using digital tools in problem solving experiences. It is recognized that students' disposition to problematize their own learning plays a fundamental role in developing resources and strategies to solve problems.

The former are questions that guide the development of an ongoing research project aiming at documenting the type of mathematical competences that students develop when they systematically use dynamic software and at understanding how mathematical concepts are transformed as they are given a new life through the digital tools they inhabit. We have worked with high school teachers who are implementing mathematical tasks in their regular mathematical classes. We would like to address relevant aspects related to:

The type of conjectures and arguments that students propose as a result of representing the problems dynamically (Cross Theorem) and the process of using simples objects (segments, lines, perpendicular bisector, etc) to ensemble geometric configurations (Conics) that lead students to reconstruct mathematical relationships. We have chosen examples that illustrate students' ways of thinking of mathematical problems that have emerged while using digital tools.

Figure 2: Exploring an angle behaviour

Potential Uses of Technology (pdf)
E-Learning for Math Class using Maple and MapleNet

In order to help students understand the meaning of mathematics used in physics, it is useful to show visualization and/or animation of expression of mathematics. We use mathematical software Maple in the class of "Mathematical method in physics" for the purpose. Maple has many kinds of functions and libraries and we would like to report about Maplets API among of them. Maplets API is one of the tools to build a graphical user interface and we can develop applications (Maplet application) which do not require students the knowledge of Maple grammar. Furthermore Maplet applications can be carried out on the web browser over the internet using MapleNet and e-Learning is possible.
Reinhard Oldenburg - University of Education/Math, Heidelberg, Germany

FeliX - a Prototypical System that links Computer Algebra and Dynamic Geometry

The learning of algebra and geometry is supported by two different classes of software tools, dynamic geometry systems (DGS) and computer algebra systems (CAS). The prototype presented in this talk proves that these two tools can be linked bidirectionally so that both representation forms (algebraic and geometric) are seamlessly integrated.

The presentation will demonstrate the system, give the basic ideas of its realization and explore how the new features.

FeliX (pdf)
From arithmetic operations with real numbers to composition of functions using dynamic number lines
Workshop

This workshop will engage participants in the construction of dynamic number lines and "dynagraphs" (parallel number lines for representing functions) using The Geometer's Sketchpad 4.06. Participants will explore relations between pairs of real numbers and their sums and products qualitatively (without computation), and more complex arithmetic combinations of the two numbers that lead to investigations of functions of two variables. The Dynagraph representation (a pair of dynamic number lines, one for the input variable and the second for the output) will be constructed and used to investigate different categories of functions (linear, polynomial, step, trigonometric, exponential and logarithmic), eventually leading to modelling compositions of two or more functions using a series of linked dynagraphs. A geometric transformation from the pair of parallel numberlines to rectangular coordinates will be constructed and used to relate the dynamic motion of the dynagraph to the coordinate graph of a function.
Using 'Educate' to pose extension problems in mathematics which students can access at home and communicate solutions to other students and teacher

We have a program at the College called 'Educate' which is used by teachers to post information re course work, assessment, assignments and notices. Students can access this at home. The program can also be used by students to enter solutions to problems posed by the teacher. My aim is to set extension problems which need consultation between students and the teacher. The aim of the lecture is to show how this is done and the successes which have been achieved in getting these young students to talk mathematics to each other.

' Educate' is able to be downloaded from the internet. I will be able to download our program from our own college website.
The Rank of a Matrix with Parameters and the Solution of a Linear System of Equations with Parameters

It seems that sometimes the \texttt{ref-} and \texttt{rref-} functions don't work in a good manner, if we consider a matrix with parameters. This is a well-known problem of the TI-CAS-calculators updated with the newest OS version 3.10 too.

In this lecture we introduce a one-step-procedure to transform a matrix with parameters, the new created \texttt{lineqsys(mat,i,k)} function, where \(i\) and \(k\) are the coordinates of the pivot. \(i\) denotes the pivot-row and \(k\) denotes the pivot-column to transform with pivoting the matrix \(mat\). During the one-step-procedure we omit the old pivot-column, i.e. from step to step we get a smaller matrix for the considered problem. Finally we can see the behaviour of the solution of the considered system in the dependence on the parameters.

