Standards-Based Learning
in an Interactive
Computer Environment

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Contents

• Standards for School Mathematics:
  The European Perspective
• Mathematical Competencies
• LeActiveMath: A Learning Tool for
  Mathematics
• Examples of Standards-Based Learning
  in a Computer Environment
• First Results from an Exploratory Study
• Synopsis
Standards for School Mathematics
The European Perspective: Germany

The federal ministries of education agreed in implementing educational standards for school mathematics with respect to

- primary education at the end of grade 4
- lower secondary education at the end of grade 9
- secondary education at the end of grade 10

http://www.kmk.org
Standards for School Mathematics
The European Perspective: Germany

Educational Standards
- articulate requirements for school-based teaching and learning
- identify goals for pedagogical work, expressed as desired learning outcomes for students
- translate the educational mission of schools into concrete terms
- draw on general educational goals
- specify the competencies that schools must impart to their students

Klieme et al. (2003)

Standards for School Mathematics
The European Perspective: Germany

Educational Standards
- are associated with a particular view of the significance of a subject or content area for personal development and of its social functions
- do not rely on lists of content and material to make educational goals concrete but identify the basic dimensions of learning activities
- should be based on an understanding of the educational mission of the given subject area

Klieme et al. (2003)

Complete reference:
To be obtained under: www.bmbf.de/pub/the_development_of_national_educationel_standards.pdf
These eight competencies were used to characterize, resp., classify all examples and exercises in LeActiveMath. In addition, we introduced a finer graduation into subcompetencies, also in accordance with Niss (2002), like - „scope“ (understand the scope of a mathematical concept, i.e., know the area where it can be applied and where not: counter-examples!)
- „formulate“ (mathematical statements), and - „generalize“ (mathematical statements)
for „Think mathematically“, or „encode“ and „decode“ for „Model mathematically“.

**Complete reference:**
M. Niss. Mathematical competencies and the learning of mathematics: the Danish KOM project.
Standards for School Mathematics
The European Perspective: Germany

- **Problem Solving**: mathematisch lösen
- **Argumentation**: argumentieren
- **Modeling**: mathematisch modellieren
- **Getting involved into Mathematics**: mathematischen Inhalten
- **Communication**
- **Use of Representation**
- **Use of symbolic, formal, and technical elements of mathematics**

Standards for School Mathematics
The European Perspective: Germany

Eine Firma bietet Vertrags-Handys und Kartenhandys zu folgenden Konditionen an:

### Vertrags-Handys
**Teddy Active**
AIKON 3410
Handy 0,00 €
Grundgebühr monatlich: 9,95 €
Gesprächskosten pro Minute 0,175 €
SMS 0,19 €
Bereitstellungsgelder: 24,95 €
(Einmalige Zahlung)
Weitere Kosten: keine

### Kartenhandys
**Teddy ExtraPlus**
AIKON 3410
Handy 129,95 € einschließlich
15 € Gesprächsguthaben
Grundgebühr monatlich: 0,00 €
Gesprächskosten pro Minute 0,412 €
SMS 0,19 €
Weitere Kosten: keine

*Bei der Errechnung der Gesprächskosten pro Minute wurde ein durchschnittlicher Wert angenommen.*

Geben Sie eine Kaufberatung, die hilfreich ist für eine Entscheidung zwischen Vertrags-Handy und Kartenhandy.

Geben Sie bei Ihren Überlegungen von einer Nutzungsdauer von 24 Monaten aus (Vertragslaufzeit) und berücksichtigen Sie die "telefonierten" Minuten pro Monat.
In LeActiveMath, we used the following subcompetencies of the competency „Deal with symbolic and formal elements of mathematics“:

- „translation“ (of natural language into formal/symbolic language, and vice versa)
- „interprete_manipulate“ (formal/symbolic terms)
- Knowing the „formal_rules“ (of symbolic language)

Instead, the mathematical competency „applying algorithms“ was integrated as a subcompetency of „Solve problems mathematically“.

Working with diagrams and tables was interpreted as a subcompetency of „Use mathematical representations“, and with respect to the mathematical tools, there exists the separate competency „Use tools and aids“.
Standards for School Mathematics
The European Perspective: Germany

(10) Fakultät

Aufgabenstellung
Das Produkt 6! berechnen.
Beispiel: 20! berechnen mit Hilfe eines elektronischen Rechners.

Allgemein:
a) Berechnen Sie 6!.
   Wie viele Ziffern hat dieser Wert?
   Wie oft ist "0" am Ende?
   Wie oft ist "0" am Ende von 20!?

b) Berechnen Sie 20!.
   Wie viele Ziffern hat dieser Wert?
   Wie oft ist "0" am Ende?
   Wie oft ist "0" am Ende von 20!?

Give a reason for the difference.

Standards for School Mathematics
The European Perspective: England

National curriculum with respect to four key stages

- Key stage 1: up to age seven (Years 1 and 2)
- Key stage 2: age seven to eleven (Years 3, 4, 5 and 6)
- Key stage 3: age eleven to fourteen (Years 7, 8 and 9)
- Key stage 4: age fourteen to sixteen (Years 10 and 11 - preparation for academic and equivalent vocational qualifications)

http://www.nc.uk.net
Standards for School Mathematics
The European Perspective: England

- Setting suitable learning challenges
  Teachers should aim to give every pupil the opportunity to experience success in learning and to achieve as high a standard as possible.

- Responding to pupils' diverse learning needs
  Teachers need to be aware that pupils bring to school different experiences, interests and strengths which will influence the way in which they learn.

- Overcoming potential barriers to learning and assessment for individuals and groups of pupils
  A minority of pupils will have particular learning and assessment requirements. Teachers must take account of these requirements and make provision, where necessary, to support individuals or groups of pupils to enable them to participate effectively in the curriculum and assessment activities.

Standards for School Mathematics
The European Perspective: England

Aim 1: The school curriculum should aim to provide opportunities for all pupils to learn and to achieve.

Aim 2: The school curriculum should aim to promote pupils' spiritual, moral, social and cultural development and prepare all pupils for the opportunities, responsibilities and experiences of life.
(Broad Concept of) Communicating
• discuss their work and explain their reasoning using an increasing range of mathematical language and notation
• use a variety of strategies and diagrams for establishing algebraic or graphical representations of a problem and its solution; move from one form of representation to another to get different perspectives on the problem
• present and interpret solutions in the context of the original problem
• use notation and symbols correctly and consistently within a given problem
• examine critically, improve, then justify their choice of mathematical presentation; present a concise, reasoned argument

All these competencies can also be classified by the encoding used in LeActiveMath. E.g., the competencies in the second point are encoded as „Use mathematical representations” and the third point is part of the „decode“ subcompetency of „Model mathematically“. The last point corresponds to the „choose/swich“ subcompetency of „Use mathematical representations“ . In addition, all the mentioned activities require the competency „Communicate”.

(Broad Concept of) Reasoning
• explore, identify, and use pattern and symmetry in algebraic contexts, investigating whether a particular case may be generalized further and understand the importance of a counter-example; identify exceptional cases when solving problems
• understand the difference between a practical demonstration and a proof
• show step-by-step deduction in solving a problem; derive proofs using short chains of deductive reasoning
• recognize the significance of stating constraints and assumptions when deducing results; recognize the limitations of any assumptions that are made and the effect that varying the assumptions may have on the solution to a problem

Here we have the following competencies (in the same order):
„Think mathematically” with its subcompetencies “scope” and “generalize” (see above)
„Argue mathematically“, subcompetency „judge“ (a given proof)
„Argue mathematically“, subcompetency „find_choose“ (a proof)
„Think mathematically“, subcompetency „scope“
Standards for School Mathematics
The European Perspective: England

General teaching requirements:
Use of information and communication technology across the curriculum

Pupils should be given opportunities to apply and develop their ICT capability through the use of ICT tools to support their learning in all subjects. They should be given opportunities to support their work by being taught to

• find things out from a variety of sources, selecting and synthesizing the information to meet their needs and developing an ability to question its accuracy, bias, and plausibility
• develop their ideas using ICT tools to amend and refine their work and enhance its quality and accuracy
• exchange and share information, both directly and through electronic media
• review, modify and evaluate their work, reflecting critically on its quality, as it progresses.
The European Perspective: England
ICT Learning in Mathematics