To get the rank of a matrix with parameters, we use a similar pivoting-procedure and omit the previous pivot-column and previous pivot-row after an exchange-step. Here we use the new created \texttt{rank(mat,i,k)} function.

During the lecture several examples are given and demonstrated with the TI voyage200.

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Paditz, Ludwig: Mathematische Modelle und wissenschaftlich-technische Anwendungen, Beispiele aus Schule und Studium mit dem grafikfähigen Symboltaschenrechner ClassPad 300, Hrg. v. CASIO Europe GmbH im BildungsverlagEINS, Norderstedt2004 (1.Aufl.), 112 S.
Discovering and proving geometric inequalities by CAS

A book by Bottema et al.: Geometric Inequalities, Groeningen 1969, contains several hundreds inequalities from elementary geometry. All the proofs are classical and in most cases the proofs are only outlined. Computer algebra approach in proving a class of constructive geometric inequalities was applied by L. Yang with the program Bottema. This program is based on the method of Cylindrical Algebraic Decomposition. Another computer algebra approach is possible by the use of Groebner bases computation. But there are problems with proving inequalities because of the field of complex numbers we are usually working with. Despite difficulties mentioned above this contribution gives the method which enables proving and even discovering some geometric inequalities by means of elimination of variables which is based on Groebner bases computation. A few examples of elementary geometric inequalities are investigated - a generalization of the parallelogram law which leads to the inequality between sides and diagonals of a quadrilateral, Euler's inequality between circumradius and inradius of a triangle, etc.

Computer solutions are accompanied by classical ones. The both methods, their strengths and weaknesses - are compared.

Using these methods in mathematics teachers education and experience from students' projects at the University of South Bohemia, Czech Republic, are given as well.
Activities with Implicit Functions and Implicit Differentiation on the TI.89/Voyage 200

Unfortunately the topic of implicit differentiation occurs early in most calculus texts (primarily so that there can be a more rigorous proof for the derivative power rule), when the student has no experience with or understanding of implicitly defined functions. Implicit function plotting on these calculators is embedded in 3D plotting, and it is generally not yet appropriate to study functions of several variables either. We present instead methods of working with implicit functions using the CAS solver, the numeric solver, tables, and function graphing all tools that are comfortable for a beginning calculus student. This work can naturally lead to a greater understanding of implicit differentiation, both when the implicit differentiation is done by hand and when carried out using a CAS process.

Activities with Implicit Functions (pdf)
Using the Voyage 200 (OS 3.10) in the Classroom: Surprising Results

We teach a variety of math topics (review of College Algebra, Calculus of one and several variables, Differential Equations, applied probability and statistics, ..) in a Technical Engineering School. The Voyage 200 (or TI 89 Titanium) is mandatory for all new fulltime students. We make use of this calculator on a regular basis, for exploring with students, in the classroom, the entire classical curriculum in mathematics.

Over the years, using as often this tool as led us to encounter examples where the answers given by the calculator may be unsatisfactory, or strange or even frankly false. On some occasion, this CAS system couldn't resolve a problem which should obviously be easy to solve. Our goal isn't to criticize the Voyage 200 calculator. All CAS system will produce, on occasion, some strange results. This can be explained by choices made in programming the system or by assumptions made to ensure that a result can be obtained. On other occasions, a better understanding of the way this CAS works can help understand results which can be surprising. We will illustrate with examples some of these strange behaviours, trying to give an explanation for why this happens or suggesting some modifications which could eliminate them.

This technology, on every student's desk, is a powerful inexpensive tool for exploring mathematics. Users will end up, solving more complex problems, doing more math than with the classical approach. But the examples showed here illustrate the fact that students still have to know and understand the entire basic math curriculum. The CAS system will not think for them!
Numerical methods with the Voyage 200: to function or to program, that is the question!

We teach a variety of math topics (Calculus of one and several variables, Differential Equations, ..) in a Technical Engineering School. The Voyage 200 is mandatory for all new full-time students. We encounter in these courses a few classical numerical methods (Newton's method, numerical integration, Euler or Runge-Kutta methods). Beside the theoretical aspects, these methods are usually illustrated using functions or programs already defined in a CAS or using files or worksheets where students only have to modify some parameters and observe the results. Having to teach a course where numerical methods would be the main subject for about 4 weeks, we decided to ask students to program some of these algorithms using their TI symbolic calculators.

We will explain briefly the differences between a function and a program on these calculators and show examples of the functions used normally in classrooms for the illustration of the classical methods. We will then see how to incorporate important aspects (number of iterations, tolerance, etc.) in the programming of these functions. Why should we want to create a program instead of a function? We will illustrate this with more complex algorithms, using examples from linear algebra (Cholesky and Gauss-Seidel methods) and even show examples of programs done by students.

The aim here was to give students the interest and the ability to create their own programs on their calculator, showing them the advantage of programming in an environment where a great number of math-functions already exist. We were also able to create exam questions based on these programs and methods; examples will be shown in our presentation. We have observed more interest from the students for this topic when we asked them to program their calculators with these algorithms. Many of them were surprised how easy it was. Perhaps, teachers could see with their students slightly more complex functions or programs illustrating these numerical methods.

Numerical Methods (pdf)
Animation of LAGRANGE Multipliers Method and MAPLE

Experience shows that IT is a powerful tool for visualization of hidden mathematical concepts. This article presents how to use animation produced with the help of CAS for demonstration of constrained maximum and minimum of function of two variables with subject to one implicit given constraint. The rule of Lagrange multipliers is demonstrated in this case, too. In this particular - the easiest - case there exist two vectors that have the same direction and Maple allow us to show it.

Animation of Lagrange Multipliers (pdf)
Die Kurvendiskussion ist tot - es lebe die Diskussion über Kurven

Workshop

Mit dem Einsatz Neuer Technologie im Mathematikunterricht ist die Kurvendiskussion, früher ein zentrales Thema im Analysisunterricht, (fast) obsolet geworden. GTR und CAS liefern die Graphen und damit die wichtigen Eigenschaften von Funktionen auf Knopfdruck.

Durch die Wahl geeigneter Funktionen kann man erreichen, dass ihr Plot mit GTR, CAS-Handheld oder mit Derive in einem Standardfenster einen (fast) völlig falschen Graph liefert. Dies wird als Ausgangspunkt genommen, um charakteristische Eigenschaften (Nullstelle, Polstelle) der Funktion mit dem gesunden Menschenverstand, d.h. OH NE Rechner zu suchen. Dann kommt man zu einer recht ungewöhnlichen Kalibrierung des Koordinatensystems, mit dessen Hilfe man alle Eigenschaften der Funktion aus dem Graph ablesen kann.

Ändert man die Funktion nur ganz geringfügig, so erhält man, entgegen der landläufigen Erwartung, einen (fast) völlig anderen Graph. Und dies wird als Ausgangspunkt zur Charakterisierung einer ganzen Klasse von Funktionen genommen.


Im Workshop wird ein Beispiel mit Hilfe des Voyage 200 konkret durchgearbeitet.
Eight Wishes about Computer Algebra Systems

Some weeks ago we received a message from Bernhard Kutzler asking for 7 proposals to improve Derive. We have thought about it, but we have realised that our improvement suggestions are conceptual and general. Based on our experience as software developers, computer algebra systems users and teachers, we have described the possibilities and characteristics of our ideal computer algebra system.

We begun using them for algebraic research in 1987 and have taught undergraduates and postgraduates using Reduce, Derive, MuPAD, Mathematica and Maple since 1990. We have implemented applications in mathematics, bioinformatics and engineering (including a contract with the Spanish Airport Authority).

CASs offer many possibilities, but are characterised by using exact arithmetic and the capability of manipulating non-assigned variables. There are some amazing applications, like symbolic integration, differential equations solving, polynomial system exact solving ...

When thinking about the "ideal" environment for the CAS programmer/user, we have thought about the following desirable possibilities:

♦ Cross-CAS Portability
♦ Cross-Operating System Portability
♦ DGS to CAS Connection
♦ Remote Control of a CAS Session from the Web
♦ Documents and Plots Handling
♦ Possibility of Menu-Oriented Access
♦ Easy GUI Development
♦ Kernel Availability for Developing External Applications.

Some of them are fully or partially available in some systems and some in no system at all nowadays.

The article includes comments and details about the authors' opinions and experiences in these fields (for instance they have designed different connections between DGS and CAS, one of them accessible from the web; they have also implemented some GUls for expert systems that call different CAS...).