Using ICT can help pupils to

- access, select and interpret information
- recognize patterns, relationships and behaviors
- model, predict, and hypothesize
- test reliability and accuracy
- review and modify their work to improve the quality
- communicate with others and present information
- evaluate their work
- improve efficiency
- be creative and take risks
- gain confidence and independence

The European Perspective: England
ICT Learning in Mathematics

The availability of ICT can enable pupils to

- experiment and learn from feedback
- think logically and develop problem-solving skills
- observe, explore, and explain patterns in number, shape, and data
- make and test hypotheses and predictions, which can be based on large amounts of data
- make generalizations that can be based on experimental evidence
- develop mathematical vocabulary and language.
The European Perspective: ICT Supported Learning

In a nutshell: The availability of ICT may foster students’

MATHEMATICAL COMPETENCIES.

Mathematical Competencies

What does mathematical competency mean?

Competency is a construct that encompasses cognitive abilities and skills possessed by or able to be learned by individuals that enable them to solve particular problems, as well as the motivational, volitional and social readiness and capacity to utilize the solutions successfully and responsibly in variable situations.

Weinert (2001)

Complete Reference:

Prerequisites for Competency Development

OPTIMAL MATHEMATICS INSTRUCTION

A Model for the Learning of Mathematics: Process – Mediation – Product

Context: Teacher
- Mathematical knowledge
- Mathematics education knowledge
- Pedagogic and diagnostic knowledge
- Monitoring beliefs
- Mathematical emotions
- Motivation and interest

Process: Mathematics instruction
- Implemented curriculum
- Classroom management
- Time spent on teaching and learning
- Cognitive activation
- Emotional and motivational support

Product: Mathematical literacy
- Cognitive competencies
- Affective personal disposition
- Behavioral personal disposition

Mediation: Students
- Use of time for teaching and learning
- Emotional / motivational processes
- Cognitive construction

Pekrun & Reiss (2004)
LeActiveMath: A Learning Tool for Mathematics

Language-Enhanced, User-Adaptive, Interactive eLearning for Mathematics

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Some background information to each of these five statements:

1. Students can adapt the system to their individual needs, and the system is also adaptive to the user’s behaviour.

2. This is a consequence of the fact that it considers all the mathematical competencies, not just only one, like learning to apply the usual calculus algorithms to find the derivative of some function.

3. LeActiveMath combines many adaptable, resp., adaptive components and tools, like a search engine, a course generator, an integrated CAS, a function plotter, etc. It also contains an Open Learner Model, which the user can (resp., should) inspect from time to time in order to check how the system rates his achievements.

4. In particular, the user can choose from many predefined calculus books or he/she can use the course generator to create an individual book.

5. This competency model is analogous to similar models from the PISA findings. In particular, it consists of 4 competency levels, which are shown on the next slide.

The weblink at the bottom leads to the homepage of the LeActiveMath project. From here one can in particular access a demo version of the software.
Every example and every exercise is encoded with a metadatum „competency_level“ that displays one of the four values „elementary“, „simple_conceptual“, „multi-step“, or „complex“.
This screenshot shows a section of the page „Differential quotient“ in the predefined book „Up and Down“.

Some explanations of the user interface:

On the left, one can see the table of content for this book. In the upper right corner, the user finds the navigation bar of the system, and below that the navigation bar of the book, which contains the title of the actual page and the page number (11 out of 15). Below, in the main window one can see the titles of 3 content elements. Coloured triangles with a letter inside indicate the type of each element: the first one is a note („N“), the second a definition („D“) and the last one an example („E“). Just as for every example and exercise, the title bar of the third element also displays 3 yellow stars that indicate that this example is considered to be of medium difficulty (on a five digit range).

Using the pencil buttons, the user can adapt each element to his/her individual needs. In particular, he can leave a note or comment here. If the element is part of an individually created book, he/she also has the opportunities to remove this element or to let the system search for similar elements and add them.

The coloured squares in the table of content give a feedback on the user’s achievement for each page; the colour scheme ranges from red to green.
Starting Point: A Problem

A hiking tour

Mary and Michael were on a hiking tour. Their hiking booklet contains a profile of their tour.

From their starting point (the famous arena) they have first reached some mountain hut, then walked a steep fixed rope route to reach the mountain top. From there they descended to a restaurant and then to their final destination, the bus stop in the next village. Now they care for the slope they have overcome.

This is the first step into a mult-step problem-based introduction of the mathematically concepts „average slope“ (as an example of a „difference quotient“), „actual slope“ (as an example of a „derivative“), and finally „derivative function“.

Competencies: Mathematical Modeling (Example)

Modeling the problem ★

In order to grasp the hiking tour mathematically, Mary and Michael need to develop some adequate mathematical model. In this case, this is not difficult: they copy the profile into some coordinate system. They give symbolic names to the characteristic points of the tour: The arena becomes A, the mountain hut becomes H, the (middle of the) fixed rope route they call R, the summit is S, the restaurant Z, and the bus stop becomes B.

Also, the concept of the slope (of an arbitrary curve) needs to be made mathematically precise. This is done by referring to the slope of a straight line.

Here, for the solution of their problem, Mary and Michael need to establish a new mathematical concept (the slope of an arbitrary curve).

This is the second step, showing the necessity to encode the real-world problem into mathematics language. The „E“ in the triangle indicates that this element is encoded as an example, and the one star indicates that is is considered to be „very easy“ (for an high school student).
This is one of many exercises (indicated by the „X“) of varying difficulty. The four stars indicate that this one is considered to be difficult (for a high school student). Others are considered to be easier, especially when they contain a picture for illustration, etc.

In LeActiveMath (i.e., not on this slide here), the underlined phrase „slope of this straight line“ carries a link to another content element, in this case, to the definition of the slope of a straight line. Clicking on that phrase displays this definition in another window. The whole calculus content of LeActiveMath is networked by more than 30.000 of these textual links, offering the student easy access to concepts he/she needs to recapitulate.
The average slope of the curve between the points (-1,0) and (1,0) is 0. Do you think that this is adequate? If not, what would you suggest to get around this problem? Try to find arguments for your opinion and discuss them with your colleagues.

This is the text of an exercise in LeActiveMath. It trains the competencies „Think mathematically“, „Model mathematically“, and „Communicate“ (with respect to the new concept „average slope of a curve“). It is encoded as a simple-conceptual exercise of medium difficulty.
Average Slopes

This is an example of a multiple-choice-single-answer exercise. The one star indicates that it is rated to be very easy. The reason for this rating is the fact that the relevant formula for the average slope is already shown in the picture. Other versions of this exercise where the formula is not displayed are rated to be more difficult.

Competency: Producing Simple Solutions

This is the invitation to a (simple conceptual, easy) exercise that trains the competency „Solve mathematically“.
Competency: Representing

This is a (simple conceptual, easy) „fill-in-the-blank“ exercise that trains the competency „Use mathematical representations“, more precisely the subcompetency „decode/understand representations“.

Competency: Conceptual Understanding

These are pictures from examples on the mathematical concept of a (general) function. In particular, they illustrate domain, range, and image set of a function.
These pedagogical strategies are implemented in the course generator, which creates individual books, tailored to the user's needs. In particular, the user can adapt the book by choosing a specific scenario, i.e., learning situation. Underlying to each of these scenarios is a different pedagogical strategy. Of course, the user can also choose the topic(s) of the book. Once he/she has made these choices, the course generator adaptively creates the book, depending on the data stored in the Learner Model.

E.g., suppose that the user wants a book on derivatives. Since the derivative is the limit of a function of difference quotients, the concept of a derivative depends on the mathematical concepts „difference quotient“ and „limit of a function“ (which, in turn, depends on the concept of a function). Hence the course generator will check if, according to the Learner Model, the user is already familiar with the concepts of difference quotients and limits. If this is not the case, the course generator will add prerequisites pages on each of these concepts.