Eight Wishes (pdf)
Using technology in math teaching and learning becomes more and more popular in Poland. Teachers can observe that lessons with technology are more attractive for students, students are more active and motivated during these lessons. Now the following questions are very important: How does using technology influence on effectiveness of educational process (rather some selected aspects of this effectiveness): understanding mathematical knowledge by students and level of their skills for applying this knowledge in problems solving? What is the meaning of "students' achievements" in computer-supported teaching and learning? How should these achievements be assessed?

I try to answer (at least partially) these questions in the research project "Assessment of students' achievements in technology-supported math education in secondary school" that is executed now in some classes of secondary schools in Bialystok. This paper presents the project (aims, problems, methods, investigation tools and procedure). Also the early results of investigation (after first stage of research) are described.
Innovamatica is a project of the Department of Mathematics and Informatics of the University of Perugia which has promoted, during the last twelve years, a strict interaction between mathematics and real world as a central theme in mathematical education. One of the main Innovamatica's activities is Orientamatica, a project addressed to secondary and undergraduate education which has involved more than 6.000 students of 50 different schools with the active participation of about 60 teachers. The present communication will illustrate Mathematics&Reality a new proposal of Innovamatica, in collaboration with Centro PRISTEM of Bocconi University, which develops and expands Orientamatica as a national project.
Das Rechnen mit Größen in Anwendungsaufgaben

CAS und andere Programme sollten in der Sek 1 nicht nur in rein mathematischen Problemen eingesetzt, sondern auch in Anwendungsaufgaben mit Größen genutzt werden. Es wird dargestellt, welche Probleme beim Einsatz von CAS in solchen Aufgaben auftreten können und in wie weit die Nutzer die Rechenergebnisse nachbearbeiten müssen, um eine praxisgerechte Ausgabe zu erhalten.

Das Rechnen mit Größen (pdf)
Throwing a die is a popular example used to illustrate the concept of a random variable in teaching statistics since everybody is familiar with games involving dice. After defining a random variable describing one throw of a single die students can apply the respective formulae for discrete random variables to calculate manually its expected value (3.5) and variance (35/12, approx. 2.92). These findings are compatible with students' experience: repeated throws of a die lead to scores of about 3 to 4 on average. The fact that there is a non-zero variance is also no surprise. An average score of 3, for example, after throwing a die four times could, of course, be the result of throwing a "3" four consecutive times (the case of no variation), but there are many more possibilities to get 12 as the sum of four throws (cases of non-zero variance).

We will define DERIVE-functions for the calculation of the expected value and variance of discrete random variables. We will also investigate if or how these values change if we consider linear transformations of such a random variable. Moreover, several other examples from gambling will be investigated using the available functions.

We will then proceed to the simulation of series of actual throws of a die in DERIVE, using the built-in pseudo random number generator, and calculating the mean and variance for each sample. By comparing the sample mean and variance with the expected value and variance of the random variable, students will see that they differ in general, but also that increasing the size of the sample results in diminishing deviations from the theoretical values.
Amorous Bugs and Pursuit Problems

We begin with the classic Four Bug Problem. Suppose there are four bugs: B1, B2, B3, B4, who begin on the corners of a square. After the starting time t= 0. B1 chases B2, B2 chases B3. B3 chases B4, and B4 chases B1.

The problem is to accurately describe the paths of these bugs. This problem is easy to understand, yet it requires knowledge of differential equations to solve. We use DERIVE to aid our calculations and to plot the paths of the bugs. We consider many variations of this classic pursuit problem and establish some interesting results. We address the following questions, and more.

1. How far does each bug crawl?
2. Do the results of the four-bug problem generalize to the n-Bug Problem in the plane?
3. What about analogous pursuit problems in three-dimensions?

Amorous Bugs (pdf)

Bugs.Helper.Functions.dfw
Bugs.Helper_2D-Standard.dfw
Bugs.Helper_2D_skip-1.dfw
Bugs.Helper_2D_skip-2.dfw
Cubic Spline functions, Matrices and Derive

A spline function is a piecewise polynomial function defined on an interval.

If you construct a cubic spline function of an interval with N-1 knots you will have 4 N linear equations. Linear systems can be represented in matrix form as the matrix equation. The matrix is non singular, and with "Derive" we get the unique solution with the matrix inverse.

My program constructs the matrix by a recursive method for a number N, and it constructs also the column vector of solutions.