Note that not only the course generation itself, but also its result, the individual book, is again adaptable and adaptive. Namely, the user can further adapt it to his/her needs by adding or removing content elements (using the pencil button described on a previous slide). Moreover, most of the books are also adaptive, since they contain dynamic items (examples and exercises) that are created „on the fly“. These dynamic items are integrated into a page of the book just at the moment when the user opens that page for the first time, such that the course generator can take the actual state of the Learner Model into account and choose, e.g., the difficulty of the exercises appropriately.

The different strategies the user can choose from are presented on the next slides.
Scenarios and Strategies

- **LearnNew / Discover**
  - **Objective:** Learn a new concept
  - **Provision:** In-depth course; introduction – concept – examples/exercises – theorems
  - **Graduation:** number of examples/exercises, difficulty of exercises

- **Rehearse**
  - **Objective:** compensation of deficits
  - **Provision:** short repetition and self-test

Scenarios and Strategies

- **Connect**
  - **Objective:** connections between concepts
  - **Provision:** concept maps and theorems

- **TrainCompetency**
  - **Objective:** raise abilities with respect to a certain competency
  - **Provision:** mainly exercises (examples)
  - **Graduation:** individual suggestions on competency to train, difficulty levels
Scenarios and Strategies

- **Workbook**
  - Objective: Practise, various perspectives on a topic
  - Provision: mainly different types of exercises (examples)

- **ExamSimulation**
  - Objective: preparation for an exam
  - Provision: set of exercises with rising difficulty for pre-defined concepts and amount of time or number of exercises

At the moment, exam simulations are available for durations of 30 min, 45 min, 60 min, or 90 min.

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This slide shows, how a new concept is learned using the „Discover“ scenario. Some items are available on demand via a button.

The number of examples and exercises inserted into the pages, and their difficulty, depend on the state of the Learner Model, in particular, on the actual achievement level.

At the end of each unit, the learner is invited to contact the Open Learner Model in order to check (and, eventually, discuss) the values displayed there. The system even allows these values to be manually changed, e.g., in order to take learning progress into account that the user has acquired outside of LeActiveMath.
These are pictures taken during the midterm evaluation of LeActiveMath, in fall 2006. Note that the copyright for these pictures remains at the photographer, Christian Gross.
First Results from an Exploratory Study

➢ Participants
  • 50 students
  • Upper secondary level (grade 11)

➢ Method
  • Work with LeactiveMath
  • Online questionnaires concerning the program

These data refer to the midterm evaluation of LeActiveMath in fall 2005. Two classes used the program over a longer period (between 5 and 10 hours).

The online questionnaires were placed at three prominent positions, namely after each example, resp., exercise, accessible via some button „Your Opinion“, and at the end of each section, accessible from the log-out screen.

In addition to these online questionnaires, the students also fill pre- and post-questionnaires on their motivations and interests, whose results are not presented here.
The red dots mark the location of the arithmetic means of the user feedback. As can be seen, pictures and multiple-choice exercises were rated high (being “good”), whereas the hints were rated very low (“bad”). The latter should be a consequence of the fact that the authors had written scaffolded hints for each exercise, starting with a very general hint, followed by a series of more and more concrete hints, resp., partial solutions, up to the complete solution. Nevertheless, at the moment of the evaluation, the exercise system was only able to display the very first hint of this series and repeated this one hint again and again. Of course, the users considered this to be not very helpful. Meanwhile, this bug has been resolved.

One of the major aims of this exploratory study was the improvement of the components and contents of LeActiveMath. For this purpose, the user feedback turned out to be helpful, e.g., about texts that were rated as being too abstract, or about applets whose purpose was rated to be unclear.
Overview Feedback on Components

I like the features for playing around like the graphics provided for drawing parabolas.

I miss exercises. I would prefer a technically more stable system.

These feedback texts are taken from the additional pen&pencil questionnaires.
Learning environments should foster students’
• Mathematical thinking, argumentation, problem solving and modeling, use of representations, dealing with symbolic and formal elements of mathematics, communication skills, use of tools and aids.
Moreover students should
• experiment and learn from feedback, make and test hypotheses and predictions, make generalizations that can be based on experimental evidence.

Recent computer learning environments provide help in order to reach these aims. Key concepts are
• Support of the students’ autonomy
• Support of self-regulated learning
• Meaningful learning and useful applications
• Exploration and experiments
Thank You!