A column vector contains finally the N cubic polynomial functions constructed by the column vector of the unique solution. If you will use the program you will have to give data of the N-1 knots and of the interval, then you will get the spline function as a column vector of N cubic polynomial functions.

Cubic Spline functions (pdf)

spline.mth
spline_example.dfw

Wie schwer ist die „Gloriosa“, die größte Glocke des Frankfurter Doms?

Workshop

Die Glockenrippe, das ist der senkrechte Halbschnitt durch den Glockenkörper, wird durch Geraden oder Parabeln angenähert.

Durch Rotation der Näherungsfunktionen wird das Volumen und somit die Masse der Glocke berechnet.

Gloriosa (pdf)
Mathematik lehren - Ein Übergang vom Frontalunterricht zur individuellen Beratung durch den Einsatz von Online-learning


In der Wahrscheinlichkeitsrechnung gibt es große Unterschiede zwischen den Schülern im Verstehen und beim Lösen der Aufgaben. Stochastische Phänomene erfordern eine ganz andere Art des Denkens, um sie mathematisch erfassen und beschreiben zu können. Die vom Lehrer speziell für die Schüler entwickelte Lernumgebung soll die Schüler unterstützen, denen die Wahrscheinlichkeitsrechnung schwerer fällt und sie motivieren, sich mit der Thematik auseinander zu setzen. Schüler, die stochastisch denken können, sollten die Möglichkeit haben, sich mit mathematischen Anwendungen und Problemstellungen zu beschäftigen.


Empirische Untersuchungen im vergangen Schuljahr zeigten, dass sich das Unterrichtskonzept zur Begabtenförderung als tragfähig erweist.

Mathematik lehren (pdf)
Mathematik lehren (ppt-Präsentation)

Viele weitere Materialien finden sich in \DES-contribs\Schurig\Demo_study
PREDICTING THE VOLUME OF A VOLCANIC ERUPTION: The case of Mount Hekla

Hekla is a well known volcano in southern Iceland which has been active at least since the last Ice Age, 10,000 years ago. It has erupted 18 times during the last 900 years and in the Middle Ages it was considered to be an entrance to Hell. Historical records of the frequency and duration of the volcanic activity are available for the majority of these eruptions and volcanologists in the last century identified the lava fields and tephra layers for many of them and measured the volume of the erupted material. In some cases the environmental effects on the health of animals and humans is known. In this presentation three linear regression models are specified with the volume as a dependent variable (Y) and either the repose period (X1) before the eruption or the duration (X2) of it as an independent variable or both:

\[
Y = \alpha_0 + \alpha_1 X_1 + \varepsilon; \quad Y = \beta_0 + \beta_2 X_2 + \varepsilon; \quad Y = \gamma_0 + \gamma_1 X_1 + \gamma_2 X_2 + \varepsilon
\]

The project is multidisciplinary: combining statistical analysis, geology, geophysics, geochemistry, history and medicine. The univariate models should be suitable for students in upper high school classes. In addition the bivariate model gives an opportunity to discuss at the college level sophisticated statistical problems such as multicollinearity, heteroscedasticity and misspecification. The results of the estimated parameters, easily performed in a spreadsheet or with handheld calculators, turn out to be highly significant and fairly precise and evaluation on the basis of past events behind the historical horizon is supported by geological evidence. Therefore the models could be useful in predicting the volume of a future eruption when the repose period gradually lengthens. Also, similar models might be applicable to other volcanos in the world.

Predicting the Volume (PowerPoint presentation as pdf)
Interactive Investigations with Slider Bars in DERIVE 6

We will present some examples of interactive investigations, where we use slider bars in DERIVE 6. These examples are adapted for secondary school students. We will present the following examples and others.

1. Investigate common tangents to two quadratic curves. Plot two quadratic curves and the common tangents of the curves.

2. Find all circles tangent to three given circles. Use slider bars to investigate under what conditions such circles exist.

3. Find all circles tangent to three given lines. Use slider bars to investigate under what conditions such circles exist.

4. To inscribe in a given acute-angle triangle the triangle of a minimum perimeter.

Interactive Investigations (pdf)
apollonius.dfw
common tangents.dfw
insk vidsk cirk.dfw
straight line.dfw
The Distance Course on Discrete Mathematics for High-school Teachers

We are of the opinion, that the discrete mathematics should to start be taught since younger school age (at the secondary school level). Therefore it is expedient to strengthen the preparation of the teachers in pedagogical higher school. Traditionally the course of discrete mathematics has been oriented on the students of the fifth semester of training for specialities of mathematics & computer science. Now, in the connection with the changes in the program of higher school, this discipline begins in the second semester. Thus, the discipline became "younger". Discrete mathematics includes elements of mathematical logic, sets, functions and relations, matrices, combinatorics, the graph theory, the grammar theory etc. The course is mainly practical. For preparation perfection of teachers the course «Discrete mathematics in the examples and problems» was developed, with embedded problems system, where the problems with different level were posed. The problems decision examples with the use of programming languages, and Maple mathematical package were included. Some problems were illustrated with the help of Derive mathematical package. The distance course was developed in «Web-class KhPI» environment.

The Distance Course (pdf)
Distance Course (ppt-presentation)
Distance Course (html)
(Un)expected infinities in the CAS answers at school.

In school mathematics (textbooks) the concept of infinity is touched upon (and usually not very thoroughly) in the topics of sequences, series, limits, etc. On a computer algebra system it is possible to encounter infinity also in the case of such topics where the textbook does not mention infinity at all. For example, if one (accidentally or willingly) calculates $1/0$. Derive and Mathematica show infinity in the answer while Maple, MuPAD and TI-92+ give an error or "undefined" message. Derive gives infinities in the sets of solutions of some fractional equations. One may encounter infinity in the case of a logarithm or a tangent, for example ...

This presentation discusses the answers for problems where the student might not expect infinity, although it is provided by some of the mentioned CAS-s. The authors of different CAS-s have intentionally decided to use different styles. In the presentation, motives and consequences of the decisions are analysed from a mathematics teacher's point of view. The styles are comparatively analysed in view of accordance with pure mathematics and school mathematics. The issues of the internal consistency in following a style within a system and the degree of liberty given to the user to change the style of infinity usage are studied.

Different styles create different possibilities and requirements for connecting the school material (textbook) and the usage of CAS-s. It is important to consider several aspects of pure mathematics, school mathematics and CAS-s. At first, it may seem that "error-based" styles are more suitable for the school (in essence the schools do indeed focus on error identification). However, the "infinity based" approach can also offer very interesting didactic possibilities.

<table>
<thead>
<tr>
<th>Expression</th>
<th>School</th>
<th>Derive 6</th>
<th>Maple 8</th>
<th>Mathematica 4.2</th>
<th>MuPAD 3.1</th>
<th>TI-92+</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lim_{x \to 0} \frac{1}{x}$</td>
<td>$\infty$</td>
<td>not defined</td>
<td>$\pm \infty$</td>
<td>undefined</td>
<td>ComplexInfinity</td>
<td>undefined</td>
</tr>
<tr>
<td>$1/0$</td>
<td>not defined</td>
<td>$\pm \infty$</td>
<td>Error</td>
<td>ComplexInfinity</td>
<td>Error</td>
<td>undef'</td>
</tr>
</tbody>
</table>

(Un)expected Infinities (pdf)
Intelligent Problem Solving Environment T-algebra Workshop

T-algebra is an intelligent learning environment for step-by-step solving of algebra problems in four areas of school mathematics: calculation of the values of numerical expressions; operations with fractions; solving of linear equations, inequalities and linear equation systems; simplification of polynomials. It can be used to solve more than 50 problem types.

Solving of problems in T-algebra takes place step-by-step like in pencil-and-paper solutions. To achieve this, T-algebra uses our own action-object-input dialogue scheme where each step consists of three stages: selecting a transformation rule, marking the parts of the expression, entering the result of the application of the selected rule. The program is able to diagnose errors made at each stage of the step: when selecting the rule or the objects of the rule or when applying the rule, i.e., inputting the resulting expression. The program can also display appropriate error messages and error positions to the student immediately after a mistake was made. The student can ask the program for advice at any moment. To help the teacher in assessing the students' knowledge, T-algebra counts the errors of different types (separately for individual tasks and in total).

The participants of the workshop have the opportunity to try out solving tasks in all four areas, examine the action-object-input dialogue for different transformation rules and test the error diagnosis at all stages of a solution step.

This workshop is the first opportunity for us to demonstrate T-algebra to the international community. The authors are expecting critical feedback during the conference and after that.

Intelligent Problem Solving Environment (pdf)
Calculus lab Transformation: from DERIVE to DERIVE and Biology

The speaker has used DERIVE software at Benedictine University since 1990. As the software evolved from DOS to Windows to a high-level CAS, so have the educational goals and methodology to achieve those goals in the beginning calculus curriculum.

What was once an integrated lecture-with-lab experience has now been modified into an independent lab experience. Students from both traditional and biology-applications calculus courses collaborate in the laboratory setting to learn how to explore and articulate mathematics using a CAS assistant. Some of the traditional calculus content is "discovered" in the laboratory setting, and many applications feature biological applications. The speaker will outline the "before and after" situations of the lab experience, including how DERIVE and other tools are employed. and highlight some of the surprising benefits of this new course.

Calculus Lab Transformation (pdf)
Lisa Townsley - Benedictine University /Mathematics, LISLE, USA
Manmohan Kaur - Benedictine University/Mathematics, LISLE, USA

DERIVE: A Mathematical Assistant
Workshop

At varying levels of college study, the students are capable of modeling, calculations and visualization, but when they have to perform all this together, they need a mathematical assistant. In this workshop, will illustrate problems that require all these skills and explain how DERIVE can assist the student to successfully solve them. We will provide opportunity for the participants to explore problems of this nature from linear algebra, multivariable calculus, differential equations, elementary calculus and discrete mathematics.

Biography:
Lisa Townsley has used DERIVE at Benedictine University since 1990. She has attended many TIME conferences. Manmohan Kaur has experience with multiple CAS platforms, and has pioneered online placement testing and quizzing for mathematics at Benedictine University. Both are always delighted to share and learn about innovative educational ideas involving CAS and/or the internet.

Format (Appended April 3rd, 2006):
The workshop will be roughly divided into two parts:

1. Presentations by organizers (~30 minutes)
   We will begin the workshop by presenting a range of problems, including fitting a line, understanding functions and limits, visualizing surfaces, optimization and systems of equations. Solving these problems is aided by a mathematical assistant, as a tool, but not as a pinnacle.

2. Lab work by participants (~60 minutes)
   We will hand out a collection of projects from which the participants can pick some to work on during the remaining part of the workshop. The topics covered include:
   a. Center of mass (DERIVE solution)
   b. Continuity (DERIVE solution)
   c. The gradient and directional derivative (DERIVE solution)
   d. Investigating the Closure of Relations (DERIVE solution)
   e. Linear systems (Traffic flow) (DERIVE solution)
   f. Weather problems (Vectors) (DERIVE solution)
   g. Romeo and Juliet (ODE) (DERIVE solution)

A Mathematical Assistant (pdf)
Exploring Mathematics and Physics Concepts using TI graphing calculators & Applications in conjunction with Vernier Sensors

Calculator Software Applications (APPS) are powerful technological tools. They allow students to explore, discover and extend mathematical ideas. The graphical calculator becomes a kind of IT -platform that can be used throughout the curriculum. The author is investigating the mathematical and pedagogical potential of using this technology in combination with Vernier sensors as devices to collect various kinds of data and of using the graphical calculator to serve as a powerful analysis tool, helping students build mathematical models. Her interest is the impact of new technologies in the curriculum and the study of the consequences of this impact on the teaching and learning processes. Students' cognitive processes are analyzed when they are confronted with an open problem (i.e. when not provided with a predefined algorithm to resolve it). The interest is on the evolution of students' cognitive abilities from an empirical approach to various mathematical activities, which involve experiences such as observing, noticing details, modifying and identifying invariants, to more abstract ones, which lead to applied mathematical knowledge enabling the student to "make sense" of the information and doing some small researches.

Exploring Mathematics and Physics – Paper (pdf)
Exploring Mathematics and Physics (ppt-presentation as pdf)
Didactical principles of integrated learning math with CAS

Integrating CAS in math education can be successful or can easily fail. One of the success factors is an effective didactic in education material and in lessons. Authors, teachers and students must be aware of the different roles CAS plays in the education and learning process. In this lecture we will focus on these different roles and the didactical implications supported with examples. Teachers who integrate CAS in their lessons and especially (potential) authors are invited for a discussion.
The Use of Technology in Flemish Mathematics Education:
Secondary versus Higher Education

Within the scope of the preparation of my dissertation we took the first steps towards the executing of two related investigations about the integration of technology in Flemish mathematics education.

The first investigation (October 2005) describes the situation in the Flemish secondary schools and the experiences and the current opinion of the mathematics teachers with respect to the use of technology in mathematics lessons. The second investigation (December 2005) makes an inventory of the present use of technology in Flemish higher education, in particular in mathematics and courses connected with mathematics (statistics, mechanics, etc).

In this lecture we will report about both investigations. In addition we will discuss the similarities, misconceptions, prejudices and some inconsistencies that we could determine. We will also compare the obtained results of the Flemish secondary schools with an analogous study of 2001.

![Figure 4: The use of ICT during the exams](image)

The Use of Technology – Paper (pdf)
The Use of Technology (ppt-presentation as pdf)
Einsatzmöglichkeiten für CAS im Mathematikunterricht der Sekundarstufe I des Gymnasiums

Im Vortrag wird der Beitrag von CAS zur Realisierung der Ziele eines zeitgemäßen Mathematikunterrichts thematisiert. An Unterrichtsbeispielen, die von einer sächsischen Arbeitsgruppe erstellt und erprobt wurden, wird verdeutlicht, wie die unterrichtliche Realisierung erfolgen kann.

Einsatzmöglichkeiten für CAS (pdf)
Solving Problems of Spherical Trigonometry with the Help of Computer Algebra Introduction and Visualization

Since early centuries people tried to find methods for accurate navigation on the surface of the Earth. The problem was to find a reliable and practical coordinate system. Eventually the search led to the known system of longitude (meridians) and latitude. Because the Earth is approximately spherical, there was a quest to solve the problems of shortest distances (great-circle distance), loxodrome (path of constant bearing) and localization on the surface of a sphere the study of angles and distances of figures on a sphere is known as spherical trigonometry.

We try to find a way of teaching spherical trigonometry with support of a computer algebra system. Using DERIVE can help to solve various problems in trigonometry, multivariate calculus and differential equations and can illustrate the navigation examples with 3D-plots. We want to show the possibilities of coordinate system conversion and parametric plots.

Solving Problems of Spherical Trigonometry (pdf)
Solving Problems of Spherical Trigonometry (ppt)
Solving Problems of Spherical Trigonometry (ppt with Viewer)
To the DERIVE files:
Elliptic Curve Cryptography with DERIVE

Elliptic curves are in a way the natural answer to the following question: What comes next after lines and conic sections, if one considers algebraic curves with increasing degree and/or genus? What is so special about them is the fact that it is possible to define an addition of points on elliptic curves, which turns them into abelian groups. In my talk it is shown, how these groups can be used for cryptographic purposes centred around the so-called Discrete Logarithm Problem.

(Note by the author: See also the workshop on the same topic.)

Elliptic Curve Cryptography (pdf)

Some Derive Tools for Computations on Elliptic Curves and Their Applications Workshop

This workshop refers to my lecture on Elliptic Curve Cryptography and consists of two parts. In the first part, using Derive 6.1 a number of tools are provided, which are needed for computations over elliptic curves, like the addition of two points, integer multiples of a point etc. They will be used to investigate the abelian group given by an elliptic curve, in particular, as regards its structure and number of points. In the second part of the workshop, we will show how this routines can be used for an implementation of the ElGamal cryptosystem, one of the most widely used public key cryptosystems, which is based on the so-called Elliptic Curve Discrete Logarithm Problem or ECDLP for short.

Elliptic Curve Cryptography Workshop (pdf)

Derive files:
Basics
Implementation of Secure Hash Algorithm
Generating Elliptic Curves (CM-Curves)
Discriminant of an Elliptic Curve
Generation of “good” Random Numbers
ElGamal Cryptosystem


MATHEMATICS BY IMAGES

Do we have to think mathematically mainly by means of symbols and formulae? Definitely not, especially because of technology, which changes the style of teaching mathematics. By using technology we may show visual side of mathematics, its lighter and brighter side. Some examples of teaching by images will be shown, some practical hints will be given.

Mathematics by Images (pdf)

Mathematics by Images (ppt - presentation)

Mathematics by Images (ppt with Viewer